INTRODUCTORY REMARKS
AND A REVIEW
OF THE BAROTROPIC AND BAROCLINIC PLANETARY BOUNDARY LAYER

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A. WIIN-NIELSEN
EUROPEAN CENTRE FOR MEDIUM RANGE WEATHER FORECASTS

1. Introduction

My first pleasant duty is to welcome all of you to the second annual seminar organised by the European Centre for Medium Range Weather Forecasts (EMCWF). Our first seminar in 1975 was devoted to the "Basic Foundation of Medium-Range Weather Forecasts", and the Proceedings of the seminar were issued to all participants and to the Member States during the autumn of 1975.

It has been our plan to arrange seminars every year and to use the seminars for this year and probably the next couple of years to cover in greater detail some of the more specialised topics connected with forecasts for 4-10 days. The choice for this year has fallen on the Atmospheric Boundary Layer. Once again we plan to publish the Proceedings of the seminar, and you can probably expect to receive your copy before the end of the year.

The seminars at ECMWF depend upon a favourable response from the scientific community and from the meteorologists of the Member States in two important ways. It is first of all necessary to gather together the high level scientists who will provide the lectures at the seminar. We are very happy that it has been possible for us also this year to convince a number of first rate scientists to serve as lecturers. This is no small task because we require not only their physical presence, but also prepared lecture notes which can be published in the Proceedings.

We welcome the following people who will give one or more lectures during the coming week:

Dr. J.-C. André, France
Dr. E. Augstein, Germany
Dr. S. Bodin, Sweden
Dr. N. Busch, Denmark
Dr. S.J. Caughey, U.K.
Dr. J. Deardorff, U.S.A.
Dr. R. Hide, U.K.
Dr. N. Thompson, U.K.
Dr. F. Wippermann, Germany.

In addition, there will be some lectures presented by members of the ECMWF staff. The following staff members will speak:

Dr. J.-F. Louis
Dr. A. Hollingsworth
Dr. L. Bengtsson

We are indeed very grateful to all of you that you have been willing to accept our invitation to participate during the coming week and to do the preparatory work in planning your contributions.

Secondly, we could not arrange the seminars without a favourable response from the Meteorological Communities in the Member States. We express our appreciation to the Directors of the National Meteorological Services for their support in sending the participants to this year's seminar. We are gratified that it has been possible for almost all of the Member States to send participants. While the majority of the participants are connected with the National Meteorological Services we find also that the university departments and other meteorological agencies in some countries are represented. Wherever you come from you are most welcome.

It is perhaps necessary to say a few words about the format of the arrangements for the coming week. As you will see from the programme we are planning to have a certain number of lectures every day. In connection with each lecture we have planned to have a period for discussion or questions and answers. We would like to have your participation in these periods. You are encouraged to ask questions about the material covered in the preceding talk. Since your lecturers are specialists in their topic they will generally assume that you know much more than you actually do. This is the general fault of the specialist. There will undoubtedly be aspects which you do not understand. Don't be afraid to show your ignorance. Your fellow participants will be grateful that somebody asked the question which they did not dare to ask. One week is a very short time for a seminar. Do not delay all your questions to the last day. In that case it will probably be too late.

The ECMWF is presently in temporary quarters in Bracknell. We expect to move to our permanent headquarters here at Shinfield Park in late 1978. Our office facilities in Bracknell do not permit us to arrange seminars and other meetings. We appreciate very much the cooperation shown by the British Meteorological Office in permitting us to use the facilities of the Meteorological Office College for the seminar this year.

Most of the practical arrangement for this year's seminar have been carried out by Dr. L. Bengtsson with the assistance of Mrs. A. Dinshawe and Mrs. J. Khoury who will be available throughout the week to attend to the practical problems. I am sure that you are going to appreciate their help.

2. Some considerations of the PBL

With the advantage of being the first speaker and without stealing any of the material to be presented later by the experts I may perhaps be permitted to take this opportunity to set the stage for the following discussion. I shall do so by first recalling the very simple, but also very essential approach which was taken in the early years of numerical weather prediction to incorporate a major effect of the planetary boundary layer on the flow of the so-called free atmosphere. When NWP got started on an experimental basis in the late 1940' s, mostly through the effort of J. Charney and his associates at the Institute for Advanced Studies, Princeton, U.S.A., they employed very simple, mostly equivalent-barotropic models. We find a very early description of these experiments by Charney and Eliassen (1949) in the first volume of then newly created journal "Tellus" in Sweden. The model is:

1) equivalent-barotropic
2) quasi-geostrophic
3) one-dimensional (longitude)

The major justification for the equivalent-barotropic model had already been developed by Charney (1947) in his study of "The Dynamics of Long Waves in a Baroclinic Westerly Current" and the quasi-geostrophic models had been obtained in his paper "On the Scale of Atmospheric Motions" (Charney, 1948) during his one-year stay in Oslo, Norway. As a parenthetical remark it is perhaps worth while to point out that these major theories of large-scale atmospheric motion were worked out by Charney when he was about 30 years old. Together with his contemporary, A. Eliassen, these theories were now to be tested by making actual predictions, but since the very first electronic computer was barely on the scene (it was used during the next year to make the first 24-hour forecasts described in a paper by Charney, Fjørtoft and von Neumann, 1950) they decided to simplify the problem by disregarding the latitudinal dependence, except for the beta effect, and obtained thus a one-dimensional model. The preliminary forecasts were without friction and mountain effects, and it turned out that the forecasts contained large errors on the large scale. To remedy the defects in their models they proposed that these errors were caused by the effects of mountains and of the planetary boundary layer. Consequently, they proceeded to incorporate these effects into the model, i.e. to parameterise the planetary boundary layer. It is thus noteworthy that the first incorporation of the planetary boundary layer in a numerical prediction model was done even before the first 1-day forecast was ever made. There are, of course, some meteorologists today who would maintain that we have not progressed very much in this area since 1949, but the remaining part of the lectures in this seminar will enable you to judge on this question yourself.

The contribution by Charney and Eliassen (1949) was not the development of an entirely new theory of the planetary boundary layer, but rather the adaption of existing theory developed at the beginning of the century by Ekman (1905) and somewhat later by Prandtl (1925) to atmospheric modelling. Without reproducing the details of the above studies we can capture the essential part of the parameterisation of the boundary layer in these early models by the following (oversimplified) considerations.

Consider the balance between the pressure force, the Coriolis force and the force of friction in a layer close to the ground:

$$\begin{aligned} 0 &= -\frac{\partial \phi}{\partial x} + fv - g\frac{\partial \tau_x}{\partial p} \\ 0 &= -\frac{\partial \phi}{\partial y} - fu - g\frac{\partial \tau_y}{\partial p} \end{aligned} \tag{1}$$

where $\phi = gz$ is the geopotential, u and v the components of the horizontal velocity, g the acceleration of gravity, f the Coriolis parameter, $\vec{\tau}$ the horizontal stress vector and p the pressure. Pressure has been used as the vertical coordinate. From (1) we can form a vorticity equation which assuming constancy of f (local theory) may be written:

$$0 = - f_s \, \nabla . \vec{v} - g\frac{\partial}{\partial p}\left[\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right] \tag{2}$$

thus eliminating the pressure force. Using the continuity equation

$$\nabla . \vec{v} + \frac{\partial \omega}{\partial p} = 0 \tag{3}$$

we get:

$$\frac{\partial \omega}{\partial p} = \frac{g}{f_s}\frac{\partial}{\partial p}(\vec{k} . \nabla x \vec{\tau}) \tag{4}$$

This equation may be integrated across the planetary boundary layer using the conditions that the stress vanishes at the top of the layer, while ω vanishes at the bottom. Denoting the values at the top by a subscript B and those at the bottom by a subscript o we find

$$\omega_B = - \frac{g}{f_s}\vec{k}.\nabla x\vec{\tau}_o \tag{5}$$

For the stress we introduce the empirical formula

$$\vec{\tau} = c_d \rho_s V_s \vec{v}_o \tag{6}$$

in which c_d is the drag coefficient, ρ_s a standard value of the density at the ground and V_s a standard value of the windspeed. As the final formula we get:

$$\omega_B = - \frac{g}{f_s} c_d \rho_s V_s \zeta_o \tag{7}$$

which shows that the vertical velocity induced by the process in the planetary boundary layer to a first degree of approximation is proportional to the surface vorticity. If you are more used to expressing the vertical velocity in the unit: cms^{-1} we may use the approximate formula

$$\omega_B = - g \rho_s W_B \tag{8}$$

giving

$$W_B = + c_d V_s \frac{\zeta_o}{f_s} \tag{9}$$

Adopting the values $c_d = 3 \times 10^{-3}$, $V_s = 10ms^{-1}$, $\zeta_o = \hat{\zeta}_o \times 10^{-5}s^{-1}$ and $f_s = 10^{-4}s^{-1}$ we find

$$W_B = 0.3\ \hat{\zeta}_o (cms^{-1}) \tag{10}$$

showing that W_B for moderate values of the surface vorticity may be of the order of 1 cms^{-1}.

In the simple approach outlined above it is clear that the boundary layer is incorporated in the model by the simple device of changing the lower boundary condition. The assumptions are that there exists a planetary boundary layer characterised by a vanishing - or at least very small - stress at the top of the layer, and that the surface stress can be expressed in terms of the surface wind.

It is clear that the application of the theory for the planetary boundary layer leading to the so-called Ekman pumping can be described in terms of the cross-isobaric flow generated by the force of friction within the layer. As is well known we obtain a systematic flow from higher toward lower pressure which in turn, because of mass continuity, must lead to induced and systematic vertical velocities, up in cyclonic regions and down in anti - cyclonic regions.

In the practical application of the theory it is naturally not necessary to make as drastic assumptions as above. For example, there is no need to assume standard values of f_s, c_d, ρ_s and V_s. The drag coefficient c_d can for example be given as a map if we have methods to determine c_d from the nature of the underlying surface.

3. The role of Baroclinicity

The original Ekman theory is a barotropic theory. However, within the same assumptions of a balanced flow in a planetary boundary layer it is possible to replace the basic barotropic state by a simple baroclinic state as has been done by a number of investigators as for example MacKay (1971). Without going through details of the mathematical derivations we shall illustrate some of the results taken from Wiin-Nielsen (1974). The conditions for the analysis were:

1. Linear variation of the geostrophic wind, i.e.

$$\vec{v}_g = \vec{v}_{go} + \vec{v}_T z \tag{11}$$

2. Actual wind approaches the geostrophic wind at great heights, i.e.

$$\vec{v} \to \vec{v}_g, \quad z \to \infty \tag{12}$$

3. Continuity in the stress vector at the interface between the Prandtl and Ekman layers, where the stress in the Prandtl layer is

$$\tau = c_d \rho V^2 \tag{13}$$

It should be noted that condition 3 applies to both direction and magnitude of the stress vector.

As can be expected it is found that the important parameters in the basic state are the magnitudes of the surface geostrophic wind V_{go} and the thermal winds V_T and the angle between them, α_T. Figures 1, 2 and 3 give examples of computed hodographs. The following two figures, Fig. 4 and Fig.5, compare observed and computed hodographs. It is seen that there is general agreement between observations and theory.

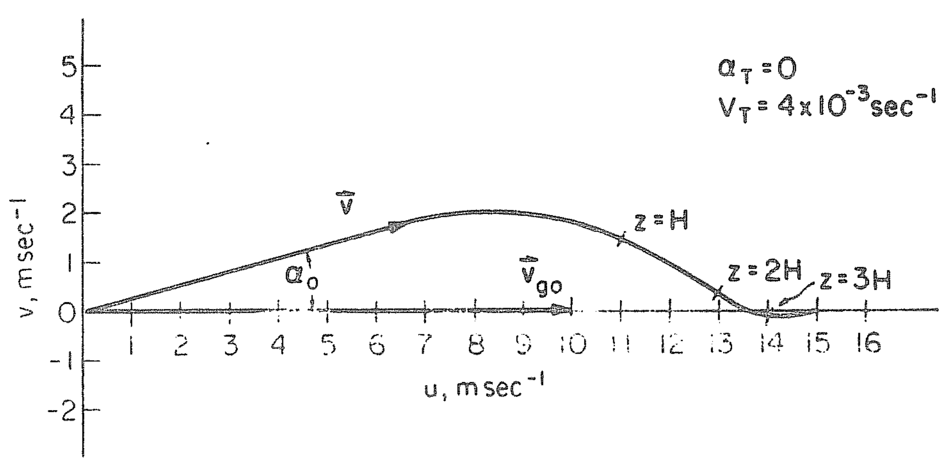

Fig.1. Computed hodograph for $\alpha_T = 0$, $V_T = 4 \times 10^{-3} s^{-1}$, and $V_{go} = 10 ms^{-1}$.

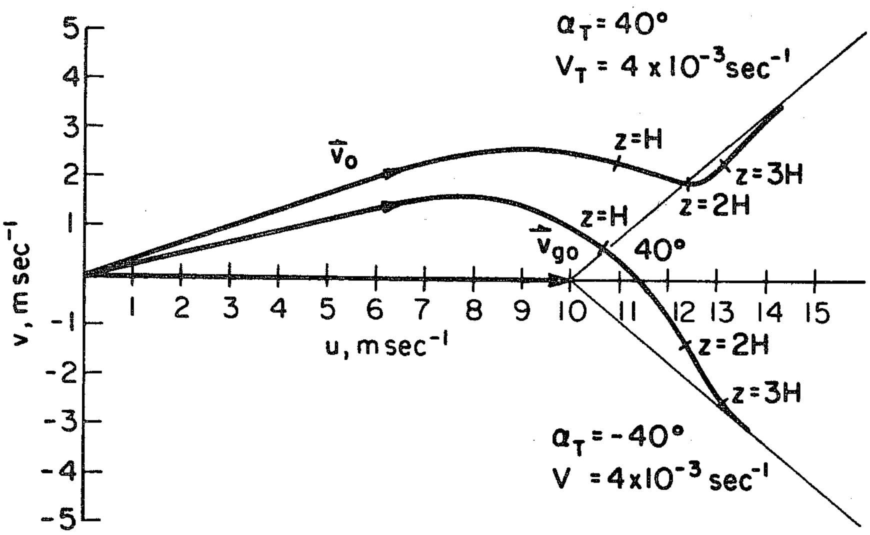

Fig.2. Computed hodographs for $\alpha_T = +40°$, $V_T = 4 \times 10^{-3} s^{-1}$, and $V_{go} = 10 ms^{-1}$.

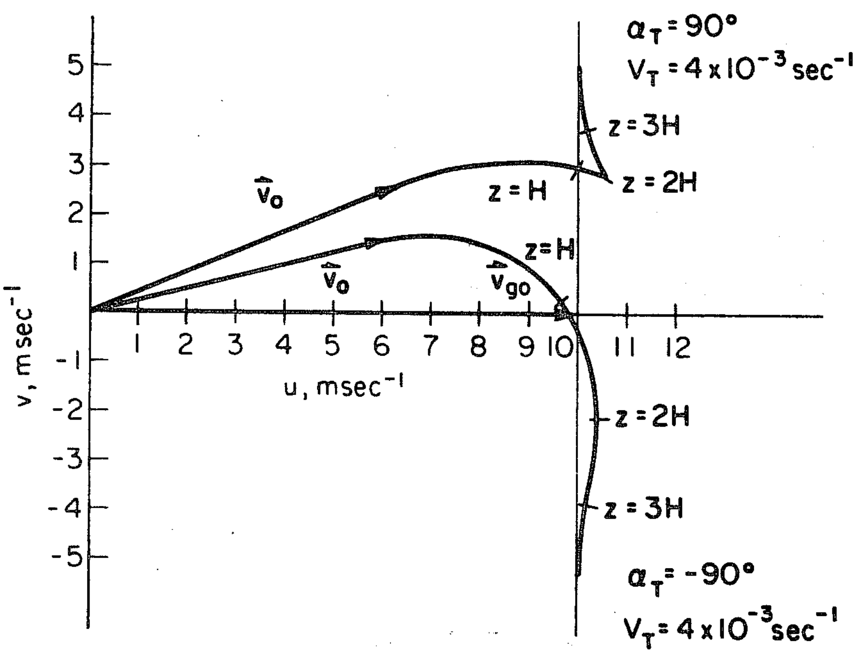

Fig.3. Computed hodographs for $\alpha_T = +90°$, $V_T = 4 \times 10^{-3} s^{-1}$, and $V_{go} = 10 ms^{-1}$.

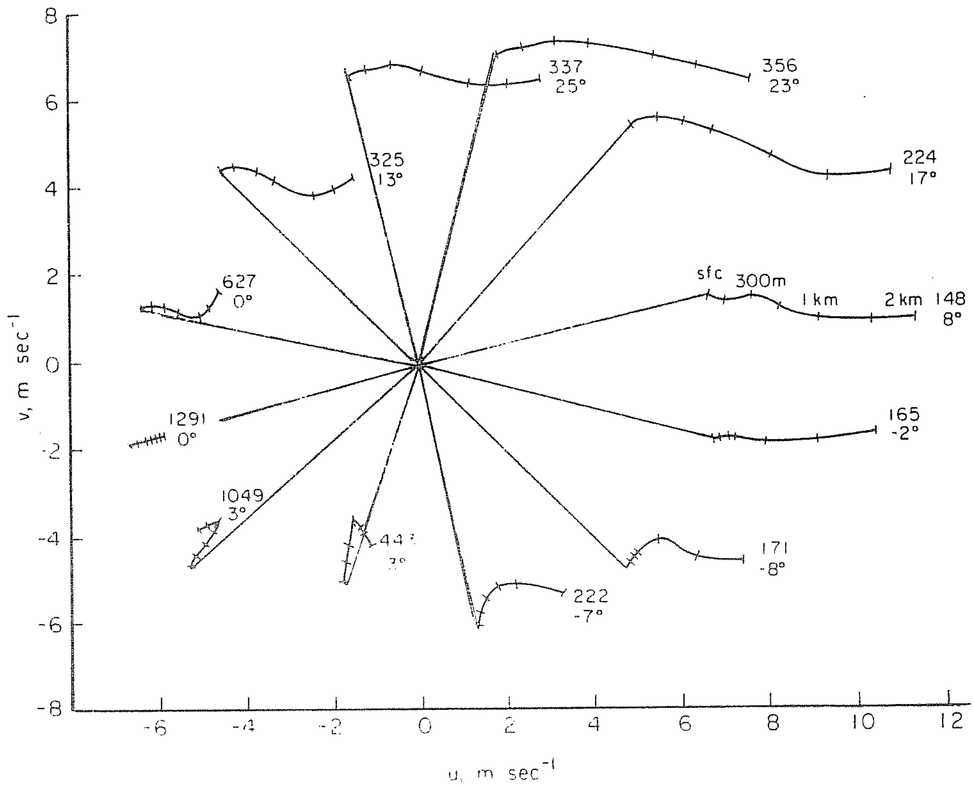

Fig.4. Observed hodographs at ship N (140^{O}W, 30^{O}N) for the years 1960-64. 12 hodographs are shown, grouped according to surface wind direction. Upper number shows the number of observations, while the lower number gives the observed veering between the surface and 1 km, counted positive in a clockwise direction. Elevation marks are shown at 0 (surface), 150, 300, 500, 1000, 15000, and 2000 m (after Mendenhall, 1967).

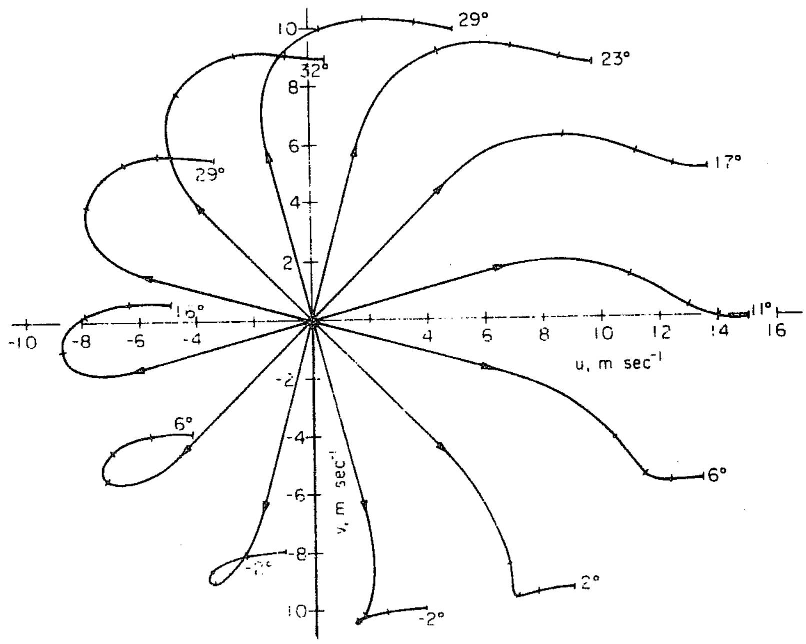

Fig.5. Theoretical hodographs computed with V_{go} = $10 ms^{-1}$, $V_T = 4x10^{-3}s^{-1}$, f = 0.729 x $10^{-4}s^{-1}$, K = $3.6m^2s^{-1}$.

In order to illustrate the major impact of the baroclinicity in the basic state we may look at the following figures. Fig.6 shows the ratio of the baroclinic and barotropic vertical velocities at the top of the boundary layer. Fig.7 compares the baroclinic and barotropic vorticities, while Fig.8 shows the ratio of convergence and vorticity at the lower interface. Finally, Fig.9 gives at the same place the ratio of the vorticity to the geostrophic vorticity.

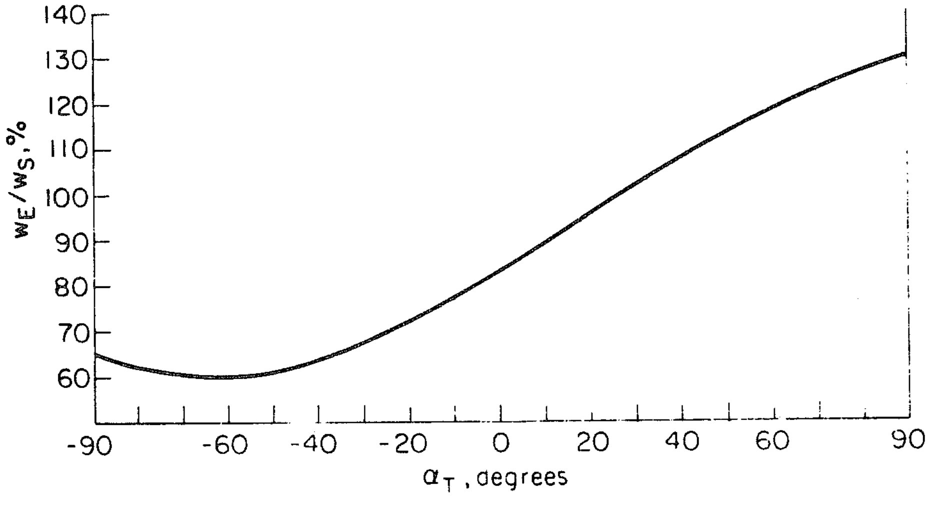

Fig.6. The ratio of the vertical velocity w_E at the top of the planetary boundary layer and the standard vertical velocity w_S, computed from the barotropic layer, as a function of α_T.

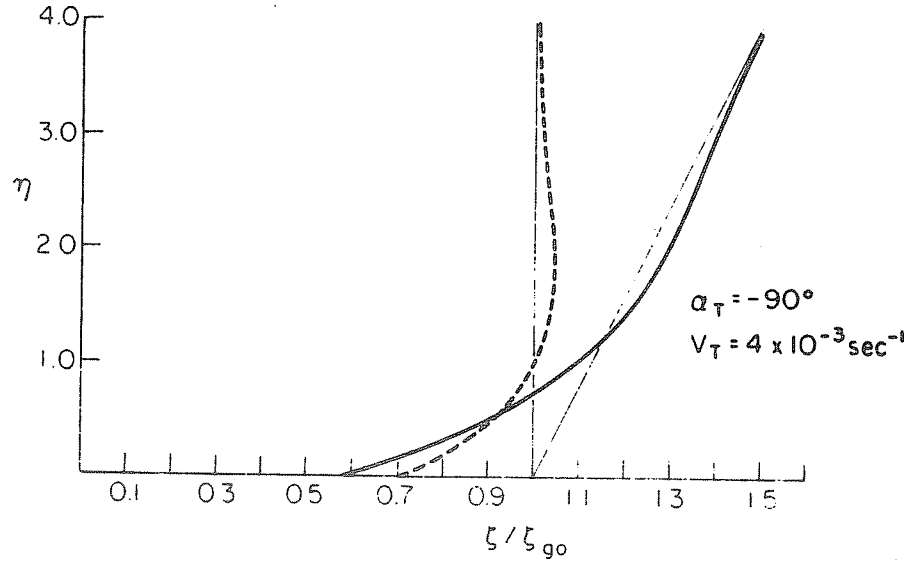

Fig.7. The vorticity of the Ekman solution as a function of height . Dashed line represents the barotropic solution.

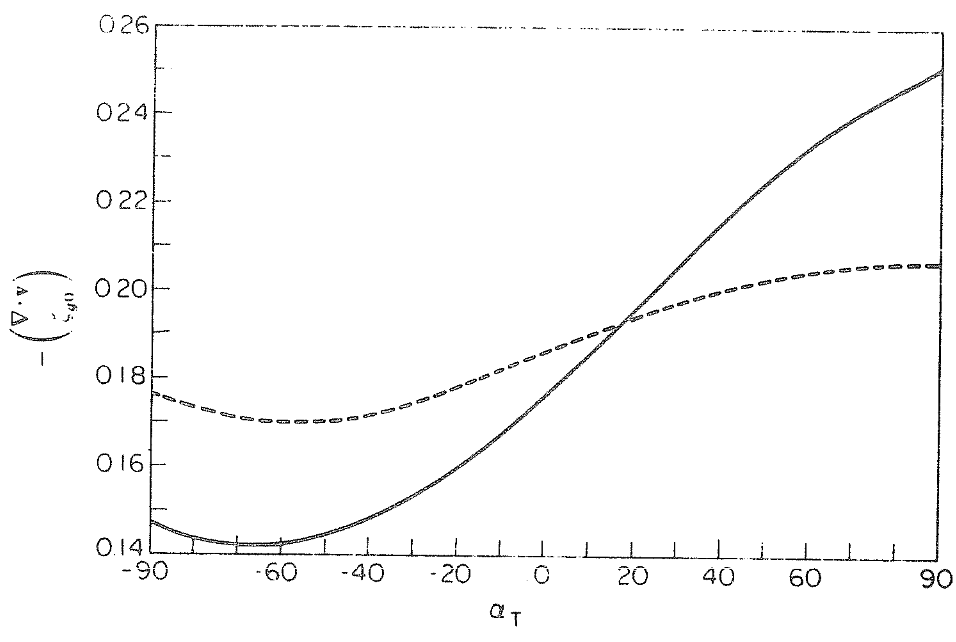

Fig.8. The ratio of convergence and vorticity at $\eta = 0$ as a function of α_T. Dashed line is the barotropic solution.

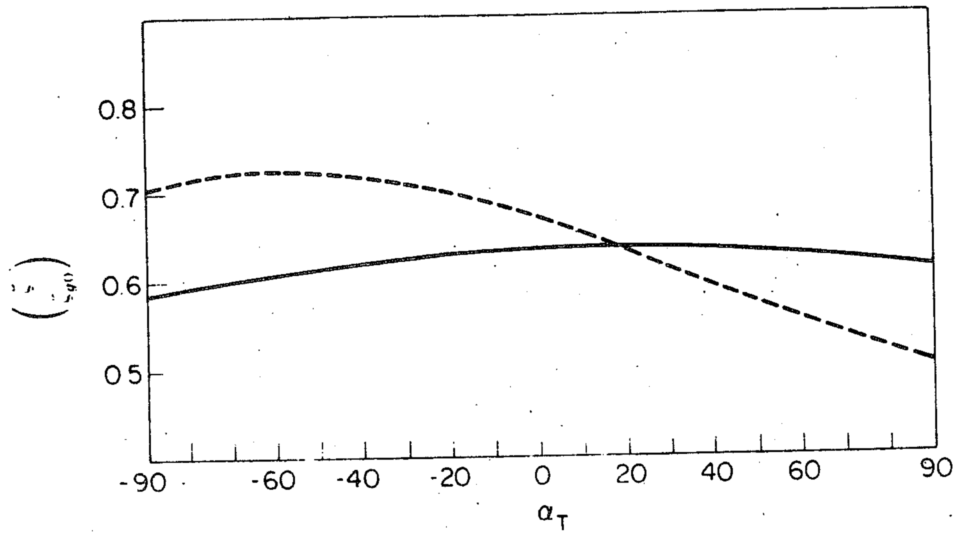

Fig.9. The ratio of the vorticity and the geostrophic vorticity at $\eta = 0$ as a function of α_T. Dashed line is the barotropic solution.

It should be pointed out for the understanding of Figures 7-9 that the barotropic solution represents the limiting case of a very small thermal wind. For details see Wiin-Nielsen (1974).

The main points to be made from the above figures are that the baroclinity in the basic state modifies the barotropic solution considerably, and that it thus is important to study the role played by baroclinicity in the planetary boundary layer.

4. Concluding remarks

The remarks made in the introductory lecture are mainly of a historical nature, and they deal entirely with the hydrodynamical aspects of the PBL, i.e. the influence of the turbulence on the forces in the equations of motion. We have not even touched on the equally important processes of a thermodynamic nature such as the turbulent transfer of heat and moisture in the PBL. I am sure that these aspects will be treated in detail by others.

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