# SOME ASPECTS OF ATMOSPHERIC PREDICTABILITY

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## 1. PREDICTABILITY, DETERMINISM, STABILITY, AND PERIODICITY

### 1.1 Introduction

Our central concern in this set of lectures will be the predictability of the atmosphere. Many of our presently accepted ideas on this subject have been gained from studies of other systems, including complex mathematical models of the atmosphere and simple sets of mathematical equations, and we shall consider the predictability of these systems also.

By the predictability of a system we shall mean the degree of accuracy with which it is possible to predict the state of the system in the near and also the distant future. The predictability which we shall consider will be qualified rather than intrinsic, in that we shall assume from the outset that the predictions will be based upon less than perfect knowledge of the system's present and past states. This qualification would not be needed if we were interested only in mathematical models, but, for real physical systems, anything else would be unrealistic. For example, our present-day instruments may be capable of measuring winds to the nearest meter per second, but we cannot imagine that instruments in the near future will routinely measure them to the nearest centimeter per second. Likewise, our present global observing network may resolve the structure of the principal cyclones and anticyclones, but we cannot visualize observations in the near future which will regularly reveal the location, size, and internal structure of every cumulus cloud.

We wish to make our treatment of qualified predictability rather general. In this lecture we shall introduce the concepts of multiple time series and processes, and determinism and randomness. We shall then show that we lose virtually no generality by considering only the predictability of deterministic time series which are realizations of processes defined by mathematical equations.

### 1.2 Prediction and its accuracy

If we are to consider the accuracy with which it is possible to predict, we must first agree on what is meant by prediction. Consider the following hypothetical predictions for tomorrow's temperature at a particular city, say London.

1. Tomorrow will be warm (i.e., warmer than normal).
2. Tomorrow will be very warm.
3. Tomorrow's maximum temperature will be 27°C.
4. The probability that tomorrow will be warm is 80 per cent.

If we disregard the occasional cases when tomorrow will be warm on one side of the city and cool on the other, for example, or warm in the morning and cool in the afternoon, it is evident that the first prediction will be either right or wrong. It is equally evident that someone who knows nothing about weather forecasting will, if he simply predicts that the following day will be warm or cool, be right half of the time. One therefore cannot demonstrate skill in forecasting simply by making a few correct forecasts and then quitting; he must show that in the long run he is right more often than wrong.

If we agree on how warm is "very warm", the second prediction is also either right or wrong. However, since a forecaster can forecast "very warm" when it turns out to be only marginally warm, or vice versa, the second hypothetical prediction will be right less often than the first, if made by a forecaster of equal and less than perfect skill. It follows that any absolute measure of skill must take into account the manner in which the prediction is worded.

If we agree that the "maximum temperature" means the officially recorded maximum, rounded to the nearest whole degree, the third prediction also is either right or wrong, and, unless the forecaster is very skillful, it is probably wrong. Yet in an obvious sense the forecaster who predicts 27°C when the temperature turns out to be 26°C has made a better forecast than the forecaster who predicts 21°C or 31°C. For forecasts of this sort an obvious measure of accuracy is the magnitude of the error, or some function thereof. To serve as a measure of skill, this quantity should be averaged over many forecasts, since one might always guess well on a single occasion. The mean-square error, and also its square root, have become rather widely used measures of the accuracy of a sequence of numerically expressed forecasts, and we shall assume that they are adequate measures for our purposes, provided that the forecasts are uniformly expressed. If we wish to compare a forecast worded in one manner with one worded in another, we must also decide how large a mean-square error for one type of forecast is equivalent to a given mean-square error for the other.

The first prediction could also be made numerical, perhaps by letting "1" represent "warmer than normal", while "0" represents "cooler than normal". The forecaster who predicts 1 or 0 then makes a squared error of one unit whenever he is wrong, and zero whenever he is right, and his mean-square error is the fraction of the time that he is wrong.

The forecaster will soon recognize that he can obtain a smaller mean-square error by frequently predicting a fraction, even though the outcome must be 1 or 0. For example, if he knows nothing about forecasting, he can reduce his mean-square error from 0.5 to 0.25 by predicting 0.5 all of the time. If he recognizes a situation which will be followed by warm weather 80 per cent of the time, he can, whenever this situation occurs, predict 0.8, thus making a squared error of 0.04 in four fifths of these cases, and 0.64 in one fifth, and hence obtaining a mean-square error of 0.16 for these cases, while he would have obtained 0.2 if he had predicted 1.0 on each occasion. He should not be informed at this point that predicting a fraction is illegal, for he is actually making the fourth hypothetical prediction, i.e., stating the probability of warmer weather, which in some instances will be the most useful statement which he could make.

The point of this discussion, as far as predictability is concerned, is that whether one is forecasting categories or numerical values, and most likely outcomes or probabilities, the forecast may be expressed in numerical terms, and the square of the difference between the predicted number and the outcome will serve as a measure of the accuracy of a given forecast. We shall therefore lose no generality in assuming in these lectures that both the forecast and the outcome are always numbers. The skill of a forecaster, or of a forecasting system, in predicting a particular quantity on many occasions can then be measured in terms of the mean-square or root-mean-square error. Since the usual weather forecast involves predicting many weather elements, the mean-square errors in predicting the different elements must be weighted in some manner before being averaged. We must decide, for example, how many meters per second of wind are equivalent to one degree of temperature. In inquiring as to predictability we shall then be asking how small a suitably weighted mean-square error can be made.

### 1.3 Time series and processes

The sequences of weather elements in whose predictability we are interested are examples of time series. By a simple time series we shall mean merely a function of time. It may be a function of continuously varying time, for example, the temperature at London, or it may be defined only for discrete values of time, for example, the daily maximum temperature at London. It may be defined in physical terms, or as a mathematical function.

A multiple time series is an array of simple time series. For the concept to be useful, the separate time series should be physically related. For example, they might be temperatures at neighboring cities or temperature and wind speed at the same city, or they might form a particular solution of a system of several coupled ordinary differential equations.

A process (simple or multiple) is an ensemble of time series; the separate members of the ensemble are realizations of the process. For the concept to be meaningful, the separate realizations must be similarly produced. For example, they may be separate time-dependent solutions of a single system of differential equations which possesses many solutions.

The global weather may be regarded as a single realization of a multiple process. We can visualize the process by imagining an ensemble of planets, each identical to the earth, and each possessing an atmosphere governed by the same laws, but with different global weather distributions at some key time. In view of our earlier discussion, we may restrict our attention to the predictability of time series, and we may assume that these series are realizations of readily definable processes.

## 1.4 Determinism and randomness

A process is deterministic if the present state of a realization completely determines the state at any specified future time, i.e., if two realizations which are identical at one time must be identical at all future times. A process is random, or stochastic, if the present state of a realization merely determines a probability distribution of states at a specified future time; two realizations may therefore be identical at present and differ in the future. If all possible present states determine the same future probability distribution at all future times, the process is completely random; in this case knowledge of the present state of a realizations tells us nothing about its future. So-called tables of random numbers are intended to be realizations of completely random processes. The weather is presumably a realization of a random process, since its behavior depends to a certain extent on human activity, which we hesitate to call deterministic. Most mathematical models of the atmosphere define deterministic processes.

Sometimes a process is considered to be deterministic if the present and past states of a realization determine the future, even though the present state alone may not completely determine the future. A process is then considered random if it is not deterministic in this sense. A process which is deterministic in this sense may be random in the former sense; an example of such a process would be one defined by a single second-order difference equation. We have chosen the former definition simply as a matter of convenience.

It might appear at this point that realizations of a deterministic process would be perfectly predictable, while those of a random process would be less than perfectly predictable, their predictability at any future time depending upon the variance of the probability distribution determined by the present state of the realization. From our point of view this is actually not the case, because of the qualification which we have imposed upon prediction, namely, that our knowledge of the present state of the realization must be imperfect.

To see how this qualification affects predictability, we note first that a deterministic process may be converted into a random process in either of two ways. First, we may round off the numerical values, thereby no longer distinguishing between states which are nearly identical. In all likelihood the set of states which are rounded off to a single state will be followed by a set of states which will be rounded off to two or more distinct states; the new process consisting of rounded-off numbers will therefore be random. Second, we may reduce the multiplicity of the process by discarding certain series from the array of series which forms each realization. For example, we may replace the weather, as it would behave if human activity and other random influences were somehow eliminated, by the weather, as revealed by actual measurements spaced a few hundred kilometers apart. These measurements resolve the cyclones and anticyclones, but omit most of the cumulus clouds, which exert considerable influence on the cyclones and anticyclones. The process with reduced multiplicity would therefore be random even if the complete weather behavior were deterministic. We should note that a random process produced in this manner is sometimes deterministic according to the second definition of determinism.

At this point we may inquire whether the converse of this result is true, i.e., given any random process, does there exist a deterministic process which may be converted into this random process by rounding off the numerical values, or by reducing the multiplicity of the time series? We suspect that it would be difficult to prove that the answer to this question is affirmative. Nevertheless, we feel that for practical purposes the answer is "yes", i.e., that any random process which we are likely to encounter behaves similarly to one which may be derived from a deterministic process. We shall therefore confine our attention to the predictability of processes which are deterministic, and therefore representable as solutions of deterministic systems of equations, with the understanding that in making our predictions we cannot determine the initial state with perfect accuracy or completeness. We can then state that the qualified predictability of the deterministic process depends upon the variance of the future probability distributions determined by present states in a derived random process.

## 1.5 Stability and instability

To pursue this matter further we require some more definitions. For our purposes we may define the difference between two realizations of the same process, at a given time, as simply the absolute value of the difference of the two numerical values, if the process is simple, or as the square root of a suitably weighted average of the squares of the differences, if the process is multiple. We shall then say that a realization of a deterministic process is stable if the difference between this realization and any other realization remains small throughout the future, whenever the difference is sufficiently small at the present. Otherwise the realization is unstable; in this case one can find realizations whose distance from the given realization is arbitrarily small at present, but exceeds some preassigned value in the ultimate future.

It therefore appears that the deciding factor in predictability is not determinism vs. randomness, but stability vs. instability. If a realization is unstable, other realizations which are close enough to it at present to be mistaken for it will eventually be found far away from it. Hence even a forecast made by perfect extrapolation, which might follow any one of these realizations, will in most instances eventually depart from the particular realization which is the outcome. If, on the other hand, the realization is stable, a good although not perfect forecast into the indefinite future may be made by making the initial error small enough.

It is to be emphasized that these conclusions are not restricted to forecasts which are made by attempting to integrate the governing equations, or some approximation to them. If there are many possible realizations, each of which eventually departs from each of the others, no forecast, whether it be dynamical or empirical, and objective or subjective, can approximate more than one of these realizations, and if a forecast should follow the correct realization it would be a mere coincidence.

In the following lecture we shall be concerned with the rate at which small initial errors amplify, as time progresses. This rate has been most frequently estimated from numerical studies with mathematical models of varying degrees of complexity. In the mean time, we can often determine whether instability is present or not by examining the past history of the realization.

We first assume that the system with which we are dealing is compact, i.e., that the number of different states, each removed from each of the others by a pre-specified amount, is limited, for any realization. This is certainly true when the system is any one of the commonly used mathematical models of the atmosphere; it is probably true when the system is the atmosphere itself, and reasonable weighting functions are chosen in defining the difference between states. It then follows that any realization will eventually acquire a state which is arbitrarily close to one which it has previously acquired. If the realization is stable, history will then approximately repeat itself, and the system will be observed to vary periodically. Equivalently, if it is observed to vary aperiodically, even though occasional temporary near-repetitions occur, the realization must be unstable (cf. Lorenz 1963, Charney *et al.*, 1966).

Observations of the atmosphere indicate that it is not a periodically varying system. It does have periodic components, notably the annual and diurnal cycles and their overtones. However, when these and any other suspected periodic components are subtracted from the total signal, a large residual is still present. The periodic components are predictable at arbitrarily long range, by mere repetition. The residual is what the weather forecaster usually wishes to predict, and its lack of periodicity indicates that it is unpredictable in the sufficiently distant future.

## 2. THE GROWTH OF ERRORS: WHY WE CAN'T PREDICT

### 2.1 General considerations

Let the equations governing a multiple deterministic process be

$$d\underline{X}/dt = \underline{F}(\underline{X}) \quad , \qquad (1)$$

where $t$ is time, and the components of $\underline{X}$ are $X_1, ---, X_N$ . Let $\underline{X}^*$ be a realization of the process. Let a second realization be given by

$$\underline{X} = \underline{X}^* + \underline{x} \quad , \qquad (2)$$

and assume that the variables $X_i$ have been normalized so that a suitable measure of the difference between the realizations is

$$D = (\underline{x} \cdot \underline{x})^{1/2} . \qquad (3)$$

Consider a case where at some "initial" time $t_o$ , $D$ is small. Then, during such time as the "errors" $x_i$ remain small, they are governed approximately by the linearized equation

$$d\underline{x}/dt = \underline{\underline{A}}\,\underline{x} \quad , \tag{4}$$

where the elements $A_{ij}$ of the matrix $\underline{\underline{A}}$ are the partial derivatives $\partial F_i/\partial X_j$ , evaluated from the solution $\underline{X}^*$ . Eq. (4) determines whether, if $D$ is initially small enough, $D$ remains small for all time, i.e., the realization is stable, or $D$ eventually grows quasi-exponentially, i.e., the realization is unstable.

In the special case where $\underline{X}^*$ is a steady solution, the elements of $\underline{\underline{A}}$ are constants, and the stability of the realization is determined by the eigenvalues of $\underline{\underline{A}}$ . If at least one eigenvalue has a positive real part $\lambda$ , the solution is unstable, and $D$ ultimately behaves like $e^{\lambda t}$; otherwise the solution is stable.

In the general case the elements of $\underline{\underline{A}}$ vary with time, and the eigenvalues of $\underline{\underline{A}}$ at any particular time do not indicate the stability of the solution. To determine from (4) whether $D$ increases we would have to solve (4) from the appropriate initial conditions. The most feasible method for solving (4), in some cases the only feasible one, is by stepwise numerical integration. This would entail first solving (1), also numerically, to obtain the elements of $\underline{\underline{A}}$ at successive time steps. At this point it would become simpler to solve (1) twice, with slightly different sets of initial conditions, evaluating $D$ at intervals to see whether it is increasing. Such a procedure has become the standard method for investigating the predictability of atmospheric models, and systems of similar complexity or irregularity. There is no need in such a study to make the initial errors as large as typical observational errors, and it is often convenient to make them much smaller. Such investigations have come to be known as "predictability experiments".

In the unstable case, once $D$ has become large, the linearizing assumptions leading to (4) are no longer valid, and (4) cannot be correct in any event, since it would imply that $D$ would grow forever. If (1) is realistic enough to force each variable $X_i$ to remain within bounds, $D$ must also remain bounded. Ultimately $D$ should oscillate about a value no greater than the difference between two randomly chosen states of the system governed by (1), although it may exceed this value for extended periods of time. If $D$ fails to reach this value, there is some predictability even at very long range, and from what we have said earlier, the system presumably varies with a periodic component.

## 2.2 A sample predictability experiment

In order to demonstrate how a predictability experiment works, we shall choose a simple system, which is, in fact, the simplest nonlinear system which can be used for such an experiment. The governing equation is a single first-order quadratic difference equation, which may be written

$$Y_{n+1} = a\,Y_n - Y_n^{\,2} \quad , \tag{5}$$

where $a$ is a constant. If $0 \leq a \leq 4$ , and $0 \leq Y_o \leq a$ , Eq. (5) generates a sequence $Y_o, Y_1, Y_2, ---$ with $0 \leq Y_n \leq a$ for all $n$ . For any particular value of $a$ , the set of all such sequences constitutes a deterministic process. We have previously studied this equation in considerable detail (Lorenz 1964), and this and similar equations have recently received much attention from mathematicians (e.g., Guckenheimer 1977, Collet and Eckmann 1980). No claim is made that this process is a model of the atmosphere.

In Table 1 the first column of "data" is a segment of a particular solution of (5), with $a = 3.75$ and $Y_o = 1.5$ . Each value has been rounded off to four decimal places before being used to compute the next value. There is no evidence of periodicity in the behavior.

In the next column a small error $y_o = 0.001$ has been added to $Y_o$ , changing it from 1.5 to 1.501. The difference between the columns is observed to grow rather irregularly as $n$ increases. We have underlined the first item in this column differing from the corresponding item in the former column by at least 0.001, 0.01, 0.1, and 1.0; on the average about 7 or 8 steps are needed for the error to grow by a factor of 10. While the error $y_n$ is small it is governed approximately by the linearized equation

$$y_{n+1} = (a - 2Y_n)y_n \quad , \tag{6}$$

but of course it never becomes larger than $a$ , since both columns are bounded by 0 and $a$ .

In the next column we have set $Y_o$ back to 1.5 but have raised $a$ to 3.751, in order to simulate the effects of predicting with an imperfectly known governing equation or physical law. Here too we find that the error increases irregularly, at a comparable rate. It appears to make little difference whether the original error is in the initial conditions or the governing equations. One can argue that, if the governing equations are wrong, an error will immediately be introduced, and will then grow just as if it had been present initially.

TABLE 1. Numerical solutions $Y_n$ of the difference equation $Y_{n+1} = a\,Y_n - Y_n^2$, for indicated values of a and $Y_o$. Computation of each value has been carried to indicated number of decimal places.

| n | a | | | |
|---|---|---|---|---|
| | 3.7500 | 3.7500 | 3.7510 | 3.750 |
| 0 | 1.5000 | 1.5010 | 1.5000 | 1.500 |
| 1 | 3.3750 | 3.3757 | 3.3765 | 3.375 |
| 2 | 1.2656 | 1.2635 | 1.2645 | 1.266 |
| 3 | 3.1443 | 3.1417 | 3.1442 | 3.145 |
| 4 | 1.9045 | 1.9111 | 1.9079 | 1.903 |
| 5 | 3.5148 | 3.5143 | 3.5165 | 3.515 |
| 6 | 0.8267 | 0.8283 | 0.8246 | 0.826 |
| 7 | 2.4167 | 2.4200 | 2.4131 | 2.415 |
| 8 | 3.2222 | 3.2186 | 3.2285 | 3.224 |
| 9 | 1.7007 | 1.7104 | 1.6869 | 1.696 |
| 10 | 3.4852 | 3.4885 | 3.4819 | 3.484 |
| 11 | 0.9229 | 0.9122 | 0.9370 | 0.927 |
| 12 | 2.6091 | 2.5886 | 2.6367 | 2.617 |
| 13 | 2.9767 | 3.0064 | 2.9381 | 2.965 |
| 14 | 2.3019 | 2.2356 | 2.3884 | 2.328 |
| 15 | 3.3334 | 3.3856 | 3.2544 | 3.310 |
| 16 | 1.3887 | 1.2337 | 1.6161 | 1.456 |
| 17 | 3.2791 | 3.1044 | 3.4502 | 3.340 |
| 18 | 1.5441 | 2.0042 | 1.0378 | 1.369 |
| 19 | 3.4061 | 3.4989 | 2.8158 | 3.260 |
| 20 | 1.1714 | 0.8786 | 2.6333 | 1.597 |
| 21 | 3.0206 | 2.5228 | 2.9432 | 3.438 |
| 22 | 2.2032 | 3.0960 | 2.3775 | 1.073 |
| 23 | 3.4079 | 2.0248 | 3.2655 | 2.872 |
| 24 | 1.1658 | 3.4932 | 1.5854 | 2.522 |
| 25 | 3.0127 | 0.8971 | 3.4333 | 3.097 |
| 26 | 2.2213 | 2.5593 | 1.0908 | 2.022 |
| 27 | 3.3957 | 3.0474 | 2.9017 | 3.494 |
| 28 | 1.2031 | 2.1411 | 2.4644 | 0.894 |
| 29 | 3.0642 | 3.4448 | 3.1707 | 2.553 |
| 30 | 2.1014 | 1.0514 | 1.8400 | 3.056 |

In order to simulate the effects of being unable to devise a perfect mathematical procedure for solving a system of equations, even if we should know the equations perfectly, we have added a final column to Table 1 in which everything is rounded off to three instead of four places. Again the error increases irregularly, at nearly the same rate. By coincidence all four solutions are reasonably close again at step 27, but by step 30 they are noticeably farther apart again. Our general conclusion is that as long as the error is initially not too great, its origin is immaterial; the time required for it to become large will depend upon the typical rate at which separate solutions of the appropriate system of equations diverge from one another.

### 2.3 Experiments with atmospheric models

Long before predictability experiments were conceived, Thompson (1957) had deduced theoretically that small errors should tend to amplify. As far as we can determine, the first predictability experiments were made with highly simplified atmospheric models which attempted to capture some of the gross features of atmospheric behavior. In the first of such studies to be written up in detail (Lorenz 1965), the process was governed by a set of 28 coupled ordinary differential equations. These equations contained quadratic terms representing the advection of vorticity and potential temperature, linear terms representing mechanical and thermal damping, and constant terms representing thermal forcing. The equations were derived from the familiar two-layer quasi-geostrophic model. The 28 variables made possible the representation, in each layer, of a zonal flow with two north-south modes, and superposed waves of three different east-west wave lengths, each with two north-south modes and sine and cosine east-west phases.

A period of 64 days was simulated. The growth rate of small errors was found to fluctuate wildly with the "synoptic situation"; during some four-day periods there was virtually no error growth, while, during others, the errors increased tenfold, but, on the average, small errors in wind velocity or temperature doubled in about four days.

Let us see what a four-day doubling time would imply regarding practical weather forecasting. A typical observational error in temperature may be as low as 1°C; it is probably not much less. In eight days, such an error would grow to 4°C, which would usually be considered tolerable. Reasonably good forecasts a week in advance should therefore be possible. In twenty days, however, the error would grow to 32°C, which would presumably be intolerable. Forecasts three weeks in advance should therefore be impossible. Even allowing for the slowdown of error growth as the errors become larger, one-month or several-month forecasts would seem to be out of the question.

As these results became disseminated through the meteorological world, it was recognized that the conclusions were too important to be trusted to models composed of a few dozen ordinary differential equations. Predictability experiments were soon made with the few large global circulation models then in existence (Smagorinsky 1963, Mintz 1964, Leith 1965); with the facilities of the middle 1960's these represented a large expenditure of computer time. As might have been anticipated, the models were sufficiently dissimilar to one another for the predictability studies performed with them to give contradicting results. Leith's model indicated no growth of errors at all; Smagorinsky's indicated a 10-day doubling time, while Mintz's showed a 5-day doubling time. For various reasons Mintz's result came to be the most generally accepted one (see Charney _et al_. 1966). The prospects for forecasting a month in advance began to grow dim, while an explanation seemed to be needed for our failure at that time to make good forecasts a week ahead.

As larger global circulation models came into being, with greater horizontal and vertical resolution, and more accurate representation of some of the physical processes, predictability experiments continued (e.g., Smagorinsky 1969, Jastrow and Halem 1970, Williamson and Kasahara 1971). The experiments contained numerous variants; sometimes, for example, the initial errors were random, while at other times they were confined to specific scales of motion or specific geographical regions. Provided that they were small enough in amplitude, however, their spectrum or location had relatively little effect upon the growth rate. Gradually the generally accepted doubling time decreased to three days or less.

Since the zonally averaged flow in the middle-latitude troposphere tends to be baroclinically unstable, the question arises as to whether the instability of time-variable flow, which gives rise to error growth in the global circulation models, is also a manifestation of baroclinic instability. The typical growth rate of errors does appear to be comparable to the growth rate of perturbations superposed upon baroclinically unstable zonal flows. However, time-variable flows possessing migratory waves can also be barotropically unstable (Lorenz 1972). Predictability experiments with models of two-dimensional turbulence (e.g., Lilly 1969) have yielded growth rates comparable to those given by global circulation models. We suspect that either barotropic or baroclinic instability alone would be sufficient to bring about a three-day doubling time, and that actually both types of instability are ordinarily present.

By and large the results of predictability experiments seem to have been more applicable to middle and higher latitudes than to the tropics. The physical processes in the tropics which lead to instability, or perhaps stability, and in particular those involving water in the atmosphere, have not always been well modeled. Moreover, variances tend to be small in the tropics, and, unless some special weighting function is introduced, the behavior of errors in the tropics will tend to be excluded from the behavior of some over-all mean-square error.

## 2.4 Evidence from observations

We could have greater confidence in our estimate of the doubling time if we could perform predictability studies with real observational data instead of numerical models. Unfortunately for this purpose, the atmosphere is not a controlled experiment; we can perhaps introduce some disturbances and see what happens, but then we shall never know what would have happened if we had not introduced the disturbances. We could demonstrate that predictability exists at a certain range by showing that a particular forecasting procedure consistently gives good forecasts, but, at least at short range, the numerical studies which indicate that predictability is limited nevertheless acknowledge the possibility of better forecasts than our present schemes produce.

The closest attainable approximation to a controlled experiment seems to be a study based on analogues. If within the historical records of past weather we can find two global weather patterns which are nearly identical, one pattern may be regarded as equal to the other pattern plus a small error, and, by examining what happened following the occurrences of the two patterns, we can see how rapidly the error grows.

Unfortunately, in the only study of this sort of which we are aware (Lorenz 1969a), based upon five years of data, we found only two pair of patterns whose differences were as small as 62 per cent of the difference between randomly chosen patterns; we estimated that 130 years of data would be needed to reduce this difference to 50 per cent. We were forced, therefore, to study the growth of errors which were already rather large, and which would never double again.

We found that, on the average, the smaller errors tended to amplify by about 9 per cent in one day, which would correspond to an eight-day doubling time if this amplification rate could continue. The most reasonable extrapolation of the results to very small errors, however, indicated that the latter would double in about 2.5 days - a result which is gratifyingly close to the doubling times suggested by the numerical models.

It would be possible, of course, to find weather patterns in the historical data which closely resemble one another over limited areas. There is no reason, however, to believe that these patterns will evolve similarly if the patterns in the surrounding areas are not similar. The results of a limited-area analogue study would therefore be hard to interpret.

## 2.5 The influence of smaller scales

The errors which have been estimated to double in about three days are of necessity the errors in the larger scales of motion. Features with horizontal scales of less than a few hundred kilometers are not resolved by the grids in the global circulation models, nor by the network of observing stations. Nothing in these studies indicates how rapidly the errors in the smaller scales would grow, if these scales were resolved by the models or the map analyses. It seems likely, however, that they would grow much faster; errors in describing the structure of a thunderstorm, for example, should grow as rapidly as the thunderstorm itself, doubling in perhaps half an hour or less.

It is beyond the capability of any computer to handle a global model which resolves individual thunderstorms and features of similar size. Accordingly, we derived a system of equations whose dependent variables were the variances of the errors in velocity in successive octaves of the horizontal spectrum (Lorenz 1969b). The coefficients in these equations depended upon the variance spectrum of the velocity itself, which we specified in advance. We found that errors in the larger scales doubled in a matter of days, while those in the smaller scales doubled in hours or minutes. More importantly, however, the errors in any scale, once established, soon induced errors in adjacent scales. Thus, even if there were no initial errors in the larger scales, the initial errors in the smallest scales would progress rapidly to slightly larger scales and thence more slowly to still larger scales, so that, after a day or so, there would be appreciable errors in scales of 1000 kilometers or greater. These would then proceed to double in a few days, just as if they had been present initially.

The model which yielded these results was much too crude to be accepted uncritically. It was derived from the barotropic vorticity equation governing flow over a homogeneous plane region, rather than the complete atmospheric equations over a sphere with tropics and polar regions, oceans and continents, and mountains and plains. There were no baroclinic effects nor effects of moisture, and no forcing nor damping. Finally, it was necessary to introduce a closure approximation of questionable validity.

Subsequent studies, however, using more sophisticated closure assumptions (e.g., Leith 1971, Leith and Kraichnan 1972), have yielded qualitatively similar results. Robinson (1967), on the other hand, had arrived at similar conclusions from the premise that the equations usually considered to govern the atmosphere were actually inapplicable to an atmosphere with a continuous spectrum of motions.

What then, do we mean when we speak of a three-day doubling time? We could mean the rate at which errors in the larger scales would double if there were no superposed smaller scales, or if the statistics of the smaller scales were exactly determined by the larger scales which they accompanied. Alternatively, we could mean the rate at which the large-scale errors will grow once they have attained a size somewhat smaller than present-day observational errors, which, in view of the smaller-scale errors, will be no more than a day or so after a hypothetical time when the large scales might be error-free.

If the smaller scales were completely absent, we could extend the range of acceptable forecasting by three days simply by cutting the observational errors in half; this we could conceivably do two or three times, thereby adding a week or so to the range of practical predictability. As things stand, however, we can gain something by improving our observations, but we must eventually reach a point where the observational errors will be no larger than the errors which would be present anyway after a day or so, because of the smaller scales. After that point is reached, further improvements in observations will yeild only minor improvements in medium and long range prediction.

## 3. REGIMES AND EXTERNAL INFLUENCES: WHY WE CAN PREDICT

### 3.1 General considerations

The results discussed in the previous lectures suggest that further progress can be made in short-range forecasting, but they are quite pessimistic for medium-range and especially long-range forecasting. Such complete pessimism may not be warranted.

In many mathematical models of the atmosphere, the only important nonlinear terms are those representing advection. These terms are quadratic; they contain the product of the advected quantity with the wind which does the advecting. These are the terms which are responsible for the growth of small errors. They are well formulated in the models.

Once the errors have become moderately large, the processes which are responsible for slowing and eventually stopping their growth may not include advection. Whatever these processes are, they need not be well represented in a model which represents advection properly.

One possible manner of behavior is that the errors, as they become large, may rapidly approach their limiting magnitude. On the other hand, they may temporarily cease or nearly cease to grow long before they reach this magnitude. Still another possibility is that the errors, although large, may rapidly oscillate in sign, so that averages over periods of a few days or longer may be predictable even when instantaneous states are not. This would be the case, for example, if we could predict the passage of a sequence of storms over a period of days, without being able to say on which days during the period a storm would cross a given longitude. It is possibilities of the latter sort which offer hope for medium-range and long-range forecasting, and perhaps for predicting changes in climate. Since the cause of behavior of the latter sort, if it exists, is uncertain, the ideas presented in this lecture are necessarily more speculative than those in the previous ones.

### 3.2 Slowly varying features

There are two factors which might cause these possibilities to be realities. One is the existence of features which by their physical nature tend to vary slowly. These may be external to the atmosphere, but they exert an influence upon the atmosphere.

Probably the most frequently mentioned feature of this sort is the ocean-surface temperature field. This tends to vary slowly because of the ocean's large heat capacity. It also exerts a direct thermal influence upon the atmospheric temperature field. The atmosphere in turn exerts a direct thermal influence upon the ocean temperature, but it cannot alter it so rapidly. Thus, for example, if a certain area of the ocean is observed to be warmer than normal, there may be a physical basis for predicting that the atmospheric temperature a few weeks in advance, in the area influenced by this part of the ocean, will be somewhat warmer than normal rather than simply normal. Studies which have compared observed oceanic temperatures with subsequent atmospheric temperatures have not all agreed. Davis (1976), for example, obtained negative results, while Shukla and Misra (1975) obtained positive results. It appears possible that the phenomenon is rather important in the tropics but less so in higher latitudes.

Other features of this sort include the abundance of sea ice and continental snow cover. Qualitatively, sea ice and snow cover would seem to have the same effect as abnormally low ocean temperature, but the physical mechanism involved is different, since, in addition to directly cooling the atmosphere, ice and snow reflect much of the solar radiation which would otherwise warm the atmosphere.

Still another feature is the variability of solar radiation. In particular, solar anomalies associated with the sunspot cycle vary rather slowly. The general proposition that the atmosphere should respond to variations in solar activity is entirely reasonable, although the precise nature of the expected response is hard to deduce. Here also, observational studies have been suggestive but inconclusive.

### 3.3 Atmospheric regimes

The other factor favoring medium-range and long-range predictability is the existence of what might be called atmospheric regimes. During a regime the atmosphere tends to behave in one particular anomalous manner; when the regime changes, the atmosphere behaves for a considerable time in a different anomalous manner. The extreme case of regime behavior is a phenomenon called "almost intransitivity". This exists if there are two or more sets of possible weather patterns, and, as a result of its internal dynamics, the atmosphere can evolve readily from one pattern to another one in the same set, but only with difficulty to a pattern in another set. Almost intransitivity is easily produced in some of the simpler atmospheric models, but it is not certain that it occurs in the more realistic global circulation models, and its presence in the real atmosphere has not been verified observationally.

One of the most frequently mentioned regime phenomena is "blocking". This is characterized by the continued presence of troughs and ridges at preferred longitudes over extended periods, even though the long-term average circulation may not possess such troughs and ridges. Blocking has at times been called an illusion, but present evidence (Dole 1981) indicates that a blocking anomaly, once established, has a greater-than normal probability of persisting over any fixed time interval. Forecasting that blocking will persist is therefore somewhat better than guesswork, i.e., some extended-range predictability exists in such situations.

Another phenomenon, which is better documented than blocking, is the quasi-biennial oscillation, which is an oscillation between persistent easterly and persistent westerly winds in the equatorial stratosphere, generally taking somewhat over two years to complete a cycle. Here there is no question but what one can forecast a year in advance, and do much better than guesswork, simply by forecasting the upper winds to be westerly if they are presently easterly, and vice versa.

More generally, there appears to be on the average a slight tendency for local positive or negative anomalies to persist; this persistence does not completely die out in fifteen days (Lorenz 1973). Whatever the cause of the persistence may be, its presence indicates that there is some predictability fifteen days in advance. Likewise, there is evidence that anomalies in averages over a given period persist over a considerably longer period; seasonal averages, for example, show some persistence a year ahead (cf. Madden 1977). Possibly we should not have considered the regime phenomena separately from the external influences. Some investigators would maintain that regimes are externally caused.

### 3.4 Beyond persistence

What are the prospects for extended-range forecasts superior to those based upon pure persistence? One phenomenon which suggests that they are favorable is the Southern Oscillation, which is a dominating feature of both atmospheric and oceanic behavior in the tropical Pacific area. This is characterized by a set of events whose beginning may be hard to predict, but which, once established, seems to proceed in a rather predictable sequence (see Bjerknes 1969, Horel and Wallace 1981).

As for forecasting procedures, if we can identify the factors which are responsible for the cessation of the growth of larger errors, we can incorporate them into our models, just as we now include advection, which is responsible for the growth of small errors. In any event, we can immediately include some of the suspected factors. We can use models in which the ocean surface temperature field and the fields of sea ice and continental snow cover are variables. We can experiment with models where the solar output is variable, to see whether they behave differently from those where it is constant. Other candidates for incorporation or more refined formulation in future models appear to include various phenomena associated with water in the atmosphere, including clouds and precipitation, and the effects of these upon short-wave and long-wave radiation. These may be particularly important in establishing our ideas regarding the predictability of tropical weather.

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