TROPICAL ASSIMILATION PROBLEMS:
CROSS-EQUATORIAL FLOW,
FRICTION AND EQUATORIAL WAVES

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1. INTRODUCTION

Presentations and discussions in this workshop have made clear the difficult but important problem of improving our techniques of data assimilation in the tropics. Current normal mode techniques are not able to fully represent the physics of many tropical systems; this is especially true for the monsoon, where the strong mean flow in low levels is inertially accelerated and subject to complex frictional forces, and where there are regions of strong, moist convective heating. These processes also cause mass adjustments and time dependence which are different from those expected for idealized linear waves near the equator.

The work reported in this paper is aimed at improving our understanding of the ways in which cross-equatorial advection and friction can influence low latitude flows in which both mass and momentum adjust. The simplest approach of linearized dynamics is chosen as an appropriate starting point, and yields (a) altered wave dispersion characteristics and (b) distortions in the wind and pressure structures for the gravest equatorial wave modes. From this free mode information, one can begin to assess the modified wind-pressure spatial relations expected when data is inserted into a tropical model.

The beginning point is the linearized system of non-dimensional equations for transient perturbations in a one-layer model on an equatorial beta plane:

$$\frac{\partial u'}{\partial t} + F\frac{\partial u'}{\partial y} - yv' = -\frac{\partial \phi'}{\partial x} - bu'$$

$$\frac{\partial v'}{\partial t} + F\frac{\partial v'}{\partial y} + yu' = -\frac{\partial \phi'}{\partial y} - bv'$$

$$\frac{\partial \phi'}{\partial t} + F\frac{\partial \phi'}{\partial y} + \left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) = 0 \qquad (1)$$

Here F is a constant Froude number $\overline{V}/c$ for mean meridional flow $\overline{V}$ (positive is northward) and b is a non-dimensional Rayleigh damping coefficient which approximates frictional drag in the layer. Assumption of traveling wave solutions with argument $(kx + \omega t)$, ω being non-dimensional frequency and k being zonal wavenumber, defines an eigenvalue problem to be solved for the complex non-dimensional wave variables $u(y)$, $v(y)$, $\phi(y)$. For simplicity, the combined problem (non-zero F,b) is split into two parts, each treated separately in the following section.

2. EFFECT OF FRICTION ON EQUATORIAL WAVES

Here, F is set equal to zero, and the resulting equations are combined to yield:

$$\frac{d^2v}{dy^2} + \left(\omega\Omega - k^2 + \frac{k}{\Omega} - \frac{\omega}{\Omega}y^2\right) v = 0 \tag{2}$$

where $\Omega = \omega - ib$ is a frequency modified by damping. Eq. (2) can be put into a form analogous to its frictionless (b=0) form by making a complex coordinate stretching

$$\hat{y} = (\omega/\Omega)^{\frac{1}{4}} y \tag{3}$$

in which case trapped waves satisfy the dispersion relation

$$\left(\omega\Omega - k^2 + \frac{k}{\Omega}\right) \left(\frac{\Omega}{\omega}\right)^{\frac{1}{2}} = 2n = 1, \quad n = 0, 1, 2 \ldots\ldots \tag{4}$$

For given k and n, this is a sixth degree polynomial for ω, for which three spurious roots may be rejected. The dispersion and damping curves for Yanai (n=0), Rossby (n=1), and Kelvin (n=-1) waves are shown for the case of strong (b=1) damping in Fig. 1. The Yanai (also known as "mixed Rossby-gravity") and Rossby modes damp rapidly, but slower than simple damping would give, with small wavenumber dependence. The Yanai mode propagates only slightly more slowly and with slightly less dispersion than in the frictionless case. The Rossby mode is slowed considerably at long zonal wavelengths, and its group velocity dispersion is altered strongly, as seen by the slope of the curve $Re(\omega)$. The Kelvin mode is moderately damped for $k > 0.5$, but is only slightly damped for longer zonal wavelengths. Interestingly, in the latter case, the waves are stationary, and there is a strong modification from frictionless dispersion for $0.5 < k < 1.0$.

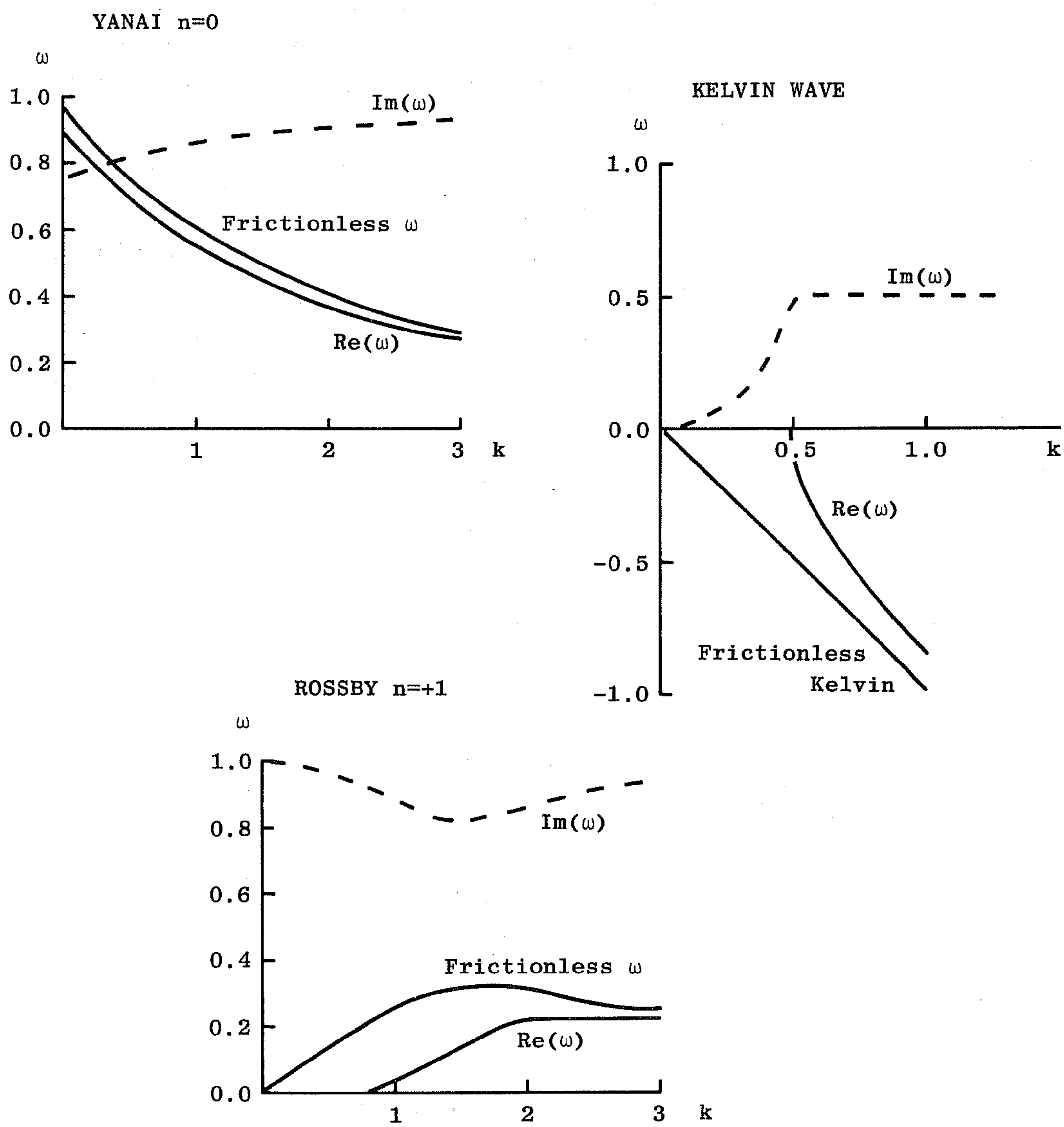

Fig. 1 Dispersion curves Re(ω) and frictional damping Im(ω) for three key equatorial wave modes for b=1. The frictionless (b=0) dispersion is also shown for comparison.

The eigenfunctions for these modes are mathematically analogous in form to those for the frictionless case. For example, $v(\hat{y})=C\,H_n(\hat{y})e^{-\frac{1}{2}\hat{y}^2}$. However, since $\hat{y}$ is complex, the exponential function may introduce latitudinal phase shifts as well as damping, while the Hermite function $H_n(\hat{y})$ may alter both phase and amplitude. Fig. 2 shows frictional eigenfunctions for two modes. The Yanai mode has been changed, as v_R, u_I, ϕ_I have been "produced", and ϕ_R is now weak compared to u_R. The phase of ϕ varies significantly with latitude, in contrast to the frictionless case. The magnitudes are not radically different from the frictionless case. The Kelvin modes retain v=0 even in the presence of friction. The phases of u and ϕ are significantly different near the equator due to friction. The Rossby mode is not shown in Fig. 2, for it has been substantially untrapped and oscillates more rapidly with latitude: it has essentially become a damped mid-latitude wave.

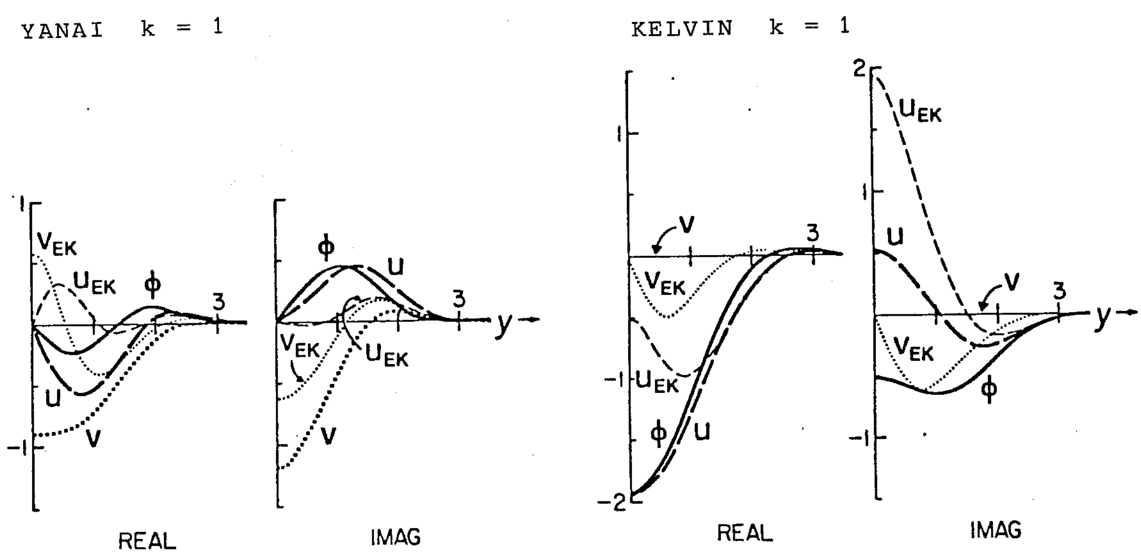

Fig. 2: Frictional eigenfunctions for the Yanai and Kelvin modes for b=1. Dotted lines show local Ekman relations for the geopotential field shown.

In summary, it appears that, of the three gravest equatorial modes, the Yanai wave is least affected by friction (apart from its temporal decay). The structure of the Kelvin wave is not strongly affected, but its time evolution is a slower decay which is dispersive at longer wavelengths. Finally, the Rossby wave is strongly altered in structure and dispersive characteristics at longer wavelengths. Thus, data inserted at larger zonal scales is likely to follow different routes during assimilation in the presence of friction compared to frictionless flows aloft.

3. EFFECT OF MEAN CROSS-EQUATORIAL FLOW ON EQUATORIAL WAVES

Here, b is set equal to zero and we examine the dependence on F. In most cases the equations from (1) are too complex to render analytical treatment, and so the eigensolutions have been determined using finite differences and QR iterations with a 21 point grid ($\Delta y=0.5$) which staggers ϕ a half interval from u and v points. Tests for the case F=0 showed good agreement with the known analytical results. Results for non-zero F were obtained by identifying analytic continuity as F was changed by small increments. These results also confirmed the expectation (derived from the equations) that a reversal of mean meridional flow (sign change in F) changes only the wind and geopotential phases; their frequencies and amplitude distributions are unchanged by a reversal in $\overline{V}$. Thus, the influence of $\overline{V}$ is far more complex than that of a simple Doppler shifting.

Fig. 3 shows the effect of $\overline{V}$ on wave frequencies. (Note that the sign of the frequency is opposite that used earlier.) It is seen that as $|F|$ increases:

(a) the eastward propagating modes (K, E_0, E_1) have a have a reduction in frequency;

(b) the Rossby mode has its frequency increased;

(c) the Yanai mode has its frequency increased, except at long zonal wavelengths;

(d) the distinction between Rossby and Yanai frequencies tends to disappear.

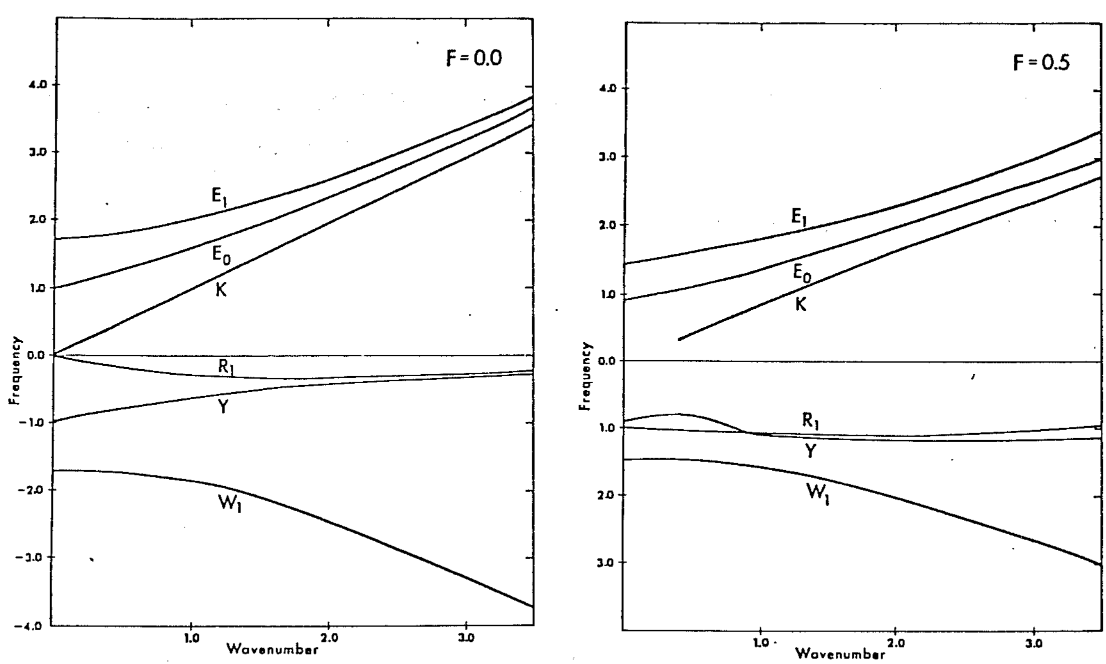

Fig. 3: Dispersion curves for equatorial waves in the absence (F=0) and presence (F=0.5) of mean cross-equatorial flow. (K is Kelvin, Y is Yanai, R_1 is Rossby, and E (W) imply eastward (westward) propagating inertia-gravity waves.)

The eigenfunctions showed considerable alteration by $\overline{V}$, especially for the Rossby, Yanai and Kelvin modes. For example, it can be shown analytically that $\overline{V}$ causes increased (decreased) trapping of the Kelvin mode when $|F|<1$ ($|F|>1$). The Yanai and Rossby modes were less trapped when $\overline{V}$ was introduced. Fig. 4 shows that these two modes are strongly distorted in response to the mean cross-equatorial flow, as judged by their streamline and geopotential patterns. The Kelvin distortion is a comparatively simple tilt of unidirectional wind and geopotential. The Yanai modification is more complex. It is seen that the mean flow $\overline{V}$ induces a net momentum transport $\overline{u' v'}$ which is opposite for the two modes; this transport is zero in the absence of $\overline{V}$.

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YANAI

KELVIN

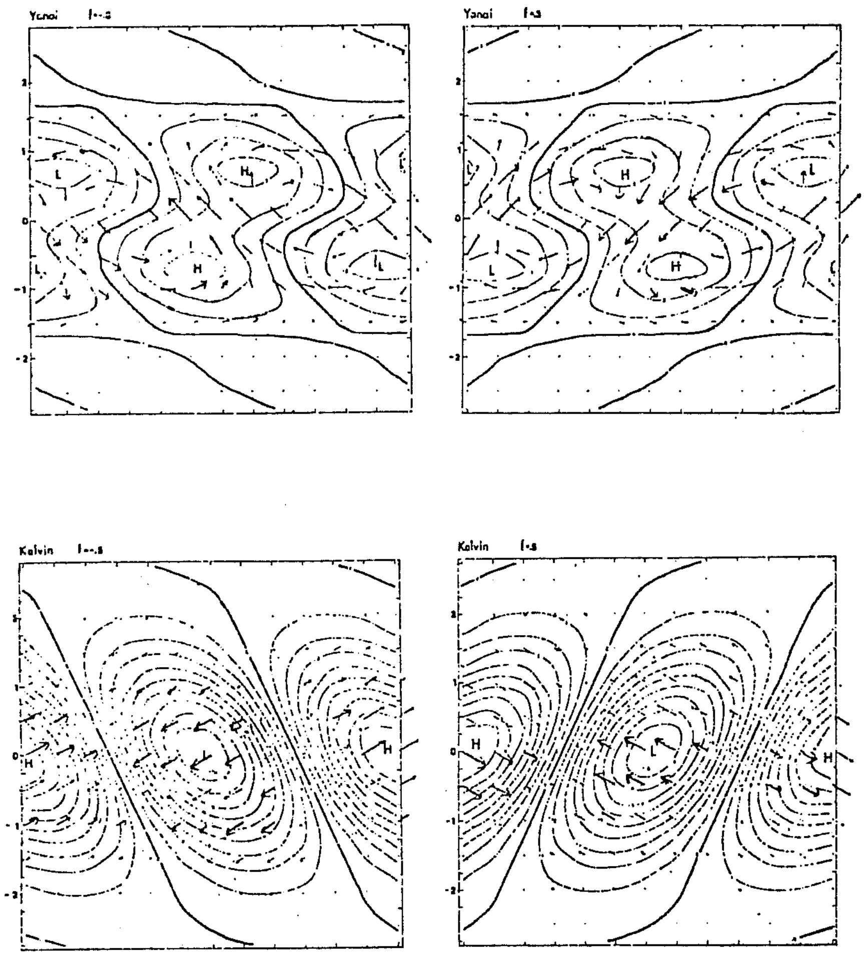

Fig. 4: Wind vectors and geopotential patterns for Yanai and Kelvin modes for the case of northward ($F > 0$) and southward ($F < 0$) mean cross-equatorial motion.