MESOSCALE ANALYSIS AT U.K. METEOROLOGICAL OFFICE

R.J.Purser
Meteorological Office, Bracknell, U.K.

1. INTRODUCTION

An experimental non-hydrostatic mesoscale forecasting model has been under development at the U.K. Meteorological Office for a number of years and has been described by Tapp and White (1976) and Carpenter (1979). The model has a uniform horizontal grid with a resolution typically of 10 Km and has up to 20 levels of variable spacing. If a model of this type is ever used for operational forecasting for the British Isles it will be necessary to develop methods of fine-scale data analysis that are reliable and efficient. I shall describe a technique of analysis that has been used successfully for producing detailed analyses, mainly of surface observations, by using spatially-recursive numerical filters to provide the correction fields of a 'successive-correction' scheme. The method used to date is essentially empirical, but I intend to indicate how a variation on the successive correction method might be employed to provide an iterative solution to the problem of determining an optimal analysis.

2. RECURSIVE FILTERS

In any analysis method it is necessary to find a technique that enables the influence from a localised observation to be spread smoothly and consistently across the analysis domain. In the case of the empirical analysis I shall describe how this is done through the repeated application of spatially recursive filters. It is worth discussing the nature of these filters in a little detail, although a more thorough description is given in Purser and McQuigg (1982).

The basic form of such a filter is exemplified in a one-dimensional uniform grid by the following simple algorithm that generates a smoothed output, B_n, from an input, A_n, at grid point, n:

$$\begin{aligned} B_0 &= A_0 \\ B_n &= \alpha B_{n-1} - (1-\alpha) A_n \qquad n > 0 \end{aligned} \tag{1}$$

α is a smoothing parameter, taking a value between 0 and 1, that controls the spatial scale of smoothing of the filter. By expanding the recursion in (1) we may express the result:

$$B_n = (1-\alpha)\left(A_n + \alpha A_{n-1} + \cdots \alpha^m A_{n-m} + \cdots \alpha^n A_0\right) \tag{2}$$

or symbolically by the convolution operation,

$$B = E * A \tag{3}$$

with

$$E_n = (1-\alpha)\alpha^n \qquad n \geq 0$$
$$= 0 \qquad n < 0 \tag{4}$$

This filter has two notable properties: firstly its convolution kernel, E_n, takes the same shape (a decaying exponential distribution) for different scale parameters, α, and secondly its characteristic scale may take any value. Together these properties make it a simple matter to reproduce scale-dependent effects of a filter applied to a distribution on a grid of one resolution with a corresponding filter applied to the same distribution resolved by a different grid. In fact all filters of this form have a continuous analogue in the convolution:

$$B(x) = -\log\alpha \int_0^\infty A(x-x')\,\alpha^{x'}\,dx' \tag{5}$$

where the continuous variable, x, replaces the discrete index, n, of equation (2). In the subsequent discussion I shall neglect the effects of the grid's finite resolution and its finite extent so that equation (5) serves as an adequate model of the actual discrete filters used.

The convolution kernel of (5) may be written,

$$E(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}} \qquad x > 0$$
$$= 0 \qquad x < 0 \tag{6}$$

where

$$\lambda = -\frac{1}{\log\alpha} \tag{7}$$

First and second moments of the distribution, $E(x)$, are:

$$M_1(E) = \int_0^\infty x\,E(x)\,dx = \lambda$$
$$M_2(E) = \int_0^\infty x^2 E(x)\,dx = 2\lambda^2 \tag{8}$$

The natural measure of the spread of the convolution is the variance of $E(x)$ about its mean:

$$M_2'(E) = M_2(E) - M_1^2(E) = \lambda^2 \tag{9}$$

Thus λ may be regarded as the characteristic scale of the associated recursive filter.

By itself the basic filter has an asymmetric effect on the field to which it is applied. To remove the bias it is necessary to apply the conjugate filter, E^+, which entails reversing the direction of operation of the algorithm. The resulting symmetric filter may itself be expressed as a convolution operation, $S*$, where

$$S * A \equiv E^{+} * E * A \tag{10}$$

with

$$E^{+}(x) = E(-x) \tag{11}$$

and hence,

$$S(x) = \frac{1}{2\lambda} e^{-\frac{|x|}{\lambda}} \tag{12}$$

Some insight into the behaviour of these filters is obtained by examining their spectral response. Defining the Fourier transform of $A(x)$ to be $\tilde{A}(k)$ so that,

$$\tilde{A}(k) = \int_{-\infty}^{\infty} A(x)\, e^{-ikx}\, dx \tag{13}$$

$$A(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{A}(k)\, e^{ikx}\, dk \tag{14}$$

then the convolution theorem is,

$$H = F * G \quad \Rightarrow \quad \tilde{H} = \tilde{F}\tilde{G} \tag{15}$$

From the definitions of $E(x)$ and $E^{+}(x)$

$$\tilde{E}(k) = \frac{1}{1 + i\lambda k}$$

$$\tilde{E^{+}} = \tilde{E}^{*} = \frac{1}{1 - i\lambda k} \tag{16}$$

Hence,

$$\tilde{S}(k) = \tilde{E}(k)\, \tilde{E}^{*}(k) = \frac{1}{1 + \lambda^2 k^2} \tag{17}$$

A higher degree of smoothing is effected by repeating the operation. That is, by applying $S^{(2)} *$ $(\equiv S * S *)$. $S^{(2)}(x)$ is continuous in two derivatives at $x = 0$ and possesses a spectral response that is clearly more effective at suppressing high wave-number structures. This may be seen from Figures 1 and 2 which illustrate the forms of S and $S^{(2)}$ together with their respective Fourier transforms.

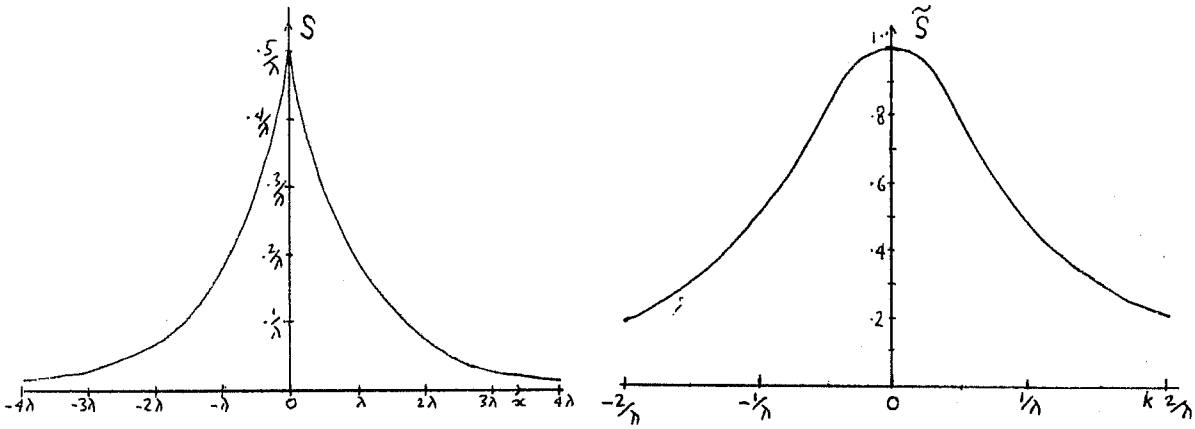

Figure 1. Smoothing kernel, $S(x)$, and its Fourier transform, $\tilde{S}(k)$.

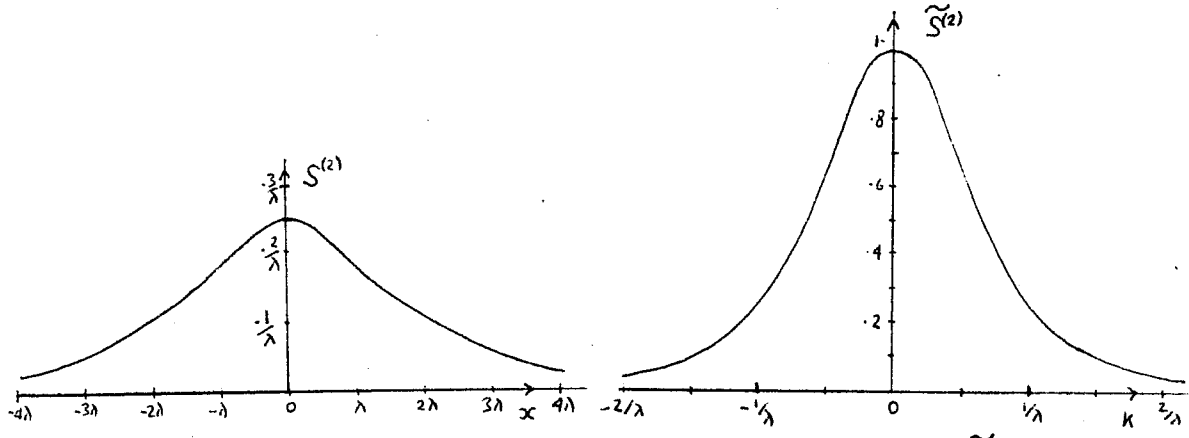

Figure 2. Smoothing kernel, $S^{(2)}(x)$ and its Fourier transform, $\tilde{S}^{(2)}(k)$

In the general case of repeating the application of filter, S, L times the spectral response of the equivalent filter, $S^{(L)}$ is

$$\tilde{S}^{(L)}(k) = \frac{1}{(1 + \lambda^2 k^2)^L} \tag{18}$$

As L increases we observe the central limit theorem applies:

$$\tilde{S}^{(L)}(k) \simeq e^{-L\lambda^2 k^2} = \tilde{G}(k) \tag{19}$$

where

$$G(x) = \frac{1}{\sqrt{2\pi(2L\lambda^2)}}\, e^{-\frac{x^2}{4L\lambda^2}} \tag{20}$$

As expected from the additive property of variances of convolved functions, the variance of the resulting smoothing kernel is

$$M_2'(G) = 2L\lambda^2 \tag{21}$$

Thus we have at our disposal a simple method of smoothing a field as if we had convolved it with Gaussian of a chosen width. A two-dimensional Gaussian filter is simulated by repeating the application of the basic filters in both the x and y directions. The small edge effects on a finite grid have little deleterious effect in the interior.

3. ANALYSIS STRUCTURE

I shall now describe the construction of a two-dimensional successive correction analysis scheme based on the use of the composite 'pseudo-Gaussian' filters reviewed in the previous section. I shall adopt the notation that a Roman suffix denotes a value at an analysis grid point, a Greek suffix indicates observation points and a bracketed superfix denotes the iteration index. Let O_α be the observation value at position α and let $A_i^{(n)}$ be the analysis value at grid point i at the nth iteration. Assume $A^{(o)}$ takes a uniform value for the first guess (though in an operational analysis this would be greatly improved by using a forecast background field instead of a constant initial value).

For simplicity I shall assume that all observations coincide with analysis grid points (but in reality a simple interpolation to nearest grid points is required). At any stage (n) of the analysis we may construct 'observation-residuals', $(O_\alpha - A_\alpha^{(n)})$, at the observation points and associate with each one a 'weight', $Q_\alpha^{(n)}$, whose value may be used to prescribe the degree of reliability and hence, the strength of influence, of each observation. (This point will be discussed more thoroughly below). An improved fit to the observations may be obtained by adding to $A_\alpha^{(n)}$ a weighted average of the neighbouring observation-residuals. If $C^{(n)}*$ denotes the application of an appropriately gauged pseudo-Gaussian filter of the type described in Section 2, then we may generate an improved fit to the observations while preserving smoothness by the operations summarised symbolically:

$$A^{(n+1)} = A^{(n)} + \frac{C^{(n)} * Q^{(n)}(O - A^{(n)})}{C^{(n)} * Q^{(n)}} \tag{22}$$

The choice of scale parameters of the basic recursive filters that comprise $C^{(n)}$ enables the effective scale of $C^{(n)}$ to be varied not only from iteration to iteration but also from one region of the grid to another if these parameters are space-dependent variables of the scheme. This allows us to account for the variable coverage of observations. A useful measure of the observation density, and hence the characteristic separation distance, is obtained by ensuring that reliable observations have approximately unit weight,

$$Q_\alpha \simeq 1 \tag{23}$$

and extracting a scale of separation, $R_W^{(n)}$, as a biproduct of equation (22) by

$$R_W^{(n)} = \frac{1}{\sqrt{W^{(n)}}} \tag{24}$$

where

$$W^{(n)} = C^{(n)} * Q^{(n)} \tag{25}$$

$R_W^{(n)}$ may then be used to control the scale of smoothing filter of the next iteration to ensure that no attempt is being made to analyse scales inadequately resolved by the local observation density. Apart from this important restriction the characteristic scale, $R^{(n)}$, of the smoothing operator, $C*$, is permitted to decrease exponentially with each iteration from an initial large scale, $R_M^{(0)}$, towards an empirical predetermined small positive limit, $R_M^{(\infty)}$. Formally:

$$R^{(n)} = \mathrm{Max}\left(R_M^{(n)}, R_W^{(n)}\right) \tag{26}$$

where,

$$R_M^{(n)} = R_M^{(\infty)} + \left(R_M^{(0)} - R_M^{(\infty)}\right) S^n$$

The principle of this strategy is essentially to fit observations with large-scale analysis structures whenever possible (ie at an early stage) and to reserve the fitting of small-scale structures to accommodate only those features that cannot be readily analysed by large-scale perturbations alone. This follows the conventional approach of successive correction analysis, eg Cressman (1959).

The observation weights are controlled to ensure that strongly deviant observations (which are probably erroneous) have negligible effect in the final analysis. Apart from a crude preliminary climatological check, there is no explicit rejection of observations, but a weight formula based on the latest observation residual ensures that a large departure from zero causes the observation in question to acquire a very small effective weight at the next iteration. The function takes the form:

$$Q_\alpha^{(n)} = \frac{1}{1 + \left(\frac{|O_\alpha - A_\alpha^{(n)}|}{TOL^{(n)}} \right)^4} \qquad (27)$$

where the 'tolerance', $TOL^{(n)}$, decreases exponentially with each iteration towards a finite limit.

$$TOL^{(n)} = TOL^{(\infty)} + \left(TOL^{(0)} - TOL^{(\infty)} \right) \sigma^n \qquad (28)$$

the function for $Q_\alpha^{(n)}$ in equation (27) is illustrated in Figure 3.

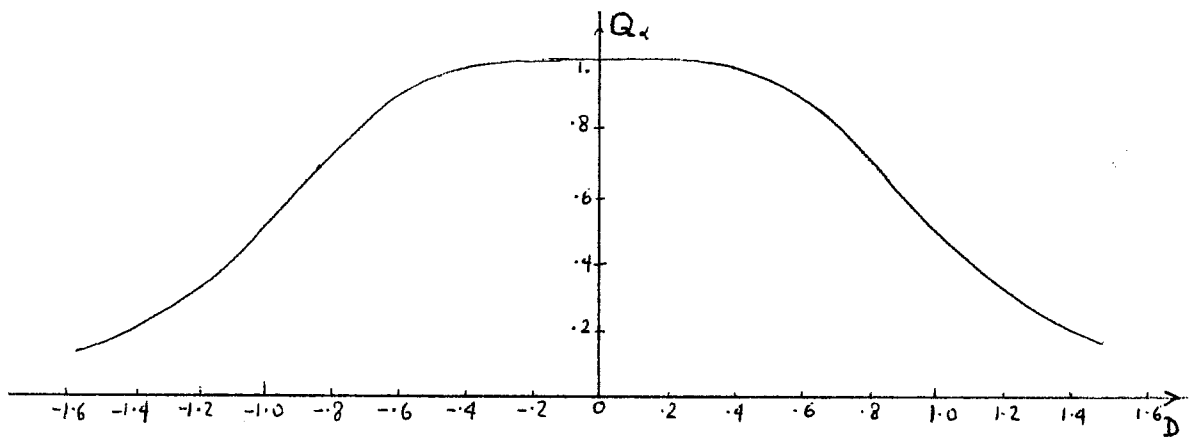

Figure 3. Q_α as a function of $D = (O_\alpha - A_\alpha)/TOL$.

The gradual reduction of TOL prevents good observations from being prematurely eliminated due to the inevitable crudeness of the analysis at an early stage of the iterations.

Figure 4 shows a sea-level pressure analysis constructed by the technique using ten corrections on a 21 x 21 grid. In this case the principal analysis

parameters were:

$R_M^{(o)} = 900\ Km$

$R_M^{(\infty)} = 22{\cdot}5\ Km$

$S = {\cdot}7$

$TOL^{(o)} = 10\ mb$

$TOL^{(\infty)} = 1\ mb$

$\sigma = {\cdot}6$

Note that in this case the anomalously high pressure observation in the middle of the English Channel has been successfully ignored by our method of quality control. Some idea of the relative scales and amplitudes of the successive corrections comprising the analysis of Figure 4 is obtained by comparing Figures 5, 6 and 7 which show the 1st, 5th and 10th corrections respectively, together with the corresponding observation-residuals from which they are derived.

4. ITERATIVE OPTIMAL ANALYSIS

The analysis scheme I have described is based entirely on empirical considerations and has required a certain degree of experimental 'tuning' to produce consistently acceptable results. It appears to perform well and can certainly be made computationally efficient, even when dealing with a very large quantity of observations, thanks mainly to the simplicity of the numerical filtering technique employed. However, it lacks any formal theoretical basis that would enable the various parameters to be prescribed objectively. I would like to complete this presentation with a somewhat speculative look at the prospects of applying an essentially iterative algorithm to the task of generating an analysis that is in some formal sense 'optimal'. By this means we may hope that some of the serious practical problems associated with the manipulation of the large matrices of conventional optimum interpolation schemes may be aleviated.

Expositions of the standard theory of optimum interpolation may be found in the publications of several investigators, eg Gandin (1963), Rutherford (1972), Lorence (1981). The central equation of the theory is the linear set:

$$A_i - B_i = \sum_{\alpha\beta} C_{i\alpha} \left(C' + E' \right)^{-1}_{\alpha\beta} \left(O_\beta - B_\beta \right) \qquad (29)$$

where A_i, B_i, O_α are analysis, background (or first-guess) and observation fields respectively. $C_{i\alpha}$ is the covariance of background error between grid point i and observation α. C' is the corresponding matrix of values between observation points and E' is the matrix of observation error covariances. In practice the correction to the background field is determined by solving a

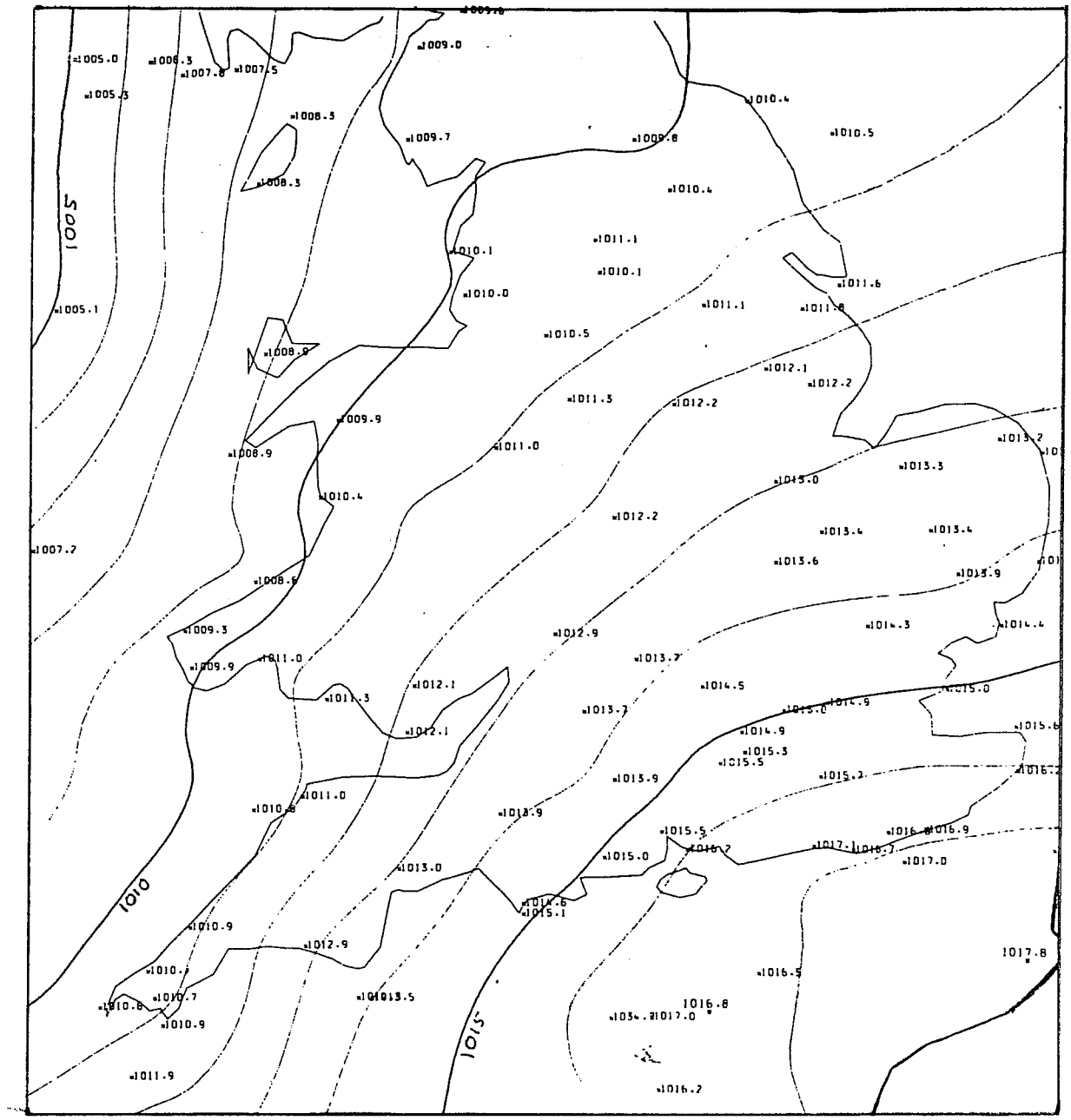

Figure 4. Analysed sea-level pressure at 12Z , 2.6.81.

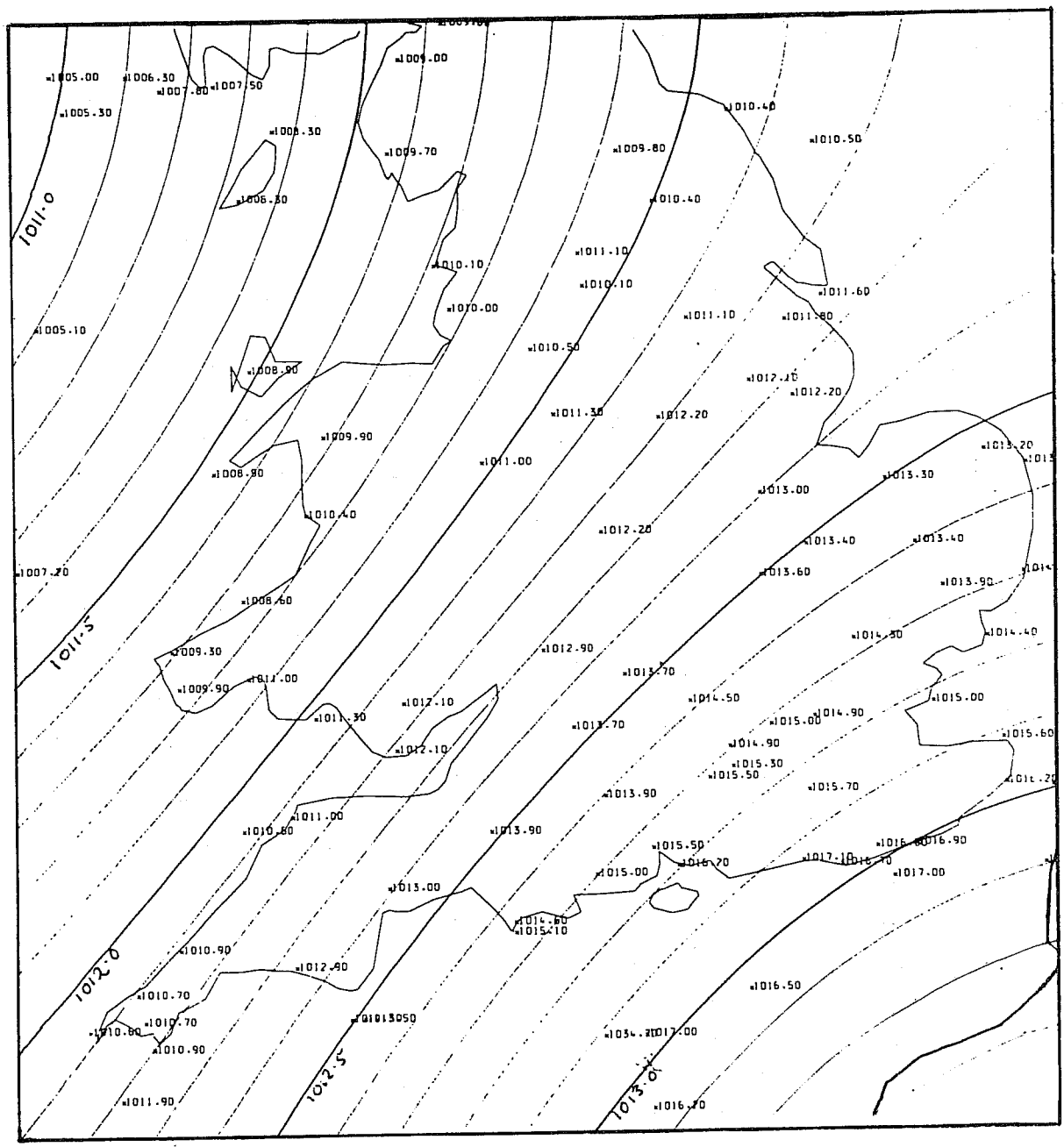

Figure 5. First correction field of the analysis of Figure 4 and plotted observations.

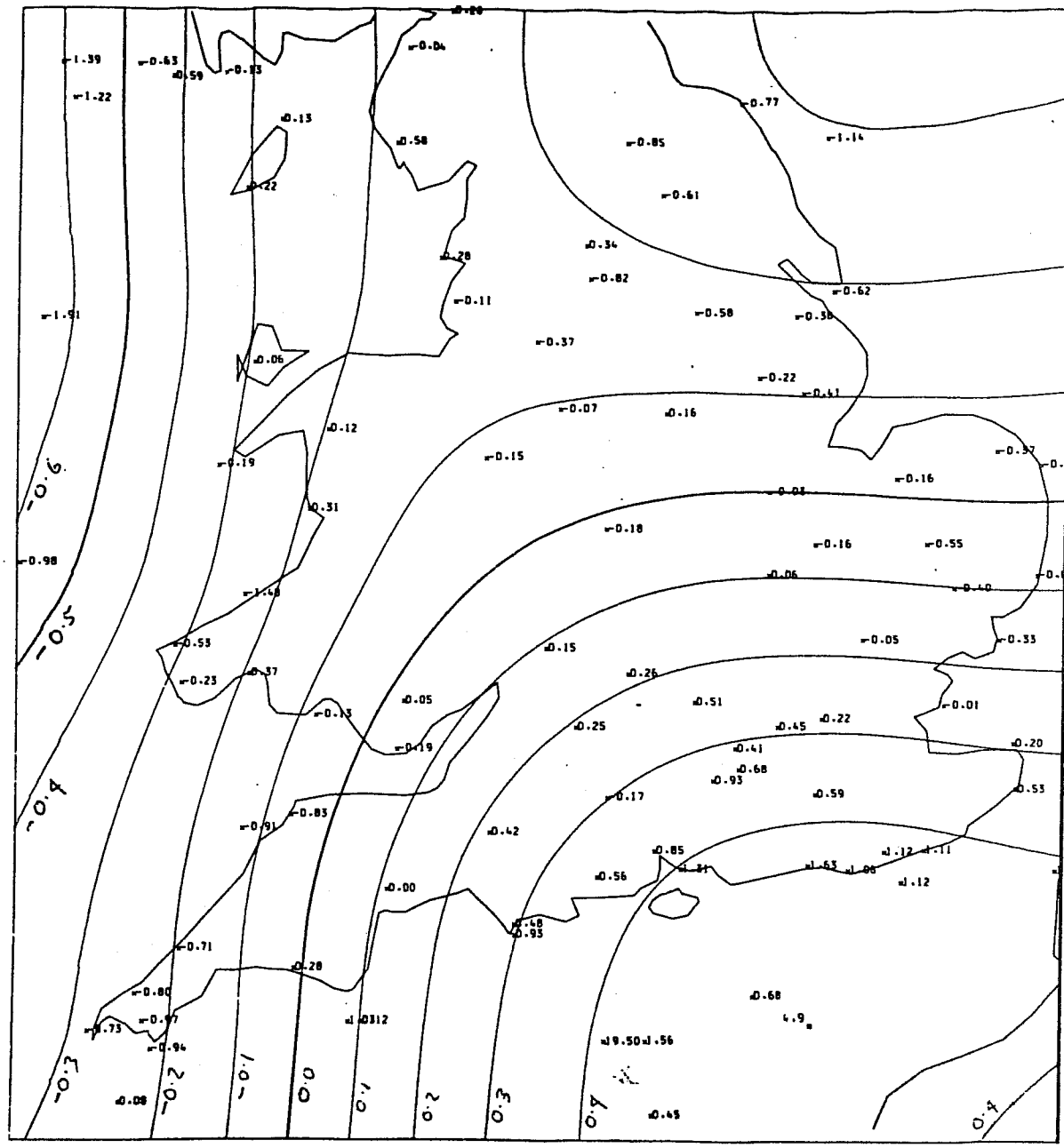

Figure 6. Fifth correction field of the analysis of Figure 4.

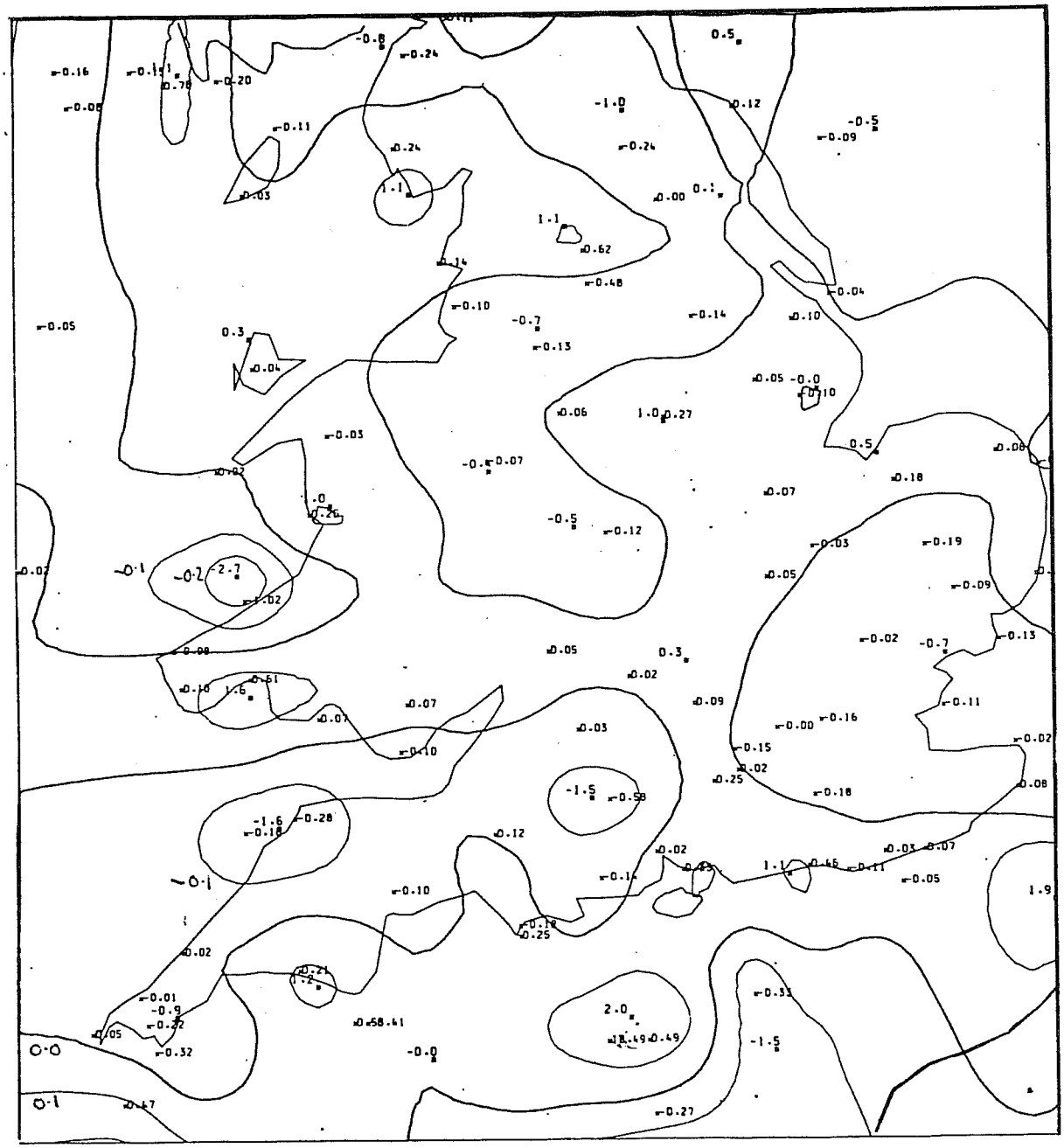

Figure 7. Tenth correction field of the analysis of Figure 4.

manageable approximation to this highly interactive linear system. Unfortunately, for a mesoscale analysis it is less easy to justify the artifical and rather drastic cut in the order of the matrices involved since a relatively large number of observations lie together within the characteristic scale of the error covariance distribution, C.

An equally valid representation of the optimum interpolation equation (29) is the implicit form,

$$A_i - B_i = \sum_{\alpha\beta} C_{i\alpha} E'^{-1}_{\alpha\beta} (O_\beta - A_\beta) \tag{30}$$

which is more suggestive of an iterative formulation. Since the covariances, C, are normally considered to be spatially quasi-homogeneous we may rewrite (30) as a convolution or pseudo-convolution (in which the convolution kernel varies slowly in space):

$$A - B = C * X \tag{31}$$

where

$$X_\alpha = \sum_\beta E'^{-1}_{\alpha\beta} (O_\beta - A_\beta) \tag{32}$$

Note that usually X_α may be easily obtained since E' is generally sparse off the diagonal.

It is likely that typical covariance functions, C, (including multivariate forms) can be well approximated by the superposition of a small number of Gaussians. If such is the case the correction to the background field can presumably be generated by applying the appropriate combination of the recursive filters of Section 2 to the impulsive distribution X. But since X itself is defined in terms of the unknown field, A, by equation (32) we must seek an iterative solution to avoid the large matrices that would otherwise have to be inverted to determine X. A possible contender might be the scheme defined by the following recursive set of equations:

$$O^{*(0)}_\alpha = 0 \tag{33}$$

$$X^{(n)}_\alpha = \sum_\beta E'^{-1}_{\alpha\beta} O^{*(n)}_\beta \tag{34}$$

$$A^{(n)} = C * X^{(n)} + B \tag{35}$$

$$O^{*(n+1)}_\alpha = O^{*(n)}_\alpha + \sum_\gamma H_{\alpha\gamma} (O_\gamma - O^{*(n)}_\gamma - A^{(n)}_\gamma) \tag{36}$$

where the matrix, H, has only a small number of non-zero elements in each row but is also an approximation,

$$H_{\alpha\gamma} \simeq \sum_\beta E'_{\alpha\beta} (E' + C')^{-1}_{\beta\gamma} \tag{37}$$

Depending on how good an approximation this is, the scheme defined by equations (33) to (36) should converge rapidly to the desired optimal state. Thus it

appears possible that by successive correction of the estimated observation-residuals, O^*, and by the spreading of analysis information by the use of recursive filters a reliable approximation to an optimal analysis might be efficiently achieved.

REFERENCES

Bergman, K. H. 1979 Multivariate analysis of temperatures and winds using optimum interpolation. Mon. Wea. Rev. 107, 1423-1443.

Carpenter, K. M. 1979 An experimental forecast using a non-hydrostatic mesoscale model. Quart. J. R. Soc. 105, 629-655.

Cressman, G. P. 1959 An operational objective analysis system. Mon. Wea. Rev. 87, 367-381.

Gandin, L. S. 1963 Objective analysis of meteorological fields. Leningrad Gidromet; Jerusalem, (Israel program for scientific translations. 1965 252pp).

Lorenc, A. 1981 A global three-dimensional multivariate statistical interpolation scheme. Mon. Wea. Rev. 109, 701-721.

Purser, R. J. and McQuigg, R. 1982 A successive correction analysis scheme using recursive numerical filters. (Unpublished) Met O 11 Technical Note No. 154.

Rutherford, I. D. 1972 Data assimilation by statistical interpolation of forecast error fields. J. Atmos. Sci. 29, 809-815.

Tapp, M. C. and White, P. W. 1976 A non-hydrostatic mesoscale model. Quart. J. R. Met. Soc. 102, 277-296.