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A B S T R A C T

In a previous paper (ECMWF Technical Report No. 4, 1977) the adiabatic part of the ECMWF's grid point model was described. This report continues the earlier and presents a description of the parameterization scheme of the ECMWF's first global forecast model. The scheme described includes :

a) large scale condensation processes (condensation and evaporation of rain);

b) turbulent fluxes of momentum, moisture and sensible heat;

c) Kuo moist convection scheme and evaporation of convective rain;

d) radiative heating/cooling;

e) prediction of surface temperature, soil moisture and snow amount over land.

The scheme has been tested in a series of 10-day forecasts using the EC grid point model, but has also been included and tested in the EC spectral model.

Notations

Independent variables

λ	longitude
θ	latitude
$\sigma=p/p_s$	vertical coordinate
t	time

Dependent variables

$\mathbb{v}=(u,v)$	horizontal wind vector with zonal and meridional components
T	temperature
q	specific humidity
p_s	total surface pressure
T_s	surface temperature
W	soil moisture
S_n	snow cover

Diagnostic variables

$\Phi \equiv gZ$	geopotential (Z = height)
$p = \sigma p_s$	pressure
$\Theta = T\left(\frac{p}{p_o}\right)^{R/c_p}$	potential temperature (p_o = 1000 mb)
$T_v = T(1+0.6077\ q)$	virtual temperature
$\rho = \frac{p}{RT_v}$	density of air
$\rho_v = \rho q$	density of water vapour
$q_{sat} = 0.622\ \frac{e_{sat}}{p}$	saturation specific humidity
e_{sat}	saturation water vapour pressure

Indices

s	suffix indicating a surface value
h	suffix indicating values at the top of the surface layer which is taken off the height of the lowest model level
k	index indicating a model level
i	" " " longitude

Indices (contd.)

j index indicating a latitude

τ " " " time level

g " " " ground value

Constants and parameters

g acceleration due to gravity

R gas constant for dry air

R_v gas constant for water vapour

c_p specific heat capacity for dry air at constant pressure

$\kappa = R/c_p$

c specific heat of water

L_1 latent heat of condensation

L_2 latent heat of ice-melting

k von Karman's constant ($k = 0.35$)

Z_o surface roughness length

ρ_w density of liquid water

T_o temperature of ice-melting ($T_o = 273.16^oK$)

$\boldsymbol{\tau}^T=(\tau_\lambda^T,\tau_\theta^T)$ downward momentum flux vector with λ,θ - components due to turbulent motion

$\boldsymbol{\tau}^c=(\tau_\lambda^c,\tau_\theta^c)$ downward momentum flux vector with λ,θ - components due to convective motion

$\mathbb{F}_M^T=((F_M^T)_\lambda,(F_M^T)_\theta)$ frictional force due to turbulent motion in the horizontal direction

$\mathbb{F}_M^c=((F_M^c)_\lambda,(F_M^c)_\theta)$ frictional force due to convective motion in the horizontal direction

H^T, H^c downward flux of sensible heat due to turbulent/convective motion

F_T^T, F_T^c heating/cooling due to turbulent/convective motion in the horizontal direction

R^T, R^c downward flux of moisture due to turbulent/convective motion

F_q^T, F_q^c change of humidity due to turbulent/convective motion in the horizontal direction

C^L, C^c	rate of condensation due to large scale/convective motion
E^L, E^c	rate of evaporation from large scale/convective rain
Q	heating/cooling rate due to radiation
B^c	change of humidity due to moist convection in the Kuo-convection scheme

1. Introduction

Processes associated with turbulent and convective transfer, condensation and radiation are important for the development of the atmospheric large-scale flow. Since these processes are related to spatially small or even molecular phenomena, they cannot be explicitely included in numerical models which only resolve scales larger than the grid size in grid point models or the truncation wave number in spectral models.

The effect of the sub-grid scale processes on the large-scale flow which we want to predict can only be considered by means of parameterization, i.e. formulating the ensemble effect in terms of the resolved grid-scale variables.

Large efforts have been made in the recent past to design reliable parameterization schemes for different processes. Based on a large amount of observational data the parameterization of the turbulent fluxes in the planetary boundary layer has been considerably improved. Progress has also been made in calculating the radiative fluxes, and a hierarchy of schemes differing with respect to the degree of approximation of the equations of radiative transfer were developed. However, the parameterization of convection is still problematic since the interaction of cumulus clouds and large scale circulation is not sufficiently understood to decide on one of the existing schemes. Finally, there are also processes which have only occasionally been the subject of parameterization studies as, for instance, mesoscale circulations, topographically induced circulations, thin layer clouds, coastal effects. None of these latter processes are parameterized in the ECMWF model.

Considering these circumstances, the ECMWF model has been designed in a rather flexible way allowing interchanges of parts of the parameterization scheme against others. Two parameterization packages have been prepared for the model.

1. The scheme originally designed for the GFDL-model (MANABE et al. (1965)) has been adapted to the ECMWF model.

2. A scheme developed at the ECMWF, which is described in this paper.

Fig.1.1 shows the processes included in the ECMWF-scheme and also shows how the different processes (printed within ellipses) depend on the variables given by the model (printed in rectangles). The thickness of the arrows represents qualitatively the importance of the interactions. For example, the effect of radiation on the ground temperature is very fast : it only takes a few hours for the sun to heat the ground; but the relationship between radiation and the temperature of the air is much weaker: condensation, diffusion and the flux of sensible heat from the ground change the air temperature more effectively. Any closed loop in the diagram (e.g. Temperature→Evaporation→Ground temperature→Sensible heat flux →Temperature) indicates a feed-back phenomenon. The reader can amuse himself by counting the number of closed loops in order to get a feeling for the complexity of the scheme. Departing from Fig. 1.1 clouds are not considered explicitely in the model at the present time. Instead, the cloud cover used for the radiation calculation is expressed in terms of relative humidity. Turbulent and convective terms are not separately specified in the graph but formally represented by "Diffusion, condensation and precipitation".

The main differences of the ECMWF scheme compared to the GFDL-scheme are:

1) Turbulent fluxes of momentum, heat and moisture are considered throughout the atmosphere, the diffusion coefficient being dependent on a mixing length which varies with height, on the windshear and on the thermal stability. Because of the stability dependence, providing large fluxes for

unstable situations, no explicit dry convective adjustment is needed in the model. In the GFDL-model the fluxes of momentum and water vapour are computed in the boundary layer only, and do not depend on stability.

2) The convection scheme designed by KUO (1965, 1974) is used in place of the moist adiabatic adjustment.

3) The radiation scheme considers the effect of clouds on the short wave and long wave radiation. The diurnal variation of solar radiation can be considered in the scheme.

4) Surface temperatures over land are predicted rather than determined diagnostically from an energy balance equation. Temperature and soil water are treated following a proposal of DEARDORFF (1978).

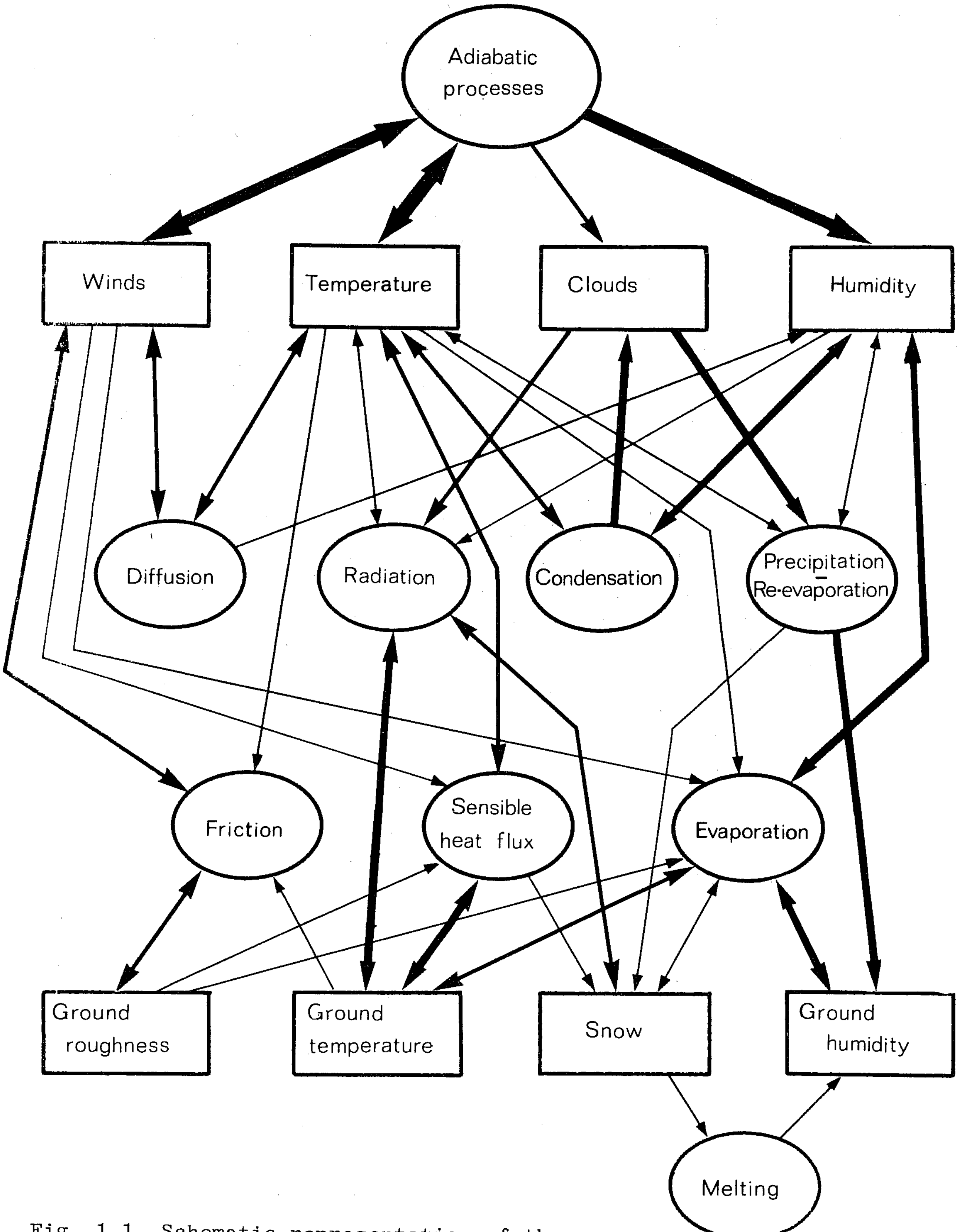

Fig. 1.1 Schematic representation of the processes included in the ECMWF model

The ECMWF scheme described in this paper is considered as a basic scheme from which further developments will start. This scheme, as well as the GFDL-scheme, has been tested recently in a series of 10-day forecasts. The results of these experiments, being reported in a separate Technical Report, indicate some principal deficiencies in the ECMWF scheme which we will try to overcome first by tuning the model before making any drastic changes.

2. The basic equations

The equations defined for the sigma system in spherical coordinates may be written as follows:

Equations of motion

$$\frac{\partial u}{\partial t} = A_u + \frac{1}{p_s}\left[- g \frac{\partial}{\partial \sigma}(\tau_\lambda^T + \tau_\lambda^c) + (F_M^T)_\lambda + (F_M^c)_\lambda\right]$$

$$\frac{\partial v}{\partial t} = A_v + \frac{1}{p_s}\left[- g \frac{\partial}{\partial \sigma}(\tau_\theta^T + \tau_\theta^c) + (F_M^T)_\theta + (F_M^c)_\theta\right] \qquad (2.1)$$

Thermodynamic equation

$$\frac{\partial T}{\partial t} = A_T + \frac{L_1}{c_p}(C^L + C^c - E^L - E^c) - \frac{1}{p_s}\frac{g}{c_p}\frac{\partial}{\partial \sigma}(H^T + H^c) + F_T^T + F_T^c \qquad (2.2)$$

Continuity equation for water vapour

$$\frac{\partial q}{\partial t} = A_q - (C^L - E^L - E^c) + B^c - \frac{g}{p_s}\frac{\partial}{\partial \sigma}(R^T + R^c) + F_q^T + F_q^c \qquad (2.3)$$

Continuity equation for mass

$$\frac{\partial p_s}{\partial t} = A_{p_s} \qquad (2.4)$$

Hydrostatic equation

$$\frac{\partial \Phi}{\partial (\ln \sigma)} = - RT_v \qquad (2.5)$$

A_u, A_v, A_T, A_q and A_{p_s} represent the adiabatic terms in the equations for u, v, T, q and p_s. B^c represents the change of humidity by moist convection as given by the KUO-scheme.

3. The vertical grid structure

The vertical structure of the model's atmosphere is represented by K layers in the sigma coordinate system. The state of the soil is described by one thin layer of specified heat capacity and specified field capacity for moisture and by one extra layer below, where temperature and soil moisture are kept constant in time.

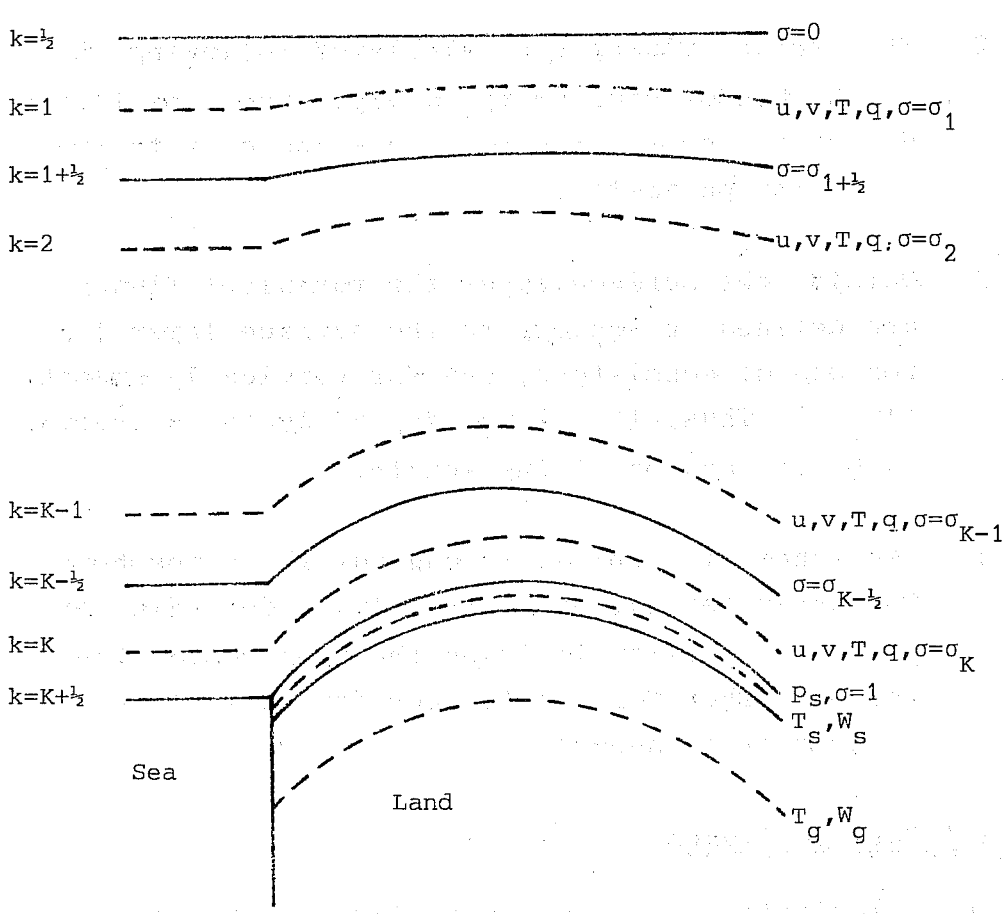

Fig. 3.1 Schematic diagram of the vertical level structure

We define

$$\Delta\sigma_k = \sigma_{k+\frac{1}{2}} - \sigma_{k-\frac{1}{2}}$$

$$\Delta\sigma_{k+\frac{1}{2}} = \sigma_{k+1} - \sigma_k \qquad (3.1)$$

4. Turbulent fluxes of momentum, heat and moisture

The parameterization of the turbulent fluxes is characterised as follows:

1. An explicit turbulent diffusion scheme is used. The planetary boundary layer (up to approximately 1000 m.) is described by three levels in the 15 level model version and by surface values.

2. The surface fluxes are calculated following the MONIN-OBUKHOV similarity theory, where the fluxes depend on the surface roughness and on a thermal stability parameter.

3. Outside the surface layer the turbulent fluxes are defined in analogy to the surface layer for reasons of consistency and for vertically smooth fluxes. Thus, the fluxes depend again on thermal stability and on mixing length.

4. The scheme is applied throughout the atmosphere. The parameterization of the fluxes depending on thermal stability includes the free convection region. Therefore no further dry convective adjustment is needed.

4.1 Surface fluxes

The calculation of the surface fluxes is based on the MONIN-OBUKHOV similarity theory where the profiles of wind and temperature depend on external parameters and on surface fluxes of momentum and heat as

$$\frac{k.Z}{u_*} \frac{\partial u}{\partial Z} = \phi_M\left(\frac{Z}{L}\right) , \quad \frac{\partial v}{\partial Z} = 0$$

$$\frac{k.Z}{\Theta_*} \frac{\partial \Theta}{\partial Z} = \phi_H\left(\frac{Z}{L}\right) \tag{4.1}$$

where

$$L = \frac{\Theta \, u_*^2}{kg\,\Theta_*}$$

is the MONIN-OBUKHOV length. The quantities u_* and Θ_* are defined in terms of the surface fluxes of momentum and sensible heat as

$$(\tau_\lambda^T)_s \equiv - \rho_s(\overline{w'u'})_s = \rho_s u_*^2$$

$$\frac{1}{c_p}(H^T)_s \equiv - \rho_s \, (\overline{w'\Theta'})_s = \rho_s \, u_* \Theta_* \qquad (4.2)$$

The flux calculations are further based on analytical formulae for $\phi_M(\frac{Z}{L})$ and $\phi_H(\frac{Z}{L})$ empirically derived by BUSINGER et al. (1971) for stable and unstable conditions. As, however, the flux relationships by ϕ_M and ϕ_H are implicit with regard to the internal parameters u_* and Θ_* and as the numerical solution is therefore rather time consuming, analytical expressions have been derived which explicitly describe the fluxes in terms of external parameters (LOUIS (1977), (1979)).

The equations for the fluxes of momentum, sensible heat and moisture used in the model are:

<u>Unstably stratified surface layer</u>

$$(\tau^T)_s = + \rho_h a \left[|\mathbb{V}_h| - \frac{bs_h}{|\mathbb{V}_h| + bc_M \, a_h \, |s_h|^{\frac{1}{2}}} \right] \mathbb{V}_h$$

$$(H^T)_s = -c_p \sigma_h^\kappa \, \rho_h \frac{a}{d_H} \left[|\mathbb{V}_h| - \frac{bs_h}{|\mathbb{V}_h| + bc_H \, a_h \, |s_h|^{\frac{1}{2}}} \right] (T_s - \frac{T_h}{\sigma_h^\kappa}) \qquad (4.3)$$

$$(R^T)_s = -\rho_h \, \frac{a}{d_q} \left[|\mathbb{V}_h| - \frac{bs_h}{|\mathbb{V}_h| + bc_q \, a_h \, |s_h|^{\frac{1}{2}}} \right] (q_{sat}(T_s) - q_h).WET$$

Stably stratified surface layer

$$(\tau^T)_s = \rho_h\, a\left(1 + \frac{b}{2}\frac{s_h}{|\mathbb{V}_h|^2}\right)^{-2} |\mathbb{V}_h|\,\mathbb{V}_h$$

$$(H^T)_s = -c_p\, \sigma_h^{\kappa}\, \rho_h \frac{a}{d_H}\left(1 + \frac{b}{2}\frac{s_h}{|\mathbb{V}_h|^2}\right)^{-2} |\mathbb{V}_h|\,(T_s - T_h/\sigma_h^{\kappa}) \qquad (4.4)$$

$$(R^T)_s = -\rho_h \frac{a}{d_q}\left(1 + \frac{b}{2}\frac{s_h}{|\mathbb{V}_h|^2}\right)^{-2} |\mathbb{V}_h|\,(q_{sat}(T_s) - q_h).\mathrm{WET}$$

The function s_h representing the thermal stability and the constants used in (4.3) to (4.4) are defined as

$$s_h = gh\left\{\frac{(\Theta_h - \Theta_s)}{\Theta_s} + 0.6077\,(q_h - q_{sat}(T_s)).\mathrm{WET}\right\}$$

$$b = 9.4,\ d_H = 0.74,\ c_M = c_H = c_q = 5.3 \qquad (4.5)$$

$$a = k^2\left(\ln \frac{h}{Z_o}\right)^{-2}$$

and $a_h = a\left(\frac{h}{Z_o}\right)^{\frac{1}{2}}$

where h is the height of the lowest model level above ground.

We allow d_q to be different from d_H in order to provide a mean for the tuning of the hydrological cycle. At present we use

$$d_q = \frac{d_H}{\alpha}$$

where the constant α is chosen either
$\alpha = 1$ or $\alpha = 1.2$.
WET is the wetness of the soil (see chapter 9).

The surface roughness length Z_o is specified as follows. Over sea the relationship proposed by CHARNOCK (see CLARKE, (1972)):

$$Z_o = 0.032\,\frac{u_*^2}{g} \qquad (4.6)$$

is used, assuming, however, that Z_o does not drop below 0.0015cm. Over land Z_o is specified as dependent on vegetation (BAUMGARTNER et al. (1977)) and on a parameter representing the roughness of the orography (i.e. a kind of variance of orography derived from U.S. - NAVY data).

4.2 Fluxes above the surface layer

The fluxes are calculated on the basis of the mixing length theory.

$$\tau^T = \rho K_M \frac{\partial V}{\partial Z}$$

$$H^T = c_p \; \rho \sigma^{\kappa} K_H \frac{\partial}{\partial Z} (T/\sigma^{\kappa})$$

$$R^T = \rho \; K_q \frac{\partial q}{\partial Z}$$

which gives in sigma-coordinates at level $k + \frac{1}{2}$

$$\tau^T_{k+\frac{1}{2}} = - \frac{g}{p_s} (\rho^2 K_M \frac{\Delta V}{\Delta \sigma})_{k+\frac{1}{2}}$$

$$H^T_{k+\frac{1}{2}} = -g \frac{c_p}{p_s} (\rho^2 \sigma^{\kappa} K_H \frac{\Delta(T/\sigma^{\kappa})}{\Delta \sigma})_{k+\frac{1}{2}} \qquad (4.7)$$

$$R^T_{k+\frac{1}{2}} = - \frac{g}{p_s} (\rho^2 K_q \frac{\Delta q}{\Delta \sigma})_{k+\frac{1}{2}}$$

As for the surface layer the thermal stratification is taken into account. The diffusion coefficient depends therefore on wind shear and on stability. Since the fluxes outside the surface layer must match the surface fluxes for similar conditions of vertical wind shear and of thermal stratification, the diffusion coefficients must be respectively defined in terms of wind shear and stability.

Again, the diffusion coefficients are defined differently for stable and for unstable conditions.

Unstable stratification:

$$(K_M)_{k+\frac{1}{2}} = \ell^2_{k+\frac{1}{2}} \left(\left|\frac{\Delta V}{\Delta Z}\right|_{k+\frac{1}{2}} - \frac{b s_{k+\frac{1}{2}}}{\left|\frac{\Delta V}{\Delta Z}\right|_{k+\frac{1}{2}} + b c_M \, a_{k+\frac{1}{2}} \left|s_{k+\frac{1}{2}}\right|^{\frac{1}{2}}} \right) \tag{4.8}$$

$$(K_H)_{k+\frac{1}{2}} = \frac{1}{d}(K_M)_{k+\frac{1}{2}}, \quad (K_q)_{k+\frac{1}{2}} = \frac{1}{d_q}(K_M)_{k+\frac{1}{2}}$$

Stable stratification:

$$(K_M)_{k+\frac{1}{2}} = \ell^2_{k+\frac{1}{2}} \left(\left|\frac{\Delta V}{\Delta Z}\right|_{k+\frac{1}{2}} \left(1 + \frac{b}{2} \frac{s_{k+\frac{1}{2}}}{\left(\frac{\Delta V}{\Delta Z}\right)^2_{k+\frac{1}{2}}}\right)^{-2} \right) \tag{4.9}$$

$$(K_H)_{k+\frac{1}{2}} = \frac{1}{d_H}(K_M)_{k+\frac{1}{2}}, \quad (K_q)_{k+\frac{1}{2}} = \frac{1}{d_q}(K_M)_{k+\frac{1}{2}}$$

The stability function s is now

$$s_{k+\frac{1}{2}} = g\left[\frac{1}{\Theta}\left(\frac{\Delta\Theta}{\Delta Z}\right) + 0.6077\left(\frac{\Delta q}{\Delta Z}\right)\right]_{k+\frac{1}{2}} \tag{4.10}$$

and a is specified as

$$a_{k+\frac{1}{2}} = \left(\frac{\ell}{\Delta Z}\right)^2_{k+\frac{1}{2}} \left(\left(\frac{Z_{k+1}}{Z_k}\right)^{1/3} - 1\right)^{3/2} \cdot \left(\frac{\Delta Z}{Z_k}\right)^{\frac{1}{2}} \tag{4.11}$$

which gives the same asymptotic height dependency of the diffusion coefficient

$$K \approx Z^2 \left|\frac{g}{\Theta}\frac{\Delta\Theta}{\Delta Z}\right|^{\frac{1}{2}} \quad \text{for } \left|\frac{\Delta V}{\Delta Z}\right| \to 0$$

as is deduced from observational data for free convection (HSU (1973)).

The diffusion coefficients depend largely on the mixing length ℓ, which is a rather uncertain parameter. The best values are likely to be those which are determined by sensitivity tests. At present we start our experiments using the mixing length profile proposed by BLACKADAR (1962)

$$\ell = \frac{k.Z}{1 + \frac{k.Z}{\lambda}} \tag{4.12}$$

where the asymptotic mixing length λ is assumed

$$\lambda = 300m$$

at present. This value may seem large compared to the one suggested by BLACKADAR (30 to 100m for neutral stratification). One must remember, however, that this formulation must account for the effect of all the motions of a scale smaller than the mesh of the model, whereas BLACKADAR considered only turbulence of a rather small scale. For the stable stratification of the standard atmosphere, the effective mixing length

$$\ell' = \ell/(1 + \frac{b}{2} \frac{s_{k+\frac{1}{2}}}{(\frac{\Delta V}{\Delta Z})^2_{k+\frac{1}{2}}})$$

reduces to a few per cent of ℓ.

4.3 Numerical solution of the diffusion equation

As the vertical grid of the model is such that the layers are rather thin near the surface, the terms including vertical fluxes must be treated implicitly in time.

The tendency due to turbulent fluxes is, taking the zonal wind for example

$$(\frac{\partial u}{\partial t}) = - \frac{g}{p_s} \frac{\partial}{\partial \sigma} \tau_\lambda^T + \ldots.$$

which in finite differences becomes

$$u_k^{\tau+1} - u_k^{\tau-1} = A_k(u_{k+1}^{\tau+1} - u_k^{\tau+1}) - C_k(u_k^{\tau+1} - u_{k-1}^{\tau+1}) \qquad (4.13)$$

where A_k and C_k depend on $u^{\tau-1}$, $v^{\tau-1}$, $T^{\tau-1}$ as follows from (4.3)-(4.12). The set of equations (4.13) is simply solved by matrix inversion using the method of RICHTMYER and MORTON (1967, section 8.5).

4.4 Flux calculations in the GRID-POINT MODEL

The ECMWF grid-point model is based on a staggered latitude-longitude grid (Fig. 4.1).

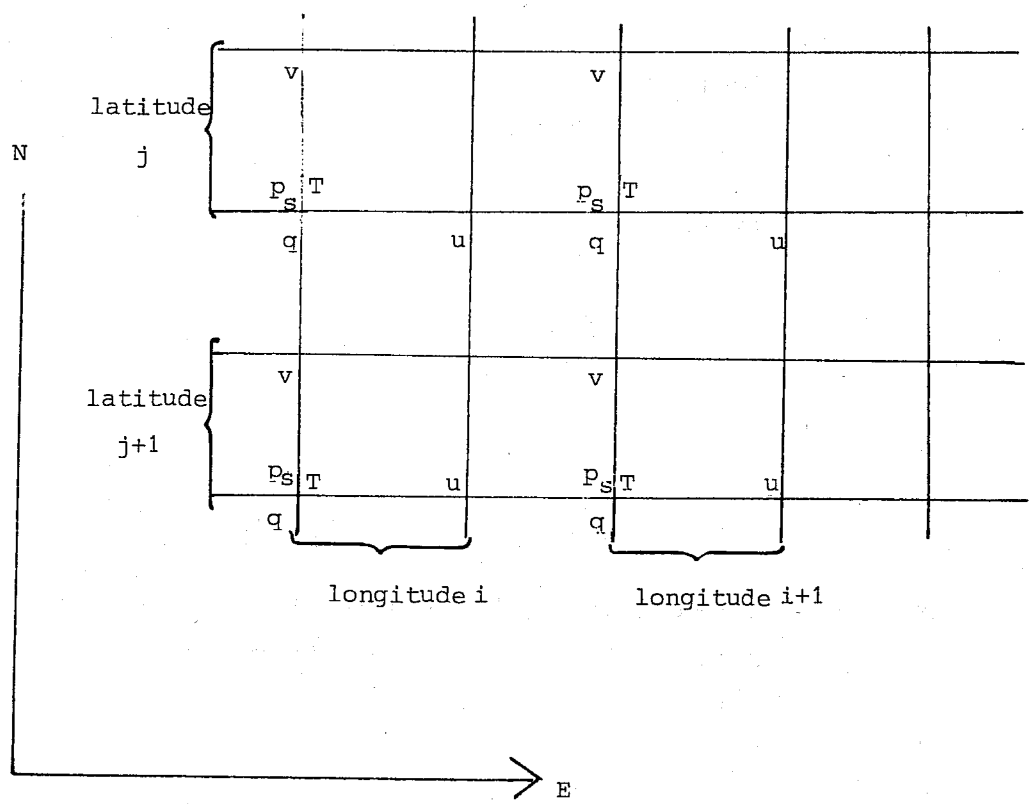

Fig. 4.1 Horizontal distribution of variables in the grid-point model

The space staggering of the variables is considered in the calculation of the turbulent fluxes in the following way. The fluxes of all quantities are calculated at (T,q)-grid points. Therefore the velocity components are first defined at (T,q)- grid points.

At non-polar grid points a linear interpolation is used

$$u^T_{i,j} = 0.5\ (u_{i,j} + u_{i-1,j}) \tag{4.14}$$

$$v^T_{i,j} = 0.5\ (v_{i,j} + v_{i,j+1})$$

and at polar grid points we use (taking the North Pole for example)

$$u^P = -\frac{2}{N}\sum_{i=1}^{N} v_{i,2}\,\sin\lambda_i \qquad (4.15)$$

$$v^P = \frac{2}{N}\sum_{i=1}^{N} v_{i,2}\,\cos\lambda_i$$

The velocity tendencies due to turbulent fluxes calculated at (T,q)- points are finally redistributed to the u- and v-points by linear interpolation

$$\frac{\partial u_{i,j}}{\partial t} = 0.5\left(\frac{\partial u^T_{i,j}}{\partial t} + \frac{\partial u^T_{i+1,j}}{\partial t}\right) \qquad (4.16)$$

$$\frac{\partial v_{i,j}}{\partial t} = 0.5\left(\frac{\partial v^T_{i,j}}{\partial t} + \frac{\partial v^T_{i,j-1}}{\partial t}\right)$$

where the v-tendencies at the polar points are defined by (taking the North Pole for example)

$$\frac{\partial v_{i,1}}{\partial t} = \frac{\partial v^P}{\partial t}\cos\lambda_i - \frac{\partial u^P}{\partial t}\sin\lambda_i \qquad (4.17)$$

5. Moist convection

At present moist convection is parameterized by a KUO-type convection scheme (KUO (1965, 1974)).

5.1 The basic feature of the KUO-convection scheme

The Kuo-scheme considers the effect of cumulus convection on the large scale flow by means of a realistic but simplified model of the cumulus clouds themselves, applying the following basic assumptions. Cumulus clouds are forced by mean low-level convergence in regions of conditionally unstable stratification. The production of cloud air is proportional to the net amount of moisture convergence into one grid point column plus the moisture supply by surface evaporation, which in a period Δt is:

$$I = \frac{\Delta t}{g} \int_0^1 A_q \, p_s d\sigma - \Delta t (R^T)_s \tag{5.1}$$

The surplus of total energy of the cloud against the environmental air is given by

$$P = \frac{p_s}{g} \int_{\sigma_{TOP}}^{\sigma_{BASE}} (c_p(T_c - T_e) + L_1(q_c - q_e)) \, d\sigma \tag{5.2}$$

where the index c refers to cloud air and the index e refers to the environmental air. The fractional cloud area a being produced by the moisture supply follows then from

$$a = L_1 \, I/P \tag{5.3}$$

The clouds produced dissolve instantaneously by artificial mixing with the environmental air, whereby the environmental air is heated and moistened by

$$(\Delta T)^c = a(T_c - T_e) \tag{5.4}$$

$$(\Delta q)^c = a(q_c - q_e).$$

The scheme described so far is the original scheme proposed by KUO (1965).

Recently, KUO (1974) has modified his scheme by introducing a parameter β such that the fraction $1-\beta$ is condensed, while the remaining fraction β is stored in the atmosphere. The parameter β can be chosen arbitrarily and can therefore serve as a useful mean to tune the convection scheme. A reasonable approach has been tried by ANTHES (1977) who proposes β depending on the mean relative humidity in the column, such that the moistening is larger in dry air than in moist air.

Instead of (5.4) we have now

$$(\Delta T)^c = a_T (T_c - T_e) \qquad (5.4)a$$

$$(\Delta q)^c = a_q (q_{sat}(T_e) - q_e)$$

where a_T and a_q are given by

$$a_T = \frac{L_1 (1-\beta) I}{c_p \frac{p_s}{g} \int_{\sigma Top}^{\sigma Base} (T_c - T_e)\, d\sigma}$$

$$a_q = \frac{\beta I}{\frac{p_s}{g} \int_{\sigma Top}^{\sigma Base} (q_{sat}(T_e) - q_e)\, d\sigma} \qquad (5.3)a$$

The parameter β which determines the partitioning of heating and moistening of the environmental air is specified following ANTHES (1977) as

$$\beta = \begin{cases} \dfrac{1 - \langle U_e \rangle}{1 - U_{crit}} & , \text{ if } \langle U_e \rangle > U_{crit} \\ 1 & , \text{ if } \langle U_e \rangle \leq U_{crit} \end{cases} \qquad (5.3)b$$

where U_{crit} is a threshold relative humidity, below which only moistening of the air occurs (U_{crit} = 0.5 at present) and where $\langle U_e \rangle$ is the mean environmental relative humidity for the cloud layer

$$\langle U_e \rangle = \frac{1}{\sigma_{Base} - \sigma_{Top}} \int_{\sigma Top}^{\sigma Base} \frac{q_e}{q_{sat}(T_e)} \, d\sigma \qquad (5.3)c$$

Both schemes have been implemented and tested in the model.

5.2 Implementation of the KUO convection scheme in the model

Several modifications have been made to the KUO scheme for use in our model. The main changes are: besides considering convective clouds originated by lifting surface air we also consider clouds which are originated at upper levels where moisture convergence is observed. This type of cloud may occur in mid-latitude frontal regions. Consequently we do not use the added moisture for the whole grid column but only the contribution which takes place in the layer between the lifting level and the top of the cloud, which in a time-step period Δt becomes

$$I = \Delta t \int_{\sigma Top}^{\sigma Lift} \left(A_q \frac{p_s}{g} - \frac{\partial R^T}{\partial \sigma} \right) d\sigma \qquad (5.5)$$

The equations (5.2) to (5.4) remain valid.

The main terms in the equation for temperature (2.2) and in the equation for moisture (2.3) due to moist convection can now be specified.

As the moisture added by mean convergence and by turbulent fluxes is converted to moistening and heating cloud layers, the convective heating given by (5.4) is only due to the release of latent heat in cumulus clouds

$$\frac{L_1}{c_p} C^c = a \frac{(\Delta T)^c}{\Delta t} \qquad (5.6)$$

The term B^c in the equation for water vapour (2.3) is given by:

$$B^c = -\delta^c(A_q - \frac{g}{p_s}\frac{\partial R^T}{\partial \sigma}) + \frac{(\Delta q)^c}{\Delta t} \qquad (5.7)$$

where δ^c is specified as

$$\delta^c = \begin{cases} 1 & \text{if } \sigma_{TOP} \leq \sigma \leq \sigma_{LIFT} \\ 0 & \text{otherwise} \end{cases} \qquad (5.8)$$

The first term in (5.7) counterbalances the adiabatic term plus the term due to turbulent fluxes in both sub-cloud layers and cloud-layers, whereas the second term gives the moistening in cloud layers.

The remaining quantities, i.e. lifting level, cloud-base, cloud-top, cloud-temperature T_c and cloud humidity q_c are determined as follows.

The surface is considered as a lifting level if the net moisture accession within the boundary layer (which besides the surface layer here only includes those layers which are neutral or unstable with respect to the dry adiabatically lifted surface air) is positive. If this is true, temperature T_c and humidity q_c at the surface are defined as

$$(T_c)_{LIFT} = T_s$$
$$(q_c)_{LIFT} = q_{sat}(T_s) \cdot U_h \qquad (5.9)$$

where U_h is the relative humidity at the top of the surface layer which coincides with the lowest level in the model. If there is no net moisture accession, the next model level above the boundary layer is checked for moisture accession. This level becomes a lifting level if there is moisture added by mean convergence and by turbulent fluxes

$$A_q - \frac{g}{p_s}\frac{\partial R^T}{\partial \sigma} > 0 \qquad (5.10)$$

Temperature and humidity for the lifting cumulus air are then specified as

$$(T_c)_{LIFT} = T_k \quad , \quad (q_c)_{LIFT} = q_k \tag{5.11}$$

In case no moisture is added at that level, the next upper level is checked for moisture convergence. Once a lifting level is found, the air is lifted adiabatically and cloud base, cloud top, cloud temperature and cloud moisture are specified.

Starting at the lifting level, air is lifted dry adiabatically up to the cloud base

$$(T_c)_k = (T_c)_{k+1} \left(\frac{\sigma_k}{\sigma_{k+1}}\right)^{\kappa} \tag{5.12}$$

$$(q_c)_k = (q_c)_{k+1}$$

The cloud base is assumed to coincide with the level $k+\frac{1}{2}$ if the air was not saturated at level $k+1$ but is saturated at the next upper level k.

$$\sigma_{BASE} = \sigma_{k+\frac{1}{2}}, \text{ if } \begin{cases} (q_c)_k > q_{sat}\left[(T_c)_k\right] \text{ and} \\ (q_c)_{k+1} < q_{sat}\left[(T_c)_{k+1}\right] \end{cases} \tag{5.13}$$

Above the cloud base the air is lifted moist adiabatically. Distribution of temperature and of moisture is found by first lifting the cloud air dry adiabatically

$$(T_c)_k^{AD} = (T_c)_{k+1} \left(\frac{\sigma_k}{\sigma_{k+1}}\right)^{\kappa}$$

$$(q_c)_k^{AD} = (q_c)_{k+1} \tag{5.14}$$

and then by correcting temperature and moisture value due to the condensation of water vapour

$$(T_c)_k = (T_c)_k^{AD} + \frac{L_1}{c_p} \frac{(q_c)_k^{AD} - q_{sat}\left[(T_c)_k^{AD}\right]}{\left(1 + \frac{L_1}{c_p} \frac{dq_{sat}\left[(T_c)_k^{AD}\right]}{dT}\right)} \tag{5.15a}$$

$$(q_c)_k = (q_c)_k^{AD} - \frac{(q_c)_k^{AD} - q_{sat}\left[(T_c)_k^{AD}\right]}{\left(1 + \frac{L_1}{c_p}\frac{dq_{sat}\left[(T_c)^{AD}\right]}{dT}\right)} \qquad (5.15b)$$

The latent heat release is determined in the same way as for the large-scale condensation (see chapter 6.1). However, for reasons of accuracy the calculation (5.15) is repeated once where $(T_c)^{AD}$ $(q_c)^{AD}$ are now replaced by the results of the first calculation $(T_c)_k$, $(q_c)_k$.

Cumulus clouds are assumed to exist only if the environmental air is unstably stratified with regard to the rising cloud parcel, i.e. if

$$(T_c)_k > (T_e)_k \qquad (5.16)$$

The top of the cloud is then defined as

$$\sigma_{TOP} = \sigma_{k+\frac{1}{2}} \text{ if } \begin{cases} (T_c)_k \leq (T_e)_k \\ (T_c)_{k+1} > (T_e)_{k+1} \end{cases} \qquad (5.17)$$

The environmental air has been checked so far for conditional instability, and base and top of the cumulus clouds, which may exist in this environmental air, have been specified. It still remains to check for moisture accession. Cumulus clouds do exist only if the net moisture accession as given by (5.5) is positive. Once the checks have been done, the fractional area a of produced clouds follows from (5.3) and the heating and moistening of the environmental air is given by (5.4) and the convective rain is given by (5.6).

The application of the KUO-scheme, as described, allows convective clouds to occur one upon another.

5.3 Convective fluxes

Vertical mixing of momentum, moisture and sensible heat due to convective motion is neglected at present

$$\tau_\lambda^c = 0, \tau_\theta^c = 0, \ H^c = 0, \ R^c = 0 \qquad (5.18)$$

5.4 Horizontal diffusion due to convective motion

Horizontal diffusion of momentum, moisture and sensible heat is also neglected

$$(F_M^c)_\lambda = 0, \; (F_M^c)_\theta = 0, \; F_T^c = 0, \; F_q^c = 0 \tag{5.19}$$

5.5 Evaporation of convective rain

The evaporation of convective rain is parameterized following a proposal of KESSLER (1969), where the evaporation is assumed to be proportional to the saturation deficit $q_{sat} - q$ and to be dependent on the density of rain M_R [g/m^3]

$$E = \alpha_1 \, (q_{sat} - q) \, M_R^{\,13/20}$$

where α_1 is a constant being zero for $q > q_{sat}$.

As the density of rain M_R is not given by the model it is convenient to express it by the rain intensity P [g/(m^{2}sec)] as

$$P = M_R \, (V_o + w) \simeq M_R \, V_o$$

where V_o is the mean fall speed of rain drops which may be parameterized as (KESSLER (1969))

$$V_o = \alpha_2 \, M_R^{\,1/8} \, / \, \sqrt{\sigma}$$

Thus we have

$$E = \alpha_1 \, (q_{sat} - q) \left[\left(\frac{\sqrt{\sigma}}{\alpha_2} P \right)^{13/20} \right]^{8/9}$$

Considering that the convective rain takes place only over a fraction a_p of the grid area the evaporation rate at level k becomes

$$E^c = a_p \alpha_1 (q_{sat} - q) \left[\frac{\sqrt{\sigma}}{\alpha_2} \, \frac{P^c}{a_p} \right]^{\alpha_3} \tag{5.20}$$

where the constants have the following values (KESSLER (1969))

$\alpha_1 = 5.44 \times 10^{-4}$, $\alpha_2 = 5.09.10^{-3}$, $\alpha_3 = 0.5777$

and the precipitation rate at level $k_{+\frac{1}{2}}$ is

$$P^c_{k+\frac{1}{2}} = \sum_{\ell=1}^{k} (C^c_\ell - E^c_\ell) \frac{p_s \Delta\sigma_\ell}{g} \qquad (5.21)$$

The fractional area of the precipitating clouds a_p may be derived from the cumulus air production rate a (5.3) following KUO (1974) as

$$a_p = \frac{a}{\Delta\tau} T \qquad (5.22)$$

where T is the half life-span of precipitating cloud air (T=10 min. in the model at present).

The evaporation is calculated in the model using slightly different values for the constants

$$\alpha_1 = 6.94.10^{-4}, \ \alpha_2 = 7.35\ 10^{-3}, \ \alpha_3 = \tfrac{1}{2} \qquad (5.23)$$

The use of the square root function rather than the power function ($\alpha_3 = 0.5777$) saves computing time and simplifies the implicit calculation of the precipitation rate in the model.

6. Large-scale condensation processes

At present condensation processes are included in the model in a rather simplified way. The main simplifications are:

1 Large-scale condensation occurs wherever the moisture exceeds the saturation value.

2. Condensed water falls instantaneously out as precipitation. Clouds are therefore not explicitly considered in the model.

Besides condensation of water vapour, the evaporation of rain drops when falling through non-saturated layers, is taken into consideration.

6.1 Condensation of water vapour

Condensation of water vapour is implemented diagnostically in the usual way. Therefore temperature and humidity are first predicted neglecting condensation, giving the values T' and q'. If the air is supersaturated, these values are changed as

$$T = T' + \frac{L_1}{c_p} C^L \Delta t, \quad q = q' - C^L \Delta t \tag{6.1}$$

where the condensation rate C^L follows from

$$C^L \Delta t = \frac{q' - q_{sat}(T')}{1 + \frac{L_1}{c_p} \frac{dq_{sat}(T')}{dT}} \tag{6.2}$$

The corrections T and q as given by (6.1) and (6.2) simply follow from

$$\Delta T = (q' - q_{sat}(T)) \frac{L_1}{c_p}$$

which by assuming a linear dependence

$$q_{sat}(T) = q_{sat}(T') + \frac{dq_{sat}(T')}{dT} \Delta T$$

yields (6.2)

The saturation specific humidity is determined from

$$q_{sat} = 0.622 \frac{e_{sat}}{p} \tag{6.3}$$

where the saturation water vapour pressure e_{sat} [Pascal] is specified by the Tetens-formula (s. LOWE (1977))

$$e_{sat} = a_1 \, e^{a_2 \frac{T-a_3}{T-a_4}} \tag{6.4}$$

with

$a_1 = 610.78$

$a_2 = 17.2693882$

$a_3 = 273.16$

$a_4 = 35.86$

From (6.3) and (6.4) we get

$$\frac{dq_{sat}}{dT} = a_2(a_3-a_4) \, q_{sat}/(T-a_4)^2$$

6.2 Evaporation of large-scale rain

The evaporation of large-scale rain is calculated in the same way as for convective rain. As the cloud cover is now one (5.20) becomes

$$E^L = \alpha_1(q_{sat} - q) \quad \left(\frac{1}{\alpha_2} \sqrt{\sigma_k} P^L \right)^{\alpha_3} \tag{6.5}$$

where α_1 and α_2 again are given by (5.21). The precipitation rate at level $k+\frac{1}{2}$ is

$$P^L_{k+\frac{1}{2}} = \sum_{\ell=1}^{k} (C^L_\ell - E^L_\ell) \frac{p_s \Delta\sigma_\ell}{g} \qquad (6.6)$$

6.3 Vertical diffusion of moisture to avoid negative humidity values

In order to avoid negative moisture values which may be caused by truncation errors due to the centered finite difference scheme, the following vertical diffusion scheme is used.

If q is found negative at level k

$$q'_k < 0$$

we set

$$q_k = 0 \qquad (6.7)$$

we change the value q'_{k+1} of the next level to

$$q_{k+1} = q'_{k+1} + \frac{q'_k \quad \Delta\sigma_k}{\Delta\sigma_{k+1}} \qquad (6.8)$$

This scheme conserves the total moisture except for a fictitious upward surface flux when the moisture of the lowest level becomes negative.

7. Radiation

The radiation processes are parameterized with emphasis on those processes which are most likely to affect the atmospheric flow in short time scales. Cloud-aerosol effects are therefore given higher priority than gaseous absorption (the opposite is appropriate when long time scale phenomena as climate are considered). Thus gaseous absorption is considered as a perturbation term against other effects i.e. Raleigh scattering and scattering and absorption by clouds and aerosols.

The computations are done in three steps:

1. The radiative fluxes are calculated first without any gaseous absorption for only a few spectral intervals (2 for the solar spectrum, 3 for the terrestrial spectrum).

2. In order to determine the gaseous absorption, we compute the derivatives of these fluxes with respect to the weak line and to the strong line absorption coefficients of the three absorbing gases H_2O, CO_2 and O_3. These derivatives, together with the fluxes themselves give the encountered amounts of absorber.

3. Finally, we use the absorber amounts to compute the gaseous transmission functions on the basis of a band model theory applied to a set of spectral subintervals.
   The final fluxes result from applying these transmission functions on the fluxes as determined in step 1.

In the following sections a brief description of the radiation scheme is given, for a complete description see GELEYN (1977) and GELEYN and HOLLINGSWORTH (1979). The notations used are as follows:

μ	cosine of the zenith angle or radiation
ϕ	azimuth angle of radiation
I_ν	monochromatic intensity of the diffuse radiation
S_ν	" " " " parallel radiation
t_ν	" optical depth taken as vertical coordinate
ω_ν	" single scattering albedo
P_ν	" normalised scattering phase function
$-\mu_o$, ϕ_o	μ,ϕ for the parallel (solar) radiation
B_ν	Planck function
T	Temperature
F_u	upward diffuse flux
F_d	downward diffuse flux
F_p	parallel flux
F,C	indices for cloud-free and cloudy areas
S_o	Solar constant
AL^d	Short wave soil albedo for diffuse radiation
AL^p	" " " " " parallel "
E	Long wave soil emissivity
$\Delta u,v$	local and effective amount of gaseous absorber
τ	gaseous transmission for short wave
τ^*	" " " long wave in the isothermal case
ε^*	flux emissivity in the long wave isothermal case
σ_B	Stefan-Boltzmann constant

The quantities I,s,t,ω,P,B represent the quantities I_ν,S_ν, etc. integrated over a spectral interval.

7.1 Computations without gaseous absorption

We start from the monochromatic equations of radiative transfer

$$\mu\frac{\partial I_\nu(t_\nu,\mu,\phi)}{\partial t} = I_\nu(t_\nu,\mu,\phi) - \frac{\omega_\nu(t_\nu)}{4\pi}\Big[S_\nu(t_\nu)P_\nu(t_\nu,(\mu,\phi,-\mu_o,\phi_o)) + \int_0^{2\pi}\int_{-1}^{1} P_\nu(t_\nu,(\mu,\phi,\mu',\phi')).I_\nu(t,\mu',\phi')d\mu' d\phi'\Big] - (1-\omega_\nu(t_\nu))\,B_\nu\,(T(t_\nu))$$

$$-\mu_o\frac{dS_\nu(t_\nu)}{dt_\nu} = S_\nu(t_\nu) \qquad (7.1)$$

If we use the two stream assumption and the Eddington approximation we can transform these equations to a set of three equations for fluxes. These equations, extended to non-monochromatic problems with spectrally averaged optical properties, are:

$$\frac{dF_u(t)}{dt} = \alpha_1(F_u(t)-\Pi B(t))-\alpha_2(F_d(t)-\Pi B(t))-\alpha_3\frac{F_p(t)}{\mu_o}$$

$$\frac{dF_d(t)}{dt} = \alpha_2(F_u(t)-\Pi B(t))-\alpha_1(F_d(t)-\Pi B(t))+\alpha_4\frac{F_p(t)}{\mu_o} \qquad (7.2)$$

$$\frac{dF_p(t)}{dt} = -\frac{F_p(t)}{\mu_o}$$

where

$$\alpha_1 = 2(1-\omega(t)A_1),\quad \alpha_2 = 2\omega(t)(1-A_1)$$

$$\alpha_3 = \omega(t)A_3(\mu_o),\quad \alpha_4 = \omega(t)\,(1-A_3(\mu_o)) \qquad (7.3)$$

and

$$A_3(\mu_o) = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{1} P(t,(\mu,\phi,-\mu_o,\phi_o))d\mu d\phi$$

$$A_1 = \frac{1}{8\pi^2}\int_0^{2\pi}\int_0^{1}\int_0^{2\pi}\int_0^{1} P(t,(\mu,\phi,\mu',\phi'))d\mu d\phi d\mu' d\phi' \qquad (7.4)$$

If we further assume that each layer between the levels at which we want to compute the fluxes is homogeneous with respect to its optical properties so that $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ can be considered as constants and that $B(t)$ changes linearly between the levels, we can integrate these equations and get:

a) for the short wave region ($B = 0$)

$$\begin{bmatrix} F_p(b) \\ F_d(b) \\ F_u(t) \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0 \\ a_2 & a_4 & a_5 \\ a_3 & a_5 & a_4 \end{bmatrix} \cdot \begin{bmatrix} F_p(t) \\ F_d(t) \\ F_u(b) \end{bmatrix} \qquad (7.5)$$

where b refers to the bottom of a layer and t refers to the top.

b) for the long wave region ($F_p = 0$)

$$\begin{bmatrix} \tilde{F}_d(b) \\ \tilde{F}_u(t) \end{bmatrix} = \begin{bmatrix} b_1 & b_2 \\ b_2 & b_1 \end{bmatrix} \cdot \begin{bmatrix} \tilde{F}_d(t) \\ \tilde{F}_u(b) \end{bmatrix} + \begin{bmatrix} b_3 & -b_3 \\ -b_3 & b_3 \end{bmatrix} \cdot \begin{bmatrix} \pi B(b) \\ \pi B(t) \end{bmatrix} \qquad (7.6)$$

with $\tilde{F} = \pi B - F$

Formally we have

$$\begin{aligned} a_i &= a_i(\Delta t, \omega, A_1, \mu_o, A_3(\mu_o)) \\ b_i &= b_i(\Delta t, \omega, A_1) \end{aligned} \qquad (7.7)$$

The functions are quite complicated but fully analytical and may be found in GELEYN (1977) or GELEYN and HOLLINGSWORTH (1979). In each layer we compute the coefficients a_i and b_i for cloudy and cloud-free areas.

At the transition between two layers (Fig. 7.1) we take into account the geometry of clouds in the following way.

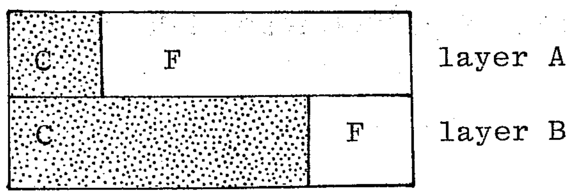

Fig. 7.1 Schematic diagram of two adjacent layers of different cloud cover

We compute the fluxes which enter the cloudy and the cloud free area of a layer as a linear combination of the fluxes which leave the cloudy part and the cloud-free part of the other layer assuming maximal possible overlapping of the cloudy parts

$$\begin{bmatrix} F^F_{p,d}(B) \\ F^C_{p,d}(B) \end{bmatrix} = \begin{bmatrix} \frac{1-\max(C(A),C(B))}{1-C(A)} & \frac{C(A)-\min(C(A),C(B))}{C(A)} \\ \frac{\max(C(A),C(B))-C(A)}{1-C(A)} & \frac{\min(C(A),C(B))}{C(A)} \end{bmatrix} \begin{bmatrix} F^F_{p,d}(A) \\ F^C_{p,d}(A) \end{bmatrix} \qquad (7.8)$$

$$\begin{bmatrix} F^F_{u}(A) \\ F^C_{u}(A) \end{bmatrix} = \begin{bmatrix} \frac{1-\max(C(A),C(B))}{1-C(B)} & \frac{C(B)-\min(C(A),C(B))}{C(B)} \\ \frac{\max(C(A),C(B))-C(B)}{1-C(B)} & \frac{\min(C(A),C(B))}{C(B)} \end{bmatrix} \begin{bmatrix} F^F_{u}(B) \\ F^C_{u}(B) \end{bmatrix} \qquad (7.9)$$

In order to calculate the fluxes throughout the atmosphere we need boundary conditions. These are:

a) For the short wave region

$$F_p = \mu_o S_o, \quad F_d = 0 \quad \text{at } \sigma = 0$$
$$F_u = AL^d . F_d + AL^p(\mu_o) . F_p \quad \text{at } \sigma = 1 \qquad (7.10)$$

b) for the long wave region

$$\tilde{F}_d = \Pi B \quad \text{at } \sigma = 0$$
$$\tilde{F}_u = (1-E)\, \tilde{F}_d \quad \text{at } \sigma = 1 \qquad (7.11)$$

Thus the radiative fluxes follow from the equations (7.5), (7.8), (7.9), (7.10) for the short wave spectrum and from the equations (7.5), (7.8), (7.9), (7.11) for the long wave spectrum. These equations form a linear system

$$\mathbb{A} . \mathbb{F} = \mathbb{S} \qquad (7.12)$$

where $\mathbb{A}$ is a matrix of coefficients like a_i, b_i,

$\frac{\min(C(A),C(B))}{C(A)}$, AL,$\varepsilon$ etc.,

$\mathbb{F}$ is the vector of fluxes $F^{C,F}_{p,d,u}$ at all levels and $\mathbb{S}$ is the vector containing the sources, i.e. $\mu_o \cdot S_o$ in the short wave case and πB in the long wave case plus some temperature gradient terms. (7.12) is solved by the Gaussian elimination back-substitution procedure.

For the long wave region the system (7.12) is solved for two cases:

1. for an isothermal state and
2. for the actual temperature distribution.

The radiative fluxes of both cases are needed later to determine the effect of gaseous absorption.

7.2 Evaluation of the amounts of gaseous absorber

The results obtained so far exclude gaseous absorption but include absorption by aerosols and clouds and also scattering by aerosols, clouds and gases. Gaseous absorption could not have been included in the first calculation because the absorption is by no means grey but has a very wide range of intensities . As mentioned before we treat gaseous absorption instead as a perturbation term with regard to the already evaluated fluxes. Therefore we first evaluate for each spectral interval the average amount of a gaseous absorber v encountered by the photons contributing to a flux. We do this by differentiating the flux with respect to the corresponding gaseous absorption coefficient k.

The principle behind this method can be found in RODGERS (1977, pp.56-60). To illustrate the method we take the simple example of a unique photon path. The dependence between the flux F with gaseous absorption and the flux F_o without gaseous absorption is for an infinitely small absorption for the short wave case

$$F = F_o e^{-kv} \tag{7.13a}$$

and for the long wave isothermal case

$$\tilde{F}^* = \tilde{F}_o^* e^{-kv} \tag{7.13b}$$

(* indicates isothermal calculated values)

Therefore we see that the absorber amount in the short wave case follows from

$$v = -\frac{1}{F}\frac{\partial F}{\partial k} \tag{7.14a}$$

and in the long wave isothermal case follows from

$$v = -\frac{1}{\tilde{F}^*}\frac{\partial \tilde{F}^*}{\partial k} \tag{7.14b}$$

In the model we determine $\mathbb{F}'$ from

$$\mathbb{A}\,\mathbb{F}' = \mathbb{S}' - \mathbb{A}'\mathbb{F} \quad . \tag{7.15}$$

which follows from (7.12) by taking the derivative with respect to the absorption coefficient k.

(7.15) is solved for $\mathbb{F}'$ in the same way as (7.12) is solved for $\mathbb{F}$. The boundary conditions are included in the matrix equation (7.15). The source term is now given by $-\mathbb{A}'\mathbb{F}$ since $\mathbb{S}'$ is zero. $\mathbb{A}'$ includes terms like

$$\frac{\partial A_i}{\partial k_j} = \frac{\partial A_i}{\partial x}\,\frac{\partial x}{\partial \Delta t}\,\frac{\partial \Delta t}{\partial k_j} \tag{7.16}$$

where x is a coefficient being either a_ℓ or b_ℓ. $\frac{\partial A_i}{\partial x}$ is determined analytically, $\frac{\partial x}{\partial \Delta t}$ is calculated from (7.7) under the condition that $\omega\Delta t$ is constant which means keeping constant the scattering optical thickness. $\frac{\partial \Delta t}{\partial k_j}$ is the actual amount of absorbing gaseous quantity within one layer, which is known from the input data u:

$$\frac{\partial \Delta t}{\partial k_j} = \Delta u_j \tag{7.17}$$

The calculations are made separately for weak line absorption and for strong line absorption. The input data are therefore the unreduced amounts in the first case and the reduced amounts (multiplied by $p/\sqrt{T}$) in the second case.

Once we have determined F' for one absorber we can compute the mean encountered amounts of gaseous absorber which, after recombination of the fluxes of the cloudy and of the cloud-free parts become

$$v_{p,d,u} = - \frac{F'^{F}_{p,d,u} + F'^{C}_{p,d,u}}{F^{F}_{p,d,u} + F^{C}_{p.d.u}} \qquad (7.18)$$

in the short wave case

and

$$v_{d,u} = - \frac{\tilde{F}'^{*F}_{d,u} + \tilde{F}'^{*C}_{d,u}}{\tilde{F}^{F*}_{d,u} + \tilde{F}^{C*}_{d,u}} \qquad (7.19)$$

in the long wave case.

It should be noticed here that the reduced amounts of absorber are computed in a slightly different way from the unreduced ones. The use of the Eddington approximation assumes an angle magnification factor of 2, which is already included in the definitions of α_1 and α_2 in (7.3). We stick to this value for non gaseous phenomena and for the calculation with unreduced amounts of gaseous absorber, which in the Curtis-Godson approximation are associated with weak absorption. However, for strong absorption associated with reduced amounts we use the factor 25/16, which is the limit case of infinite strong line absorption in the transmission function described by (7.20) below.

Thus, in the short wave case the coefficients $\frac{\partial x_i}{\partial t}\Delta u_j$ in (7.16) are multiplied by 25/32 for i = 4 and i = 5 (i.e. for diffuse radiative fluxes) and remain unchanged for i = 1 (i.e. for parallel radiative fluxes which have, of course, no angle magnification) and are multiplied by $\gamma = (1/\mu_o + 2.\ \frac{25}{32})\ /\ (\frac{1}{\mu_o} + 2)$ for i = 2 and i = 3 (i.e. for a mixed case in which we assume a single scattering in the middle of the layer). In the long wave case the coefficients $\frac{\partial b_i}{\partial x}\Delta u_j$ are multiplied by 25/32 for every i (only diffuse radiative fluxes).

7.3 Final computation of radiative fluxes

In order to take into account the effect of spectral over-lapping of gaseous absorption, the big spectral intervals are divided into several sub-intervals. In every one of these intervals transmission functions are calculated by

$$\tau_i^\ell = \frac{1}{1 + \dfrac{C_1^\ell v_i}{\sqrt{1+C_2^\ell \dfrac{v_i^2}{v_i^r}}} + C_3^\ell v_i^r} \tag{7.20}$$

where v_i, v_i^r are the unreduced and reduced encountered gaseous amounts calculated for the gas i (i=1,2,3 for H_2O, CO_2 and O_3) at spectral interval ℓ. The coefficients C_1^ℓ, C_2^ℓ, C_3^ℓ depend on temperature and are also different for the different subintervals. The effect of overlapping of subintervals by the different gases is simply considered by taking the product of the corresponding τ_i^ℓ. The total transmission function follows then from a linear combination of the transmission functions over all subintervals.

The final radiative fluxes are computed as follows.

In the short wave case the final fluxes are obtained from

$$F_{p,d,u} = F^o_{p,d,u} \cdot \tau_{p,d,u} \tag{7.21}$$

where $F^o_{p,d,u}$ are the fluxes without gaseous absorption and $\tau_{p,d,u}$ is the total gaseous transmission.

In the long wave the final fluxes follow from

$$F_{d,u} = \Pi B + (F^o_{d,u} - F^o_{d,u}/\overset{*}{\varepsilon}_{d,u})\ \tau_{d,u} + (F^o_{d,u}/\overset{*}{\varepsilon}_{d,u} - \Pi B)\frac{\tau_{d,u}-1}{\ln\tau_{d,u}} \tag{7.22}$$

where $\overset{*}{\varepsilon}$ is the emissivity corresponding to the fluxes computed in the isothermal case and ΠB the local black body flux. Here we have again assumed that the black body fluxes vary linearly with the optical depth.

7.4 The radiation-dynamics interface

The radiative fluxes will be recalculated every third or fourth hour to simulate the diurnal cycle. At present, however, the diurnal cycle is not considered in the model and the fluxes are therefore recalculated only twice a day. The zenith angle is therefore replaced by its mean value

$$\cos\bar{\xi} = \frac{1}{24} \int_d \cos\xi \, dt \qquad (7.23)$$

7.4.1 The input information

The input of the remaining quantities does not depend on the time frequency of the radiation calculation. Input values taken from the model are always instantaneous values. The input values are either climatologically preassigned or are taken from the model or are of mixed type like the surface albedo.

Carbon dioxide has a constant mixing ratio over the whole globe

$$r_{CO_2} = 5.10^{-4} \qquad (7.24)$$

Aerosol varies with height only assuming the same vertical distribution and the same optical properties everywhere. The actual values have been taken from measurement but may be slightly changed at a later stage whilst tuning the model.

Ozone varies with height, latitude, longitude and with season. The vertical distribution is characterized by two parameters a and b. The mixing ratio integrated between $p=0$ and $p=p_1$ is is given by

$$\int_0^{p_1} r_{O_3} \, dp = \frac{a \, p_1^{3/2}}{p_1^{3/2} + b^{3/2}} \qquad (7.25)$$

where a and b are determined from observational data through a latitude longitude dependence in terms of spherical harmonics and through a seasonal dependence in terms of trigonometric functions.

The geographical distribution of the surface albedo is climatologically specified but depends on the distribution of the snow cover predicted by the model.

For the long wave soil emissivity E we assume the value

$$E = 0.996 \tag{7.26}$$

to ensure stability of our flux calculation which would fail for E = 1.

The remaining input values, temperature, humidity and cloud cover, are taken from the model itself.

The surface temperature is predicted by the model (see chapter 9), whereas the temperature at infinity is at present extrapolated linearly with respect to σ all over the globe, but further investigations will be necessary to find the most appropriate values. The input temperature values which must be specified in the free atmosphere at the intermediate levels $k+\frac{1}{2}$ (Fig. 3.1), are calculated by linear interpolation from the model's temperature values at the k-level.

The water vapour mixing ratio and the saturation mixing ratio needed at k-levels are taken directly from the q- and T- values of the model.

Cloud cover and cloud liquid water content are specified at present as follows. The cloudiness is given by

$$C = \left(\max\left(\frac{U - \sigma}{1 - \sigma}, 0\right)\right)^2 \tag{7.27}$$

where U is the relative humidity and σ is the vertical coordinate. The liquid water content is assumed zero at present. However, the extinction due to liquid water is considered in the model by aerosol adsorption of water vapour according to the relative humidity.

7.4.2 The radiation output used in the model

As the radiation calculation is not made at every time step but only a few times per day, the results from the radiation calculation are used until the radiation is recalculated.

Thus, the radiative heating rates in the free atmosphere are simply kept constant between two radiation time steps.

However, the surface radiative fluxes which are used to predict the surface temperature over the continents cannot be treated as simply as that, since the surface radiative flux depends largely on the surface temperature and therefore cannot be kept constant when the surface temperature is allowed to change during the forecast.

We therefore adopted the following procedure.

The net downward solar radiative flux is kept constant.

The long wave radiative flux, which is originally given from the downward radiative flux and the upward radiative flux $\sigma_B T_s^4$ as

$$F_s^T = (F_s^T)^{\downarrow} - \sigma_B T_s^4 \qquad (7.28)$$

is assumed to be proportional to T_s^4 by

$$F_s^T = -(1-\alpha)\sigma_B T_s^4 \qquad (7.29)$$

Rather than keeping the long wave radiation fluxes constant we fix the coefficient $(1-\alpha)$ which follows from (7.28) and (7.29).

The radiation-dynamics interface described here is considered as a preliminary version. It will probably be changed in the near future in several points (input of cloudiness depending on more model parameters, extension of the surface flux treatment to the fluxes in the free atmosphere and inclusion of a variation of F^S with the solar zenith angle between two radiation time-steps).

8. Horizontal diffusion of momentum, heat and moisture

The horizontal diffusion scheme presently used in the experiments is a non-linear fourth order scheme (HOLLINGSWORTH and GELEYN, 1978). We have also coded two second order schemes (1. the scheme proposed by CORBY et al. (1972) and 2. a slightly changed version of the scheme designed by SMAGORINSKY (1963)).

The fourth order diffusion scheme is defined by

$$F_X = -k \sqrt{(\overline{\nabla^2 Z})^2 + (\overline{\nabla^2 D})^2}\ \nabla^2(\nabla^2 x) \tag{8.1}$$

where x represents u,v,T and q and F_x stands for $(F_M^T)_\lambda$, $(F_M^T)_\theta$, F_T^T and F_q^T in (2.1) to (2.3), respectively. Z is the relative vorticity and D is the divergence of the horizontal wind. The spatial averaging of $\nabla^2 Z$ and $\nabla^2 D$ under the square root function differ slightly for u,v,T and q because of the staggered grid used. The operator ∇^2 is

$$\nabla^2(\ ) = a^2\cos^2\theta(\Delta\lambda)^2 \frac{1}{a^2\cos^2\theta}\ \delta_\lambda(\delta_\lambda(\ )) + a^2\ \Delta\theta^2 \frac{1}{a^2\cos^2\theta}\ \cos\theta\ \delta_\theta(\cos\theta\,\delta_\theta(\ )) \tag{8.2}$$

using the conventional notations. The present value of the constant parameter k is k = 0.01 (8.3)

9. Surface values

To compute the heat fluxes and the moisture fluxes, the temperature and the moisture must be known at the surface.

9.1 Surface temperatures

9.1.1 Surface temperature over sea

Over the sea the surface temperatures are pre-assigned and are kept constant during the forecast period.

$$T_{s,t} = T_{s,t=0} \tag{9.1}$$

This assumption is made for open water as well as for ice covered water.

9.1.2 Surface temperature over land

Over land a thin soil layer of specified heat capacity is defined which exchanges heat and moisture with the atmosphere and with the deep soil. The equation applied for temperature is:

$$C_s \frac{\partial T_s}{\partial t} - F_s^S + (1-\alpha) . \sigma_B T_S^4 - H_s^T - L_1 R_s^T + L_2 \rho_w M_{Sn} + B_T = 0 \tag{9.2}$$

where C_s is the thermal capacity per unit horizontal area of the soil, σ_B is the Stefan-Boltzmann constant, F_s^S is the net downward solar radiation and M_{Sn} is the snow-melt. α is defined in 7.4.2. The heat conduction in the soil B_T is parameterized by

$$B_T = \lambda_T \frac{T_S - T_D}{\Delta Z} \tag{9.3}$$

where λ_T is the thermal conductivity and T_D is a fixed temperature in the soil at depth ΔZ. Melting of snow M_{Sn} (unit is m H_2O/sec) is considered whenever snow is present and the surface temperature exceeds the temperature of ice-melting T_o. In that case the surface temperature is reduced to T_o and the supplied energy is used to melt the snow instead.

$$T_s = T_o$$

$$M_{Sn} = \frac{1}{L_2 \rho_w} (F_s^S - (1-\alpha) . \sigma_B T_S^4 + H_s^T + L_1 R_s^T - B_T) \tag{9.4}$$

The constants C_s, λ_T and ΔZ must be specified depending on whether the diurnal cycle is included or not. The values used are based on the proposal of DEARDORFF (1977).

9.2 Surface moisture

9.2.1 Sea Surface moisture

Over the sea the surface humidity is equal to the saturation value at the given surface temperature and the wetness is 1.

$$q_s = q_{sat}\,(T_s)\ ,\ \mathrm{WET} = 1 \qquad (9.5)$$

9.2.2 Land surface hydrology

over land the surface hydrology implemented in the model is similar to that used in general circulation models (GFDL, NCAR).

Soil moisture and snow-cover are predicted considering precipitation evaporation, snow-melt, runoff and moisture diffusion into the ground (Fig. 9.1)

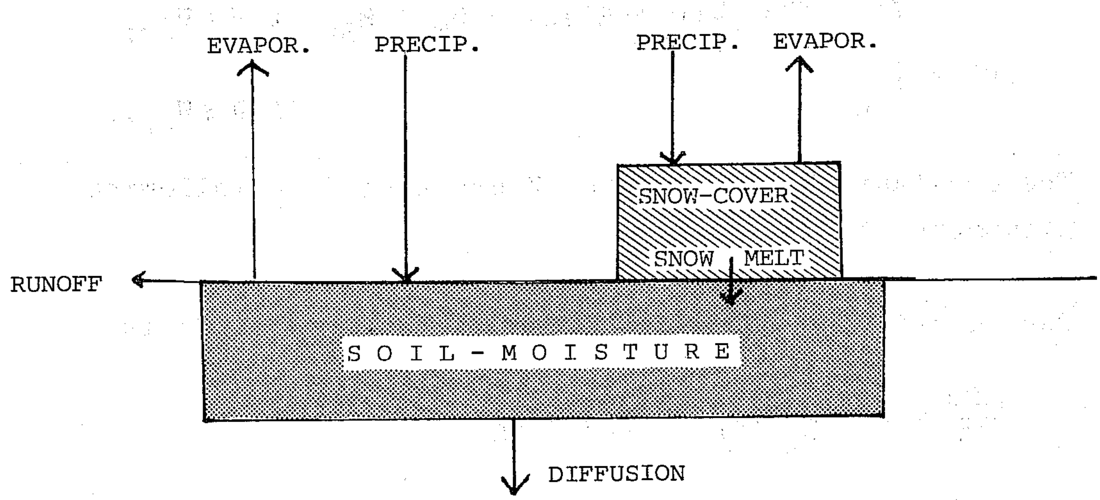

Fig. 9.1 Schematic diagram of surface hydrology

The equation for soil moisture W (unit is mH_2O) is

$$\frac{\partial W}{\partial t} = (1-\delta^{Sn})\,\frac{1}{\rho_w}\,(P_s+(R^T)_s) + B_W + M_{Sn} - RO \qquad (9.6)$$

where P_s is the total precipitation due to large-scale and convective motion

$$P_s = \frac{P_*}{g} \sum_{k=1}^{K} (C_k^L - E_k^L + C_k^c - E_k^c)\Delta\sigma_k \tag{9.7}$$

with C_k^L. E_k^L, C_k^c, E_k^c as given by (6.2), (6.5), (5.6) and (5.20). Precipitation is either rain or snow, depending on the surface temperature.

$$\delta^{Sn} = \begin{cases} 0 & \text{if } T > T_o \\ 1 & \text{if } T < T_o \end{cases} \tag{9.8}$$

B_W represents the moisture flux due to diffusion in the soil and is parameterized by

$$B_W = \lambda_W \frac{W - W_D}{\Delta Z} \tag{9.9}$$

where W_D is a fixed value of moisture in the soil at depth ΔZ.

The run-off takes place whenever the soil moisture exceeds a given critical value

$$RO = \begin{cases} (1-\delta^{Sn}) \frac{1}{\rho_W}(P_s+(R^T)_S) + B_W + M_{Sn} & \text{if } W > W_{crit} \\ 0 & \text{if } W \leq W_{crit} \end{cases} \tag{9.10}$$

The constants W_{crit}, λ_W and ΔZ are specified following DEARDORFF (1977).

The equation of the snow-height Sn (unit is mH_2O) is

$$\frac{\partial Sn}{\partial t} = \delta^{Sn} \frac{1}{\rho_W} (P_s+(R^T)_s) - M_{Sn}$$

with δ^{Sn}, P_s, M_{Sn} and $(R^T)_s$ as defined above.

The surface humidity values, which are needed for the surface flux calculations (4.3) and (4.4), are specified over land as follows. The surface humidity is

$$q_s = q_{sat}(T_s) \tag{9.12}$$

and the wetness of the soil which is defined as

$$\mathrm{WET} = \mathrm{MIN}\left\{ \left(\frac{Sn}{Sn_{crit}} + \left(1 - \frac{Sn}{Sn_{crit}}\right) \cdot \frac{W}{W_{crit}} \right) \quad 1 \right\} \tag{9.13}$$

This formula guarantees that the wetness changes smoothly with snow cover.

9.3 Specification of surface temperature and surface moisture in the Grid-Point Model

In grid point models it seems generally advantageous to smooth the fields by some kind of horizontal diffusion in order to prevent sharp horizontal gradients which may be caused by different processes. As the surface fluxes of sensible heat and of latent heat may also cause sharp gradients due to land-sea contrasts, we decided to use smoothed values of temperature and humidity rather than the original values in the flux calculations (4.3) and (4.4).

Thus, instead of using T_s as given by (9.1 and (9.2) we use

$$\overline{(T_s)}_{i,j} = (T_s)_{i,j} + \alpha\left[(T_s)_{i-1,j} + (T_s)_{i+1,j} + (T_s)_{i,j+1} + (T_s)_{i,j-1} - 4(T_s)_{i,j}\right] \tag{9.14}$$

with $\alpha = 0.125$ at present.

In the same way q_{sat} in (9.5) and in (9.12) is replaced by

$$(\overline{q_{sat}})_{ij} = q_{sat}\left[\overline{(T_s)}_{ij}\right] \tag{9.15}$$

10. References

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