On the Parameterization of a Transient Cumulus Cloud Ensemble

by

K. Fraedrich

INSTITUT FÜR METEOROLOGIE, FREIE UNIVERSITÄT, BERLIN
FEDERAL REPUBLIC OF GERMANY

1. Introduction

The latent heat released by ensembles of cumulus clouds essentially contributes to the energetics of many synoptic scale disturbances. The fact that the horizontal scale of an individual cumulus cloud is some orders of magnitude smaller than a large scale disturbance has two major consequences: (i) the collective effect of a cumulus cloud ensemble and its large scale interaction is to be investigated; (ii) if this interaction is to be included into a large scale model, it cannot be incorporated in full detail but by some parametric procedure. In the following an observational and theoretical description of these two aspects of cumulus cloud populations is discussed.

2. Observational and numerical information

2.1 Geometrical structure of individual clouds

The phenomenological picture of a cloud can be described by its maximum area a_c (or radius r) and height z_τ, and the life-time 2τ. Observational statistics allow to deduce some relationships between these parameters:

(i) Large cumulus clouds have a long life-time and vice versa:

$$\frac{a_c}{\tau} = c = \text{constant}$$

The cumulus cloud attains its maximum area a_c after the growing period τ i.e. at the state of maturity, when the cloud also reaches its maximum depth z_τ. Then the decay phase starts, during which the cloud substance is eroded into the large scale environment. Both the growing and decay periods last about a half life-time τ , i.e. with respect to area and height the clouds undergo an almost symmetrical life-cycle (see e.g. radar studies by Cruz, 1973; Singler, 1975).

(ii) Deep cumulus clouds are correlated with a large radius r and vice versa,which leads to the aspect-ratio γ:

$$\frac{r}{z_\tau} = \gamma = \text{constant}$$

This width-depth ratio γ can physically be explained using the following qualitative arguments; the entrainment factor

$$\lambda = \frac{3\alpha}{r}$$

which is observed to be inversely proportional (α: constant of proportionality) to the cloud radius, determines the dilution rate of the cumulus, its buoyancy and consequently its maximum depth z_τ (where buoyancy or kinetic energy vanish); e.g. a small dilution rate λ is coupled with a large radius and depth of the cloud.

Different classes of cumulus clouds may be characterized by different magnitudes of the parameters c, γ (and perhaps α). At least two main structures can be distinguished: non-precipitating and precipitating clouds (or shallow and deep convection). The change in structure depends on the intensity of the entropy-flow by which the clouds are related to the undisturbed or disturbed large scale situation.

2.2 The statistics for a cloud ensemble

Within a large scale unit area F a population of individual cumulus clouds can be described by the following cloud number distribution function depending on the cloud radius r:

$$n(r) = K \exp(-\beta r) \quad .$$

Observations of cloud populations under disturbed and undisturbed situations support such a relationship (see e.g. Plank, 1969; Ruprecht, 1974; or Cho, 1978; Lopez, 1976 for a log-normal distribution). The parameters K, β can simply be derived from observations of the total number N_o and area cover of clouds $A_o = \sigma F$, (σ: fractional cloud cover), assuming an infinitely large radius spectrum and a circular cloud area, $a = \pi r^2$:

$$N_o = \int_0^\infty n(r)\, dr = K \beta^{-1} \; ; \quad A_o = \int_0^\infty \pi r^2 n(r)\, dr = 2\pi K \beta^{-3}$$

or vice versa, from K and β one can determine A_o and N_o.
Introducing the width-depth ratio γ one can derive a similar profile for the cloud tops, so that an average cloud layer depth Z can be derived

$$Z = \frac{1}{N_o} \int_0^\infty z_\tau \; n(r)\, dr = \frac{1}{\beta\gamma}$$

This cloud depth distribution is also observationally verified under undisturbed situations; under disturbed conditions a bi-model distribution function, if really observed, may give a more appropriate description.

2.3 Some results from numerical modelling

Sommeria's (1976) comprehensive model simulating moist convective boundary layer processes has been evaluated to corroborate the observed relationships (sections 2.1 and 2.2) and to test their significance (Beniston, 1977). For the relationships between cloud area and half life-time (expansion rate $c = a_c/\tau$) and between radius and depth (aspect ratio $\gamma = r/z_\tau$) the linear regression is well supported at a high level of significance. The number density distribution n (r) which is exponential in cloud radius, is found to be simulated at the 99%-significance level.

3. A Parameterization

3.1 Individual cloud

In agreement with the observations (section 2) a simple one-dimensional, transient cumulus cloud model can be derived with the life-cycle being included. Assuming a vertically constant (time-integrate cloud mass flux m_c (Fraedrich, 1973) one obtains a horizontal mass balance for a rising parcel, where entrainment λm_c (= $\partial m_c/\partial z$) is balanced by detrainment, the latter being realized by the area expansion a_c/τ = c of the visible cloud. This leads to the following mass continuity equation

$$\lambda m_c - \rho \frac{a_c}{\tau} = 0$$

where turbulent fluctuations (parameterized and described by entrainment, first term) lead to the observed cloud area expansion (second term). Thus, lateral detrainment $d_l = \lambda m_c = \rho\ a_c/\tau$ = const. is completely balanced by entrainment, so that mass flux m_c remains constant; final detrainment $d_f = m_c \delta z^{-1}$ occurs within a layer depth when the ascending cloud parcel has reached the maximum height z_τ.

The cloud properties can be deduced by the well-known entrainment equation which, from the physical point of view, belongs to this typ of cloud model. Thus, conserved quantities I can directly be deduced (I_c, $\bar{I}$ refer to the cloud and the large scale environment)

$$\frac{\partial I_c}{\partial z} = \lambda(\bar{I} - I_c)$$

There are some good approximations of conserved quantities for this simple cloud model (with top-hat profile) describing the thermodynamic and dynamic behaviour; the moist static energy $h = c_p T + gz + Lq$, the total water content $l + q$ for non-precipitating clouds, and a pseudo-potential vorticity or 'Ertels Wirbelinvariante' $W = (\xi + f) \rho^{-1} \partial h/\partial z$.

Thus, given the large scale $\bar{l}$-distribution and the cloud base boundary conditions, the complete mass and l-budgets of individual clouds can be determined for any prescribed radius r (or cloud top z_T, knowing the aspect ratio γ for disturbed or undisturbed situations). The large scale $\bar{l}$-values in the subcloud layer may define the cloud base conditions $l_{co} \sim \bar{l}_o$ where, however, the cloud pseudo-potential vorticity has a negligibly small value ($\bar{W}_o \neq W_{co} = 0$ because $\partial h_c/\partial z \sim 0$)

3.2 Cloud ensemble

With the observed structural and statistical relationships (section 2), and the model of the individual cloud (section 3.1) the cloud ensemble can simply be established: the vertical variations of the mass budget (vertical mass flux, lateral and final detrainment) and the budget of the conserved quality l ("compensating subsidence", lateral and final detrainment of the cloud property l_c), by which the cloud ensemble interacts with its large scale environment (Fraedrich, 1977).

Two additional parameters of the ensemble will become important: A characteristic cloud layer depth $\hat{Z}$ which is directly related to the average depth Z of the cloud population and determined by the radius of the average cloud area A_o/N_o using the aspect ratio γ:

$$\hat{Z} = \gamma^{-1} \sqrt{\frac{A_o}{\pi N_o}} = \sqrt{2}\, Z \quad .$$

The total mass recycling rate of the cloud ensemble due to the life-cycle of the individual clouds by which the cloud ensemble modifies the environment.

$$\delta_c = F^{-1} \int_o^Z \int_o^\infty (d_l + d_f)\, dr\, dz$$

3.3 Large scale interaction

The interaction of a cumulus cloud ensemble and the large scale environment is twofold:

(i) Apparent large scale sources are produced by the cloud ensemble:

$$F\, Q_I = M_c \frac{\partial \bar{I}}{\partial z} + D_I\, (I_I - \bar{I}) + D_f\, (I_f - \bar{I})$$

due to compensating subsidence, lateral and final detrainment.

(ii) A closure condition for the cloud population statistics has to be defined in addition to the large scale boundary conditions in the cloud free environment and at cloud base to yield a complete description of the cloud ensemble or the apparent sources, respectiv By this closure the shape of the distribution function (i.e. K and β has to be determined:

(a) an observational closure provides information on the total number N_o and area cover A_o of the cloud ensemble from which K and β can be deduced (section 2.2); (b) a large scale model-closure provid the information on the characteristic depth $\hat{Z}$ of the cloud ensemble layer,(i.e. where the large scale divergence $\nabla_h \cdot \overline{\rho \mathbf{v}} = 0$ changes it sign) and on the mass recycling rate δ_c (i.e. which most efficiently modifies the large scale, if it is as large as the absolute large scale mass flux or ventilation: $\rho\, | w |_{\hat{Z}} = \delta_c$ at $\hat{Z}$).

The basic assumption for any such closure is the significantly great relaxation time of the large scale (~100 hours) compared to the clou population (~15 hours). Physical interpretations and additional in-formations on this transient cloud ensemble and its large scale inte action can be obtained from Fraedrich (1976, 1977).

4. Outlook

Several investigations can be emphasized for future work. The relationship between cloud ensemble statistics and large scale parameters, i.e. the closure conditions, should be observationally documented, numerically simulated and perhaps physically understood in order to completely establish the scale interaction. There still remains the question, whether one dimensional cumulus clouds are sufficient to describe reality. Especially, observational and numerical experiments for vorticity budgets are needed to test the momentum transfer as it is parameterized by the conservation of 'Ertels Wirbel-invariante' or pseudopotential vorticity (see Fraedrich, 1974). Then, the step to proceed to two or three dimensional convective systems might turn out to be useful (or not) from the parameterization point of view.

References

Beniston, M.G., 1977: Aspects of non-precipitating cumulus convection in a planetary boundary layer model of the tropical atmosphere.
Proceedings, 11th Technical Conference on Hurricanes and Tropical Meteorology of the American Meteorological Society, December 13-16, 1977, Miami Beach, Florida.

Beniston, M.G., 1978: Utilisation d'un modèle tridimensionnel pour étudier les cumulus des alizé et la paramétrisation de leurs effects sur le champ turbulent de la couche limité tropicalle. PhD-Thesis, L'Université Pierre et Marie Curie, Paris 6, 144 p

Cho, H.R., 1978: Some statistical properties of a homogeneous and stationary shallow cumulus cloud field.
Journal of the Atmospheric Sciences, 35, 125-138.

Cruz, L., 1973: Venezuelan rainstorms as seen by radar.
Journal of Applied Meteorology, 12, 119-126.

Fraedrich, K., 1973: On the parameterization of cumulus convection by lateral mixing and compensating subsidence, Part I.
Journal of the Atmospheric Sciences, 3o, 4o8-413.

Fraedrich, K., 1976: A mass budget of an ensemble of transient cumulus clouds determined from direct cloud observations.
Journal of the Atmospheric Sciences, 33, 262-268.

Fraedrich, K., 1977: Further studies on a transient cumulus cloud ensemble and its large scale interaction.
Journal of the Atmospheric Sciences, 34, 335-343.

Lopez, R.E., 1976: Radar characteristics of the cloud populations of tropical disturbances in the north-west Atlantic.
Monthly Weather Review, 1o4, 268-283.

Plank, V.G., 1969: The size distribution of cumulus clouds in representative Florida populations.
Journal of Applied Meteorology, 8, 46-67.

Ruprecht, E., L. Breuer, K. Bucher, E. Scheidtmann and W. Zäck, 1974: Radarbeobachtungen der Konvektionsgebiete im Bereich der innertropischen Konvergenzzone über dem äquatorialen Atlantischen Ozean.
"Meteor"-Forsch.-Ergebnisse, Reihe B, Vol. 11, 11-23.

Singler, A., 1975: Wachstumsgeschwindigkeit und Lebensdauer konvektiver Niederschlagswolken.
Arch. Meteor. Geoph. Biokl., A24, 33-39.

Sommeria, G., 1976: Three-dimensional simulation of turbulent processes in an undisturbed trade-wind boundary layer.
Journal of the Atmospheric Sciences, 33, 216-241.