COMPUTER REQUIREMENTS FOR ATMOSPHERIC MODELLING

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1. INTRODUCTION

It is a great pleasure for me to open this Workshop on 'The usage of Multiprocessors to integrate atmospheric models'. As most of you know, the meteorological forecasting problem was identified already by von Neumann as an ideal application for computers. Under his guidance and supervision a special group was set up at the Institute for Advanced Studies in Princeton in 1984 under J. Charney to undertake an integration of the vorticity equation using real atmospheric observations. The result was successful and was one of the very first attempts to use electronic computers to solve non-linear equations by numerical methods. Since then, the meteorologists have stayed in the forefront in applying the fastest available computers to solve the forecasting problem using increasingly more realistic models of the atmosphere. The meteorological community is an active user and a very substantial user for supercomputers and discussions between the meteorological modellers and the experts within the computing industry are important and I am sure mutually beneficial.

What makes the problem of weather prediction so intruiging is the fact that the atmospheric system is essentially non-linear and cannot be decomposed into independently acting modes. The inevitable error in observing the smallest scales of motion must then contaminate larger scales and finally destroy the accuracy of any predicition. The weather prediction can therefore be seen as an unstable problem in the sense that small initial differences have large final effects. Although the problem as such is deterministic, it is, for practical reasons non-deterministic since the initial state can never be perfectly known. Theoretically, the weather prediction has therefore much in common with more general non-deterministic problems, such as economical and social systems. Atmospheric prediction models may, therefore, also serve as a useful prototype for better understanding of a more general class of problems, where the dynamical laws are not yet too well understood. Although the basic physical laws governing the atmosphere have been known since the last century, no real progress took place before the advent of computers. The first integration of a simple two-dimensional atmospheric model was done by Charney, Fjörtoft and von Neumann (1950). Since then there has been a rapid development of successively more realistic models following the very fast development of computers. These models have been used for a large range of atmospheric problems from short range weather prediction to the simulation of the climate of the earth.

The problem of predicting the atmosphere from time scales from a few days to a few weeks is particularly challenging and important. It is regarded as perhaps the most difficult prediction problem because we have to rely on accurate treatment of the atmospheric observations as well as on accurate modelling. In the appreciation of the importance of this problem, the Western European countries decided to jointly set up and finance a special European Centre dedicated to medium range prediction, the European Centre for Medium Range Weather Forecasts (ECMWF). ECMWF was established in 1975. It started to make daily 7-day forecasts from August 1979. The Centre's medium range forecasts are now the best available.

2. THE PHYSICAL AND MATHEMATICAL BASIS FOR NUMERICAL MODELS

The behaviour of the atmosphere is governed by fundamental physical laws and their boundary conditions. The macrostructure of these laws has been known for over a century. What has been lacking and to some extent is still lacking, is the understanding of the interaction of the macroscales (greater than a few hundred kilometers) with processes of much smaller dimensions such as radiative transfer, turbulence fluxes, cloud and precipitation processes.

The physical laws governing the atmosphere are:

The gas law

$$p = \rho RT \qquad (1)$$

The continuity equation for dry air

$$\frac{d\rho_d}{dt} = -\rho_d \nabla \cdot \underline{v} \qquad (2)$$

The continuity equation for moist air

$$\frac{d\rho_w}{dt} = -\rho_w \nabla \cdot \underline{v} + S \qquad (3)$$

The first law of thermodynamics

$$c_p \frac{dT}{dt} - R \frac{dp}{dt} = Q \qquad (4)$$

The equation of motion

$$\frac{d\underline{v}}{dt} = \frac{1}{p} \nabla p - g - 2\underline{\Omega} x \underline{v} + \underline{F} \qquad (5)$$

For a definition of symbols and expressions see Table I. The physical parameters entering these laws are: $\underline{v}$, the three-dimensional velocity vector relative to the earth; T, the temperature; p, the pressure; ρ_d, the density of air; the ρ_w the density of water vapour. Some atmospheric models have additional conservation laws for liquid water (cloud water) and for ozone.

The equations (1) to (5) constitute a closed system which can be solved at all future times from a given initial state and with the necessary prescribed boundary conditions. However, the equations still contain some unspecified source and sink terms and it is necessary to provide a second set of expressions where we can specify these in terms of known physical quantities and/or in the basic parameters $\underline{v}$, T, p, ρ_d and ρ_w.

In the case where the source and sink terms, $\underline{F}$, Q and S are zero the system is energetically closed and can only describe adiabatic processes. It is also reversible and can be integrated backward in time as well as forward. The diabatic term, $\underline{F}$ represents dissipation of momentum and Q and S represent sources and sinks for heat and water vapour.

The complete atmospheric equations in (1) to (5) have not so far been used for operational forecasting. They are very general and represent in principle all scales of motion from the microscale to the largest planetary scale.

For practical purposes, we can only resolve the scale of motion which can be analysed by standard data and it has therefore become common practise to simplify the equations by omitting small terms and filtering out unwanted motion. The approximate equations are derived by considering the temporal and spatial scales of interest which are from about 1 hour and 100 km respectively.

With these assumptions, which are not only dictated by the data distribution but also by computational considerations, the basic equations (1) to (5) can be simplified. It is found for instance that the vertical equation of motion can be reduced to a diagnostic relation where the vertical pressure force is balanced by the gravitational force:

$$\frac{1}{p}\frac{\partial p}{\partial z} + g = 0 \qquad (6)$$

The hydrostatic equation (6) leaves us without a prognostic equation for the vertical motion, w, but the adoption of (6) will make it possible to determine w diagnostically from the remaining equations. An additional consequence of (6) is that other approximations must be made to guarantee that the resulting system, in the absence of sources and sinks, conserves energy and momentum. These approximations lead to that w is eliminated from the expression of kinetic energy and the radial distance is replaced by the average radius of the earth. The resulting equations are generally called the "primitive equations" by the meteorological community. An interesting consequence arising from the adoption of the hydrostatic relation is the removal of vertical travelling sound waves.

Observational and computation restrictions have made it necessary to confine the atmospheric model to the description of phenomena larger than a certain given scale. Present computers put this limit around 100 km in the horizontal and around one kilometre in the vertical. The dimension of such a volume is a measure of the computational resolution. What happens on scales smaller than that of the volume is known as subgrid scale processes and the simplified prescription of how they are related to the macroscale is known as parameterization. Finally, the empirical constants as well as the dependent variables of the macroscale that enter these relations are known as parameters.

The physical processes which go into the description of $\underline{F}$, Q and S consist of a manifold of subgrid processes which must be parameterized. These processes do represent a considerable fraction of the numerical calculation to be carried out in each grid point.

3. NUMERICAL ASPECTS

We will next consider the numerical and computational aspects of atmospheric modelling. If A is the integration domain then the number of grid points in a horizontal area is $A/(\Delta s)^2$ where Δs is the average horizontal grid size. For K vertical levels the total number of grid points is $KA/\Delta s)^2$. If there are n variables per point, then the total number of variables at any one time is $nKA/(\Delta s)^2$ which is a measure of the number of degrees of freedom in the model. The maximum time increment Δt, to guarantee stability depends on the integration scheme. If we, for instance, use an explicit integration scheme the condition for computational stability is given by an expression of the kind

$$\Delta t < \frac{\Delta s}{\sqrt{2}(c+U_{max})} \qquad (7)$$

where $c \sim 300$ m sec^{-1} is the speed of the fastest gravity waves and U_{max} is the maximum horizontal wind speed.

Since we have eliminated vertically propagating sound waves by using the hydrostatic relation the vertical condition is less severe than the horizontal one. However, with more economical integration techniques such as semi-implicit Kwizak and Robert (1971) a longer time step can be used. Hereby gravity waves are treated implicitly, while the slower Rossby waves are treated explicitly. For a model using a semi-implicit scheme and a staggered grid a relation of the kind

$$\Delta t < \frac{\Delta s}{U_{max}} \tag{8}$$

will hold.

For a total integration time T at least the following number of time steps are needed

$$\frac{T(U_{max})}{\Delta s}$$

Finally, if we need N number of operations per variable/time step the following number of arithmetic operations are needed to make a forecast with the length of time T for an area A:

$$\frac{A\ K\ nT\ (U_{max})\ N}{(\Delta s)^3} \tag{9}$$

4. ATMOSPHERIC PREDICTABILITY AND PRESENT PREDICTIVE SKILL

Atmospheric models are used both for weather prediction and for climate simulation. One can, in principle, as for the ECMWF model, use the same model for both these applications. Following the continuing improvements in meteorological observations, computers and associated progress in modelling technique, remarkable achievements have taken place during the last 20 years and useful predictive skill has been extended from a few days in the 1960s to 5 to 7 days at present. In the short time scale detailed simulations of intense vortices such as Genoa cyclogenesis, polar lows and tropical hurricanes have been possible.

As mentioned previously, numerical weather prediction can never be exact due to the inevitable errors in the determination of the initial state coupled with an inherent tendency for errors to grow. This error growth is not an artifact of a numerical model, but a consequence of the non-linearity and instability of the dynamics of the atmosphere. Considerable research is taking place to assess the predictability of the atmosphere, both from data and by numerical experiments. It has been found that the doubling time for small errors is of the order of two to three days and successively decreasing as the error is approaching some asymptotic value. For practical purposes this growth seems to limit the prediction of the day to day weather to something in the order of two to three weeks. However, the error growth is much lower for the largest scales of motion and it may be possible to predict large scale anomalies well beyond this time.

5. CONCLUSIONS

It is obvious that the improvement of supercomputers is crucial for a further development of the meteorological models and hence our ability to make better weather forecasts. The speed of computers has approximately increased by a factor of 10 every five to seven years and we may therefore expect computers in the Giga flops range towards the end of this decade if we are bold enough to extrapolate.

It appears that in order to achieve performances of several Giga flops and beyond, we may have to turn to multiprocessing systems. The meteorological forecasting problem has a simple logical structure and it is straightforward, at least in principle, to program an atmospheric model for a multi-processing system, due to the fact that we are essentially carrying out the same kind of calculations in each grid point. The Centre's new prediction model is simultaneously being executed on two processors of the Cray X-MP and, as will be described in following lectures, the code for the ECMWF model has been designed in such a way that the calculation can be executed simultaneously on an even number of processors. This is not the first time this has happened in meteorology. The US Navy Weather Service in Monterey uses a CDC Multiprocessor system more than 15 years ago. However, it did not start there. Lewis F. Richardson, who was the first to integrate a numerical model more than 60 years ago and long before the existence of any computers, outlined in his absolutely amazing book 'Weather Prediction by Numerical Processes' (Richardson 1922), a futuristic idea of a "weather forecasting factory" consisting of 64000 human "multiprocessors". Fig. 1. Perhaps therefore one day when we can build the 64000 multiprocessing system, we may solve the weather forecasting problem.

TABLE I Symbols and expressions used in the article

$\underline{v}$	=	(u,v,w) - 3-dimensional velocity vector relative to the earth
T	=	temperature
p	=	pressure
ρ_d	=	density of dry air
ρ_w	=	density of water vapour
ρ	=	density of moist air. $\rho = \rho_d + \rho_w$
$\underline{F}$	=	dissipation of momentum
S	=	water vapour source/sink term
Q	=	diabatic heating
R	=	gas constant for moist air
c_p	=	specific heat at constant pressure
g_a	=	acceleration due to gravity
$\underline{\Omega}$	=	earth angular velocity
$\underline{R}$	=	radial co-ordinate measure from the centre of the earth
g	=	$g_a - \underline{\Omega} x \underline{\Omega} x \underline{R}$ (local gravity acceleration)
x,y,z	=	spatial co-ordinate
t	=	time
$\frac{d}{dt}$	=	individual derivative = $\frac{\partial}{\partial t} + \underline{v} \cdot \underline{\nabla}$
$\underline{\nabla}$	=	horizontal gradient operator

Table II

Estimated computational requirements for some numerical models. The independent parameters can be reduced to 4/level plus surface pressure by a combination of the continuity equation (2) and the hydrostatic equation (6). Version I is an estimate of the present situation. Version II is a projection of the requirements around 1990.

A	=	5.10^{14} m^2	(area of the globe)
K	=	20	number of vertical levels
n	=	4	number of independent parameters
U_{max}	=	150 ms^{-1}	estimated maximum wind speed
N	=	number of operations/gridpoints/parameters	
Δs	=	horizontal grid length	
T	=	Total integration time	
M	=	Total number of operations	

(a) Short-range forecast, limited area model (1/20 globe)

I	II
N ~ 250	400
Δs = 50 km	25 km
T = 2 days	2 days
M = 10^{11}	= 10^{12}

(b) Medium-range forecast, global

I	II
N ~ 1000	1500
Δs = 150 km	75 km
T = 10 days	1 month
M = 5.10^{12}	= 2.10^{14}

(c) Climate prediction/simulation, global

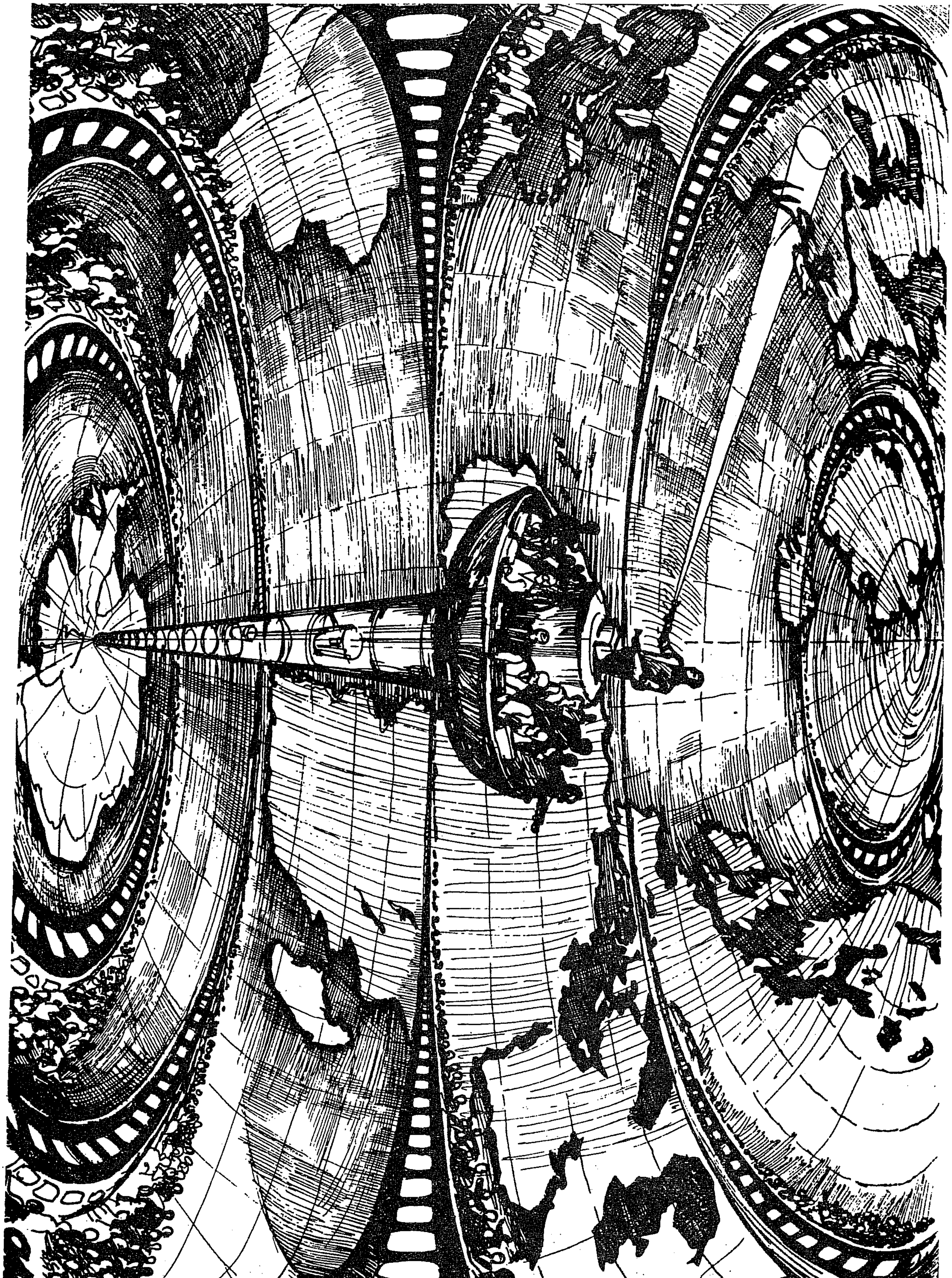
I	II
N ~ 2000	3000
Δs = 300 km	150 km
T = 10 years	100 years
M = 5.10^{14}	= 6.10^{16}

Fig. 1 Weather Forecasting by Numerical methods as envisaged in 1922 by L.F. Richardson

"Imagine a large hall like a theatre, except that the circles and galleries go right round through the space usually occupied by the stage. The walls of this chamber are painted to form a map of the globe. The ceiling represents the north polar regions, the tropics in the upper circle, and the antarctic in the pit. A myriad computers are at work upon the weather of the part of the map where each sits, but each computer attends only to one equation or part of an equation. The work of each region is co-ordinated by an official of higher rank. Numerous little "night signs" display the instantaneous values so that neighbouring computers can read them. From the floor of the pit a tall pillar rises to half the height of the hall. It carries a large pulpit on its top. In this sits the man in charge of the whole theatre. One of his duties is to maintain a uniform speed of progress in all parts of the globe. In this respect he is like the conductor of an orchestra in which the instruments are slide-rules and calculating machines. But instead of waving a baton he turns a beam of rosy light upon any region that is running ahead of the rest, and a beam of blue light upon those who are behindhand.

Four senior clerks in the central pulpit are collecting the future weather as fast as it is being computed, and despatching it by pneumatic carrier to a quiet room. There it will be coded and telephoned to the radio transmitting station".

Picture credit: A. Lannerback, Dagens Nyheter, Stockholm.



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