SPECIAL PROJECT PROGRESS REPORT

All the following mandatory information needs to be provided. The length should *reflect the complexity and duration* of the project.

Reporting year	**2021**………………………………………………………
Project Title:	**Direct numerical simulation of long-term evolution of wind waves: dynamics vs kinetics, with applications to freak waves prediction** ………………………………………………………
Computer Project Account:	**SPGBSHRI**………………………………………………
Principal Investigator(s):	**Prof V.I. Shrira**………………………………………… ………………………………………………………
Affiliation:	**School of Computing and Mathematics, Keele University, Keele ST5 5BG UK**………………………
Name of ECMWF scientist(s) collaborating to the project (if applicable)	……………………………………………………… ………………………………………………………
Start date of the project:	**2019**………………………………………………………
Expected end date:	**2021**………………………………………………………

Computer resources allocated/used for the current year and the previous one

(if applicable)
Please answer for all project resources

		Previous year		Current year	
		Allocated	Used	Allocated	Used
High Performance Computing Facility	(units)	**500000**	**497516**	**500000**	**139746**
Data storage capacity	(Gbytes)	**100**	**100**	**100**	**100**

Summary of project objectives (10 lines max)

The target of the project is to perform long term DNS simulations and to compare them with high quality observations. As shown by Annenkov & Shrira (2014), higher-order moments of a random wave field, and hence the probability of freak waves, are dependent on spectral shape, not just on integral characteristics of a wave field. Preliminary estimates show that the found discrepancies in spectral shape can strongly affect higher-order moments, in particular kurtosis, and the difference is expected to be substantial (of the order of 100%). This has huge potential implications for the prediction of extreme wave events. Specific objectives include exploring the discrepancy between the shape of the DNS (verified by observations) and the Hasselmann equation predictions, examining implications for probability of freak waves, mixing via the vortex force and other processes sensitive to the shape of spectra, getting new insights into the input and dissipation functions.

Summary of problems encountered (10 lines max)

No particular problems encountered…………………………………………………………………....

Summary of plans for the continuation of the project (10 lines max)

This is the final year of the project. In the remaining months, we plan to clarify a few important outstanding questions, for which additional numerical experiments are needed. In particular, the fundamental question of scaling of wind wave growth rates in direct numerical simulations and simulations with the generalised kinetic equation, and the role of intermittency in the averaged wind wave field. ……………………………………………………………………………………………..……

List of publications/reports from the project with complete references

Annenkov S., Shrira V., Romero L., Melville W.K., Le Merle E., Hauser D. 2021 Wave development and transformation under strong offshore winds: modelling by DNS and kinetic equations and comparison with airborne measurements. EGU General Assembly Conference Abstracts, EGU21-10437.

Annenkov S., Shrira V., Romero L., Melville W.K. 2021 Long-term evolution of directional spectra of wind waves modelled by DNS and kinetic equations, and comparison with airborne measurements. EGU General Assembly Conference Abstracts, EGU21-10435.

Annenkov, S.Y., Shrira, V.I. 2021 Effects of non-gaussianity on evolution of a random wind wave field. Phys. Rev. Letters, submitted. ………………………………………………………………………

Summary of results

If submitted **during the first project year**, please summarise the results achieved during the period from the project start to June of the current year. A few paragraphs might be sufficient. If submitted **during the second project year**, this summary should be more detailed and cover the period from the project start. The length, at most 8 pages, should reflect the complexity of the project. Alternatively, it could be replaced by a short summary plus an existing scientific report on the project attached to this document. If submitted **during the third project year**, please summarise the results achieved during the period from July of the previous year to June of the current year. A few paragraphs might be sufficient.

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In this project, we study numerically the long-term evolution of water wave spectra, using the standard model based on the Hasselmann kinetic equation (KE), widely used in operational wind wave forecasting, as well as two other models developed by us and based upon different sets of assumptions. The second model is the generalised kinetic equation (gKE), using the same statistical closure as the KE, but derived without the assumption of quasi-stationarity, therefore valid when a wave spectrum is changing rapidly (e.g. at the initial stage of evolution of a narrow spectrum, or after a rapid change of wind forcing). The third model is based on the Zakharov integrodifferential equation for water waves and does not depend on any statistical assumptions. Since the Zakharov equation plays the role of the primitive equation of the theory of wave turbulence, we refer to this model as direct numerical simulation of spectral evolution (DNS-ZE). The DNS-ZE method allows to study long-term spectral evolution (up to $O(10^4)$ periods), which was previously possible only with the KE. Thus, we are able to perform a direct comparison of spectral evolution with and without the statistical closure.

During the third year of the project, we compared the results obtained with all three models with airborne measurements collected in two different experiments: GOTEX experiment off the coast of Mexico in February 2004, and HyMeX experiment in the western Mediterranean sea in February and March 2013. In both cases, the wave field was generated by a strong quasi-stationary offshore wind jet, which is caused by pressure differences and accelerates passing through a valley into the sea. For modelling of waves off the Mexican coast, the initial conditions were taken from the measured spectrum at the moment when wind waves prevail over swell after a short initial part of the evolution. Waves in the Mediterranean Sea were modelled with constant wind forcing and zero initial condition.

The evolution of integral characteristics, e.g. significant wave height and wave steepness, is reproduced reasonably well by all modelling approaches. However, the spectral shape of developed waves demonstrates a large discrepancy between, on the one hand, the measured spectra and the DNS modelling and, on the other hand, spectra modelled by both kinetic equations (figure 1).

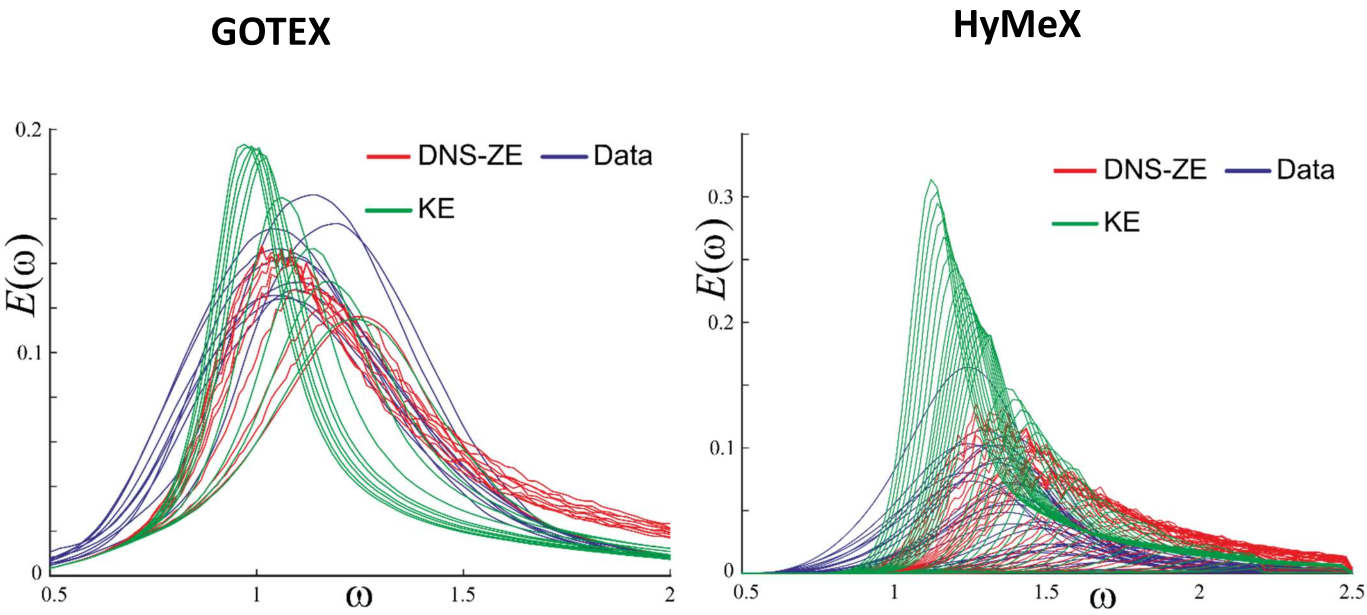

Figure 1. Spectral development in modelling of GOTEX and HyMeX data. Measured spectra (blue curves), DNS spectra (red curves), KE spectra (green curves)

At the intermediate and advanced stage of development, both measured spectra and the DNS spectra tend to Pierson-Moskowitz spectral shape, while the modelling based on the kinetic equations invariably predicts spectra with a higher, more pronounced peak. In terms of the parameter of spectral peakedness, a commonly convenient measure of spectral shape, a large (of order one) discrepancy was found (figure 2).

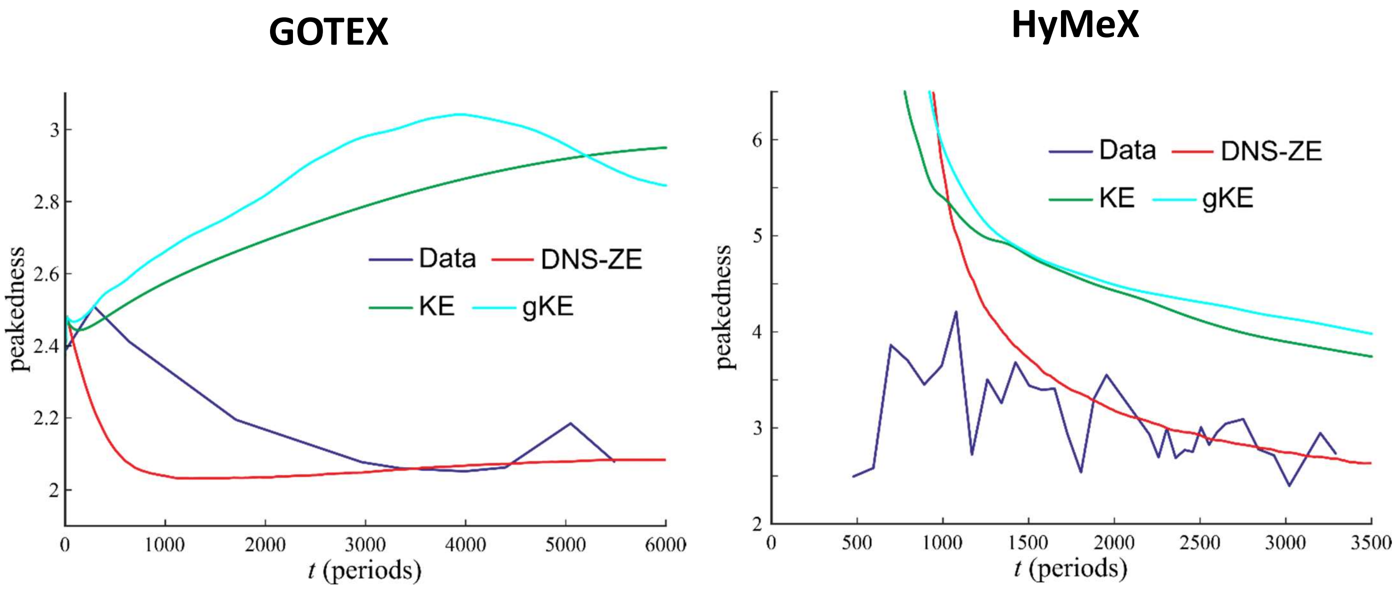

Figure 2. Spectral peakedness $Q_p = \frac{2\int \omega S(\omega)^2 d\omega}{[\int S(\omega) d\omega]^2}$ (Goda 1970) as function of time in GOTEX and HyMeX experiments, and in the simulations by DNS and the kinetic equations.

Thus, we have demonstrated that spectral shapes obtained with the DNS are in much closer agreement with measurements of mature sea states. This discrepancy between the spectral shapes of mature oceanic waves and solutions of the Hasselmann equation is long known as a major problem in wind wave modelling. For the first time, we have shown that it can be overcome with DNS, pointing to the nonlinear interaction term of the Hasselmann equation as the origin of the discrepancy. Although the discrepancy mostly concerns spectral shapes, rather than spectral integrals, it has huge significance for many applications, including freak wave prediction. In particular, it was shown that the demonstrated difference in spectral shape corresponds to an order one change of kurtosis, which dramatically affects the freak wave probability estimates. On the other hand, since the existing wind wave models are optimised against the available measurements, knowledge of systematic errors in models can drastically improve the quality of such optimisations, and thus improve the quality of predictions even for spectral integrals like significant wave height.

The origin of the discrepancy has been identified as the neglect of non-gaussianity in the derivation of the Hasselmann equation (Annenkov & Shrira 2021). While the Hasselmann equation takes into account finite nonlinearity, it assumes infinitesimal non-gaussianity, since in its derivation the six-point correlator is expressed in terms of two-point ones, neglecting the four-point cumulants in the expansion. Although this approximation, which takes into account only the leading term in the expansion of the correlator, is a prerequisite for obtaining the kinetic equation for spectral amplitudes in the closed form, it leads to the equation with the right-hand side being a homogeneous function of the spectrum, thus neglecting all effects of finite nonlinearity. We show by DNS that although non-gaussianity is weak, in the long term it leads to a considerable distortion of the spectral shape. Here, we are helped by the particular design of the DNS algorithm. The algorithm is based on the idea of coarse-graining of a wave field (Annenkov & Shrira 2018), which relaxes the resonance condition into $\boldsymbol{k}_0+\boldsymbol{k}_1-\boldsymbol{k}_2-\boldsymbol{k}_3=\Delta\boldsymbol{k}$, in contrast to the standard condition $\Delta\boldsymbol{k}=0$. Here $|\Delta\boldsymbol{k}|/k_{min}<\lambda_k\overline{\omega}/\omega_{min}$, and the crucial role is played by parameter λ_k (the coarse-graining parameter). If $\lambda_k=0$, there are no interactions (on a log-spaced grid), and a wave field is gaussian. When λ_k is increased from zero, the number of wave interactions grows quadratically, the rate of spectral evolution quickly increases and then saturates, on a certain value of λ_k depending on grid resolution. It is convenient to use λ_k as a way to create wave fields with the same level of nonlinearity, but different levels of non-gaussianity.

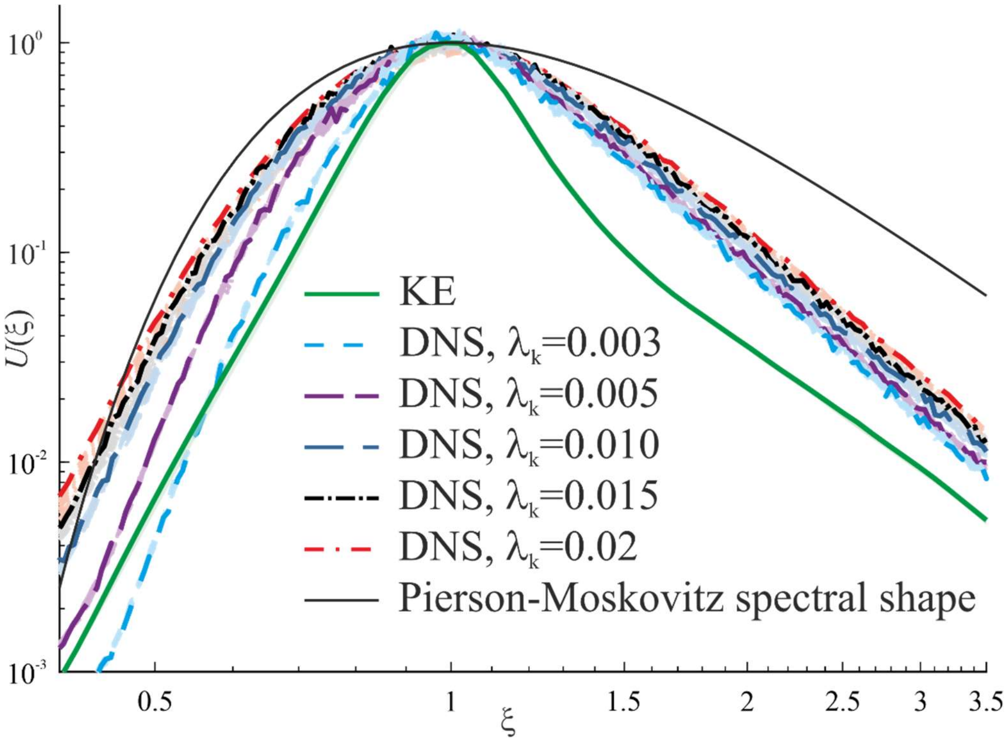

Figure 3. Self-similar shape function extracted from the numerical solutions at the last 1000 wave periods of evolution. Shapes at every 100 periods are shown in light colours, the final curve is in darker colour of the same hue. Numerical models used are the KE and the DNS on 161x71 (DNS161) and 321x71 (DNS321) grids for different values of λ_k.

We use a very refined 321x71 grid. When $\lambda_k = 0.003$, the number of interactions is relatively small (10^8), and the wave field is nearly gaussian, with very slow evolution. With the increase of λ_k, the number of interactions increases to $7 \cdot 10^9$. We are interested in spectral shape at the advanced stage of the evolution, when the self-similar shape is reached. For small λ_k the shape function is close to the KE spectral shape (figure 3). With the increase of λ_k the shape function approaches a different form resembling Pierson-Moskowitz spectral shape.

Thus, we have demonstrated that although non-gaussianity is weak, in the long term it leads to considerable distortion of the spectral shape. At the same time, integral parameters of a wave field appear to be less affected, with the error remaining within the uncertainty introduced by wave breaking, which the DNS modelling has to take into account. The spectral shape obtained by the DNS appears to be in much closer agreement with observations of mature sea states than the KE spectral shape.

The findings of this project have numerous important implications. First, they are of crucial importance for all applications where the shape of the wave spectrum is significant, rather than just its integrated description, in particular for probability estimates of extreme wave events, design or coastal hazard risk assessments, sediment transport models, etc. Second, it is well known that the wind wave models based on the KE are optimized for certain frequency and directional resolutions against the available measurements. Knowledge of systematic errors in models can drastically improve the quality of such optimizations, and thus improve the quality of wind wave modelling. Third, the findings of this study provide an insight on the role of non-gaussianity in kinetic models, which is significant for a wide context of wave turbulence in various branches of physics.

References

Annenkov, S.Y. & Shrira, V.I. 2018 Spectral evolution of weakly nonlinear random waves: kinetic description versus direct numerical simulations. J. Fluid Mech. 844, 766-795.

Annenkov, S.Y. & Shrira, V.I. 2021 Effects of non-gaussianity on evolution of a random wind wave field. Phys. Rev. Letters, submitted.