ON THE THERMAL FORCING OF EXTRA-TROPICAL PLANETARY WAVES

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1. Introduction

An important pre-requisite for successful medium and long-range numerical weather prediction is an understanding of the underlying mechanisms behind the maintenance and transition between circulation regimes, not least because it seems that the skill of the forecast may be a function of the circulation regime. For example, Palmer (1988) discusses the use of a PNA index as a predictor for the forecast skill in the medium range.

In recent years there has been a de-emphasis of the role of middle-latitude processes in low frequency variability and so here, partially to redress the balance, attention is focussed on the role of the extra-tropical diabatic heating field in the maintenance of, and transition between circulation regimes. It is suggested that there is a spectrum of thermal response ranging from the classically forced to what we shall call the thermally equilibrated response. We investigate the possibility that certain anomalous weather regimes may lie at the extrema of this spectrum of thermal response. Should this turn out to be the case then it may contribute to our understanding of how the forecast skill depends on circulation regime.

First of all it is important to stress that the diabatic heating Q is strongly controlled by the motion field and is therefore likely to be a strong function of circulation regime. For example the distribution of diabatic heating in middle latitudes is strongly modulated by the position of the storm tracks, being largely associated with latent heat release. If the oceanic storm track is interrupted by a large amplitude ridge associated with a blocked flow downstream, one might expect diabatic heating rates over the storm track to be anomalously low. Thus Q should not be regarded as a fixed function of space and time. In many theoretical studies of thermally forced planetary waves, however, Q is specified (as computed from a residual heat budget calculation, for example) and not allowed to change. Indeed Shutts (1987) has suggested that if one prescribes Q in this way then the magnitude of the thermal response may be under-estimated because the possibility of thermal equilibration is excluded.

To be more precise, consider a model in which Q is represented as a Newtonian process $Q = -\gamma(T - T^*)$, where γ is a relaxation time-constant and T^* is a radiative - convective equilibrium temperature. If T^* happens to be compatible with a solution of the unforced/undamped equations of motion (a stationary, free Rossby wave for example), then the possibility exists that T can closely approach T^*. In other words the wave may equilibrate toward a state in which the diabatic heating rates are vanishingly small. This is the process of thermal equilibration. The dangers of attributing the thermal response to a fixed forcing now become apparent, for the diabatic heating computed as a residual in the thermodynamic equation may be vanishingly small on the scale of the stationary Rossby wave and therefore dominated by noise: consequently we could have little confidence in the linear solution to fixed forcing.

As we shall see the hall mark of such a thermally equilibrated response is a phase-locking of the thermal wave with the T^* field, a dying out of the surface winds, a marked reduction in the diabatic heating rates and almost vertical phase lines. Strong candidates for this equilibrated response are the anomalous regimes identified by Dole (1986) and given the names PAC(+), ATL(+) over the Pacific and Atlantic sectors (see also the 'low index' states identified by Wallace and Hsu, 1985). It appears that these also happen to be flow regimes which are particularly difficult to forecast with skill in the medium range - see Arpe and Klinker (1986).

Here the mechanism of thermal equilibration is investigated using a quasi-geostrophic, hemispheric three level spectral model. It extends the β - plane channel study of Mitchell and Derome (1983) to a sphere.

In section 2 the form of possible equilibrated states is studied by solving for finite amplitude quasi-geostrophic free modes on a hemisphere. Solutions can be readily found if the axi-symmetric component of the flow is such that the function $\Lambda_0 = {dq_0}/{d\psi_0}$, where q_0 is the potential vorticity and ψ_0 is the stream-function, only depends on height. Furthermore, if $\Lambda_0 = \Lambda_0(z)$, then thermal forcing can excite an equivalent barotropic response (no sign changes in the vertical) if the flow is close to resonance.

In section 3 conditions on the zonal flow and the wavy structure of T^* required for equilibration are set out and contrasted with the off-resonant thermal response.

In section 4 the process of equilibration toward these solutions is studied in a three-level quasi-geostrophic hemispheric model forced by a diabatic heating parametrized as a Newtonian process and spun down by Ekman friction at the ground. There is no orography. It is shown that if T^* is chosen to have a free mode form then equilibration toward T^* readily occurs provided that the vertical structure of the wave is consistent with zero pressure perturbation at the surface. The thermally equilibrated wave, characterized by vertical phase lines, zero surface winds and vanishingly small diabatic heating rates, is contrasted with the more familiar off-resonant response.

In section 5 implications of the study are discussed for our understanding and prediction of the northern-hemisphere winter-time long wave pattern and its anomalies. Finally, based on our hypothesis, we speculate on the possible relation between forecast errors and circulation regimes.

2. Finite amplitude free modes on a sphere.

We adopt a diabatic heating of the form $Q = -\gamma(T - T^*)$ and consider those components of T^* that project onto stationary solutions of the inviscid, adiabatic equations of motion: in the present study we choose the class of stationary, free, finite amplitude Rossby waves. It is these components which are of interest here for they can be resonantly excited by thermal processes and, once excited, equilibrated. In linear theory analytical progress is made possible by linearizing about an atmosphere in solid body rotation in which $u_0 \propto \cos(\text{lat})$. However if we are interested in non-linear solutions of the unforced/undamped equations of motion, $\underset{\sim}{V}.\nabla q = 0$, it is unnecessary to neglect any higher order terms or to make such a restrictive assumption about the form of the zonal flow. All that is required is that Λ_0 should only be a function of height: there must be a linear functional relationship between q_0 and ψ_0 at each level in the atmosphere (for more details see Marshall and So, 1988)

We seek solutions to the non-linear equation (all symbols are defined in the appendix)

$$J(\psi, q) = 0 \tag{1}$$

of the form

$$\psi = \psi_0 + \psi'; \quad \psi' = F(h) \sum_{m \neq 0} A_m \bar{P}_n^m e^{im\lambda} \tag{2}$$

where ψ_0 is an axi-symmetric flow and ψ' the departure from it. Thus each wavy component has the same vertical structure F and n of the normalised associated Legendre function. It follows from this form of ψ that the non-linear equation (1) reduces to

$$\frac{G_0}{P_*}\frac{\partial}{\partial h}\left(\frac{P_*}{S}\frac{\partial F}{\partial h}\right) - [\Lambda_0(h) + n(n+1)]F = 0 \tag{3}$$

where

$$\Lambda_0(h) = \frac{dq_0}{d\psi_0} = \frac{-\cos\Theta}{u_0}\frac{dq_0}{d\mu} \quad ; \mu = \sin\Theta \tag{4}$$

Eq.3 has the form of a Schrodinger equation in which the role of the 'potential function' is played by Λ_0 . It relates the vertical structure of the stationary planetary wave to its horizontal structure.

It can be immediately seen that since F is a function of h only, Λ_0 must also be a function h only. The equation is solved by either specifying the integer n and the vertical structure F, thus defining Λ_0 , or specifying Λ_0 and solving the eigenvalue problem in F with n as the eigenvalue. The first method is preferred here since an integral value of n can be easily ensured. We further note that the vertical structure of the waves must be chosen in such a way that the extrapolated value at the surface is zero since, otherwise, Ekman friction will move the flow off resonance.

The equation for the zonal flow satisfying $q_0 = \Lambda_0\psi_0$ is

$$\nabla^2\psi_0 + \frac{G_0}{P_\star}\frac{\partial}{\partial h}\left(\frac{P_\star}{S}\frac{\partial\psi_0}{\partial h}\right) - \Lambda_0\ \psi_0 = -2\mu \qquad (5)$$

The particular integral is $F_0\mu$ (note $\mu = P_1^0(\mu)$ and $\nabla^2\mu = -2\mu$). The complementary function has the same vertical structure as the waves since the vertical structure equations are the same in both cases (the r.h.s. of Eq.5 is zero). In fact the $\bar{P}_n^0$ component could be incorporated into ψ' by allowing m=0 in the definition. Thus the solution is complete with ψ_0 defined by Eq.5. Note that the zonal flow now consists of the solid body rotation and the $\bar{P}_n^0$ component whose amplitude can be adjusted arbitrarily. In this way we overcome the necessity to adopt a solid body zonal flow, as required in the linear problem. Moreover the wavy components ψ' also have arbitrary amplitudes and phases and a rich variety of solutions can be constructed.

To illustrate the possible complexity of free-mode configurations fig.1 shows one such solution for a three-level representation of Eq.3. This planetary wave will be thermally excited in section 4. We choose n=8 and the vertical structure of the waves to be (1,3,5). Λ_0 is a function of height

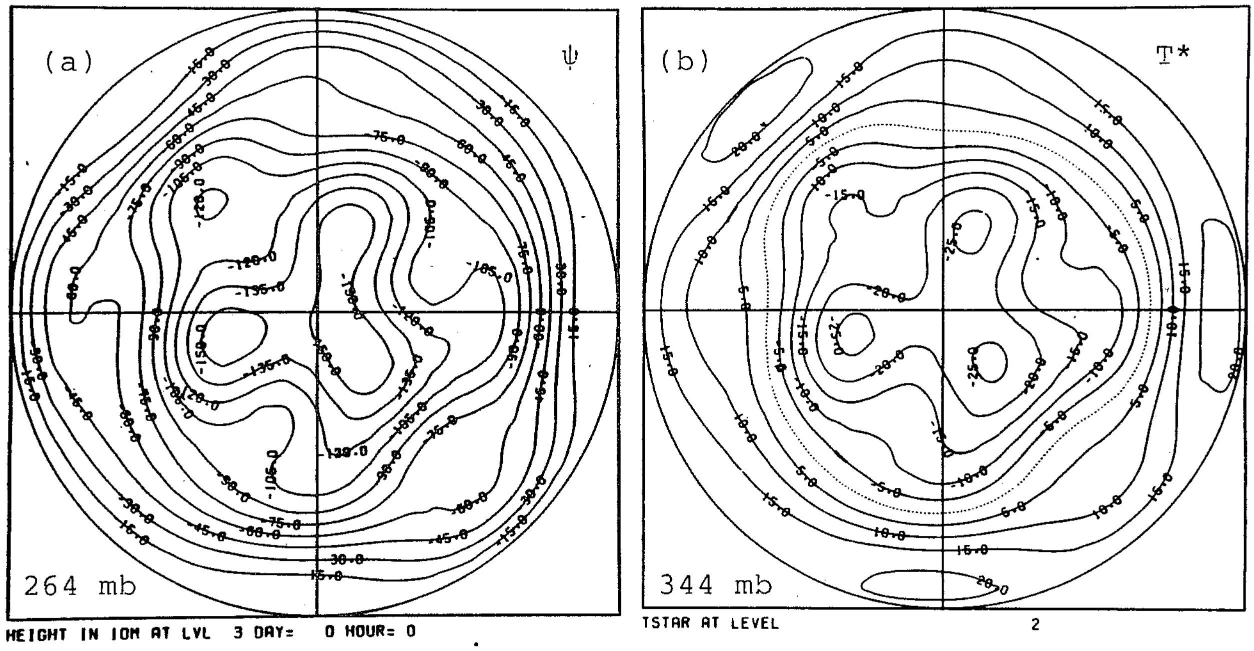


Fig 1 (a) Streamfunction of the free mode with n=8 at 264mb. Contour interval 15 decametres.
(b) T^* of the quasi-free mode at 344mb. Contour interval 5K.

only. The contribution of the particular integral to the zonal flow is the solid body rotation whose velocities at the equator are 1.5, 10.7, and 22.7 m/s at levels 1 to 3. For simplicity, we have set the amplitude of $\overline{P}^0{}_8$ to zero and therefore the flow consists of superposition of solid body rotation and wavy components only. The wavy components are $P^1{}_8$, $P^3{}_8$ and $P^5{}_8$ whose amplitudes and phases are given in the table.

	Amplitude at level 1 to 3 (in decametres)	Phase in wave space
$P^1{}_8$	0.85, 2.55, 4.3	166°
$P^3{}_8$	3.75, 11.25, 18.75	37°
$P^5{}_8$	1.96, 5.89, 9.8	129°

Table 1 Components of a free mode for n=8.

Fig.1a shows the streamfunction at 264mb with wave numbers 3 and 5 dominating at high and middle-latitudes respectively. The troughs of wave number 3 are regions of low T^*, which can be thought of as characterising the continents; the ridges are regions of high T^* characterising the oceans in winter seasons. The maximum temperature difference between 'continents' and 'oceans' along a latitude circle, 13.7 K, is found at 60°N. There are clear regions of strong diffluence and confluence which are not dissimilar to observed flow configurations.

At first sight it might seem that a ψ_0 satisfying Eq.5 places undue restrictions on the form of the zonal flow. However, there is much observational evidence that the time-mean flow is only a small departure from free-mode form and, moreover, is close to free modes in which the streamfunction is linearly related to the potential vorticity at each level in the atmosphere. Derome (1984) draws attention to this remarkable fact and presents supporting observations from the zonal-average monthly mean flow during January 1979. For further observational support and discussion see Marshall and So (1988). Thus the solutions presented here are more than of academic interest.

3. Equilibrated and thermally forced planetary waves

It is now convenient to define i) 'n_z of the zonal flow' to be the n of the free mode (as described in section 2) which has the given zonal configuration and ii) 'n_w of T^*' to be the n of the $\bar{P}^m{}_n$, $m \neq 0$, which describes the wavy horizontal structure of T^*. Obviously, if $n_z=n_w$ then a free mode is obtained. The behaviour of the wave can be studied as n_w moves away from n_z with a simple linear model which is thermally forced by a term proportional to $(T^* - T)$ - see Marshall and So (1988). The amplitude of the wave response is found to be a maximum and the diabatic heating zero, when $n_z=n_w$. This is the resonant response and is the free mode of section 2. When $n_w>n_z$ and bottom friction is applied, the wave amplitude drops sharply, the heating rate increases, the response pattern moves downstream with respect to T^*, the phase lines tilt westward and a strong surface pressure pattern appears. This is the off-resonant thermal response. Physically, the zonal flow speed is larger than the propagation speed of the Rossby wave with $n=n_w$ and so the waves are blown downstream. However, if the zonal wind weakens such that n_z increases towards n_w, or if T^* evolves such that n_w decreases towards n_z, the wave amplitudes are enhanced and the waves retrogress towards the T^* pattern, reducing the heating rate. We call this process 'thermal equilibration'. Similar behaviour occurs for $n_w < n_z$ except that now the wave response is upstream of T^* and the waves tilt eastward with height. In the following numerical experiments, we show that a free mode can be grown from an axi-symmetric flow and study how the the interplay between the strength of the zonal flow and the meridional structure of T^* controls the amplitude and phase of the planetary wave response.

4. Numerical illustrations

A three-level hemispheric spectral model at truncation T15 (described in the appendix) is employed to thermally excite planetary waves. To simplify our discussion and present the equilibration mechanism in its purest form we suppress all baroclinically unstable modes by damping unforced wave modes with m > 4 on a time scale of 1/2 day. We shall have a particular interest in the excitation of planetary waves which have an equivalent barotropic vertical structure (no sign changes in the vertical).

Numerical experiments show that free modes can be readily excited and maintained (both grown from zero amplitude on an axi-symmetric flow or maintained close to an initial free configuration) but only if a constraint is applied on the barotropic mode. We have chosen to relax the (extrapolated) surface wind to zero by introducing a surface Ekman layer $\varepsilon\nabla^2\psi_s$ where ψ_s is the surface pressure perturbation and ε is an Ekman spin-down time. The vertical structure of the planetary wave is chosen so that ψ_s is zero. This is the only flow consistent with a solution of the unforced, undamped equations of motion; not only must $T=T^*$, but also $\psi_s \equiv 0$ to ensure that frictionally driven Ekman layers at the surface are inactive. Thus as the planetary wave equilibrates toward T^* switching off the internal diabatic sources, the surface winds also spin down switching off the mechanically driven vorticity sources at the surface.

If a constraint on the barotropic mode is not applied then it is not possible to equilibrate toward or maintain our equivalent barotropic free mode states because the Newtonian thermal forcing can only constrain the differences between the response at the three levels, not the absolute values of these response.

Before going on to present results from our numerical experiments, it should be mentioned that the solution constructed in section 2 is not a perfect free mode of our numerical model and must be adjusted in two ways. Firstly, the solid body rotation represented by $\psi_0 = \sin\Theta = P^o{}_1(\sin\Theta)$ cannot be exactly represented in the model since the zonal flow is required to be zero at the equator. Thus our hemispheric model does not carry a $P^o{}_1$ component (see appendix). Instead $P^o{}_1$ is approximated by a linear combination of $P^o{}_2$ and $P^o{}_4$ giving a jet at about 30^oN not unlike the observed wind pattern. This zonal configuration is, in fact, more realistic than a solid body rotation which has an easterly maximum at the equator. Secondly, the vertical structure of the zonal flow, being different from that of the wavy components, does not extrapolate to zero at the surface. This problem is overcome by adjusting the zonal wind at level 1 to ensure that the extrapolated surface wind is indeed zero. This amounts to reducing the pole to equator temperature gradient at the lower thermodynamic level. With these two modifications, the quasi-free mode has a zonal jet at 30^oN with speeds of 3.8, 11.4 and 24.1 m/s at the three levels and pole to equator temperature differences of 31.4 K and 40.9 K at the lower and upper thermodynamic levels respectively. The T^* field of the quasi-free mode is shown in fig.1b and is hardly distinguishable from the T of the free mode (not shown here) corresponding to fig.1a.

i) Equilibration at resonance

The model is initialised with an axi-symmetric zonal flow and thermally forced towards the quasi-free mode. The relaxation timescale for the wavy components of T^* is 7 days with an Ekman spin-down time of 3.75 days. At day 40,the model has settled to

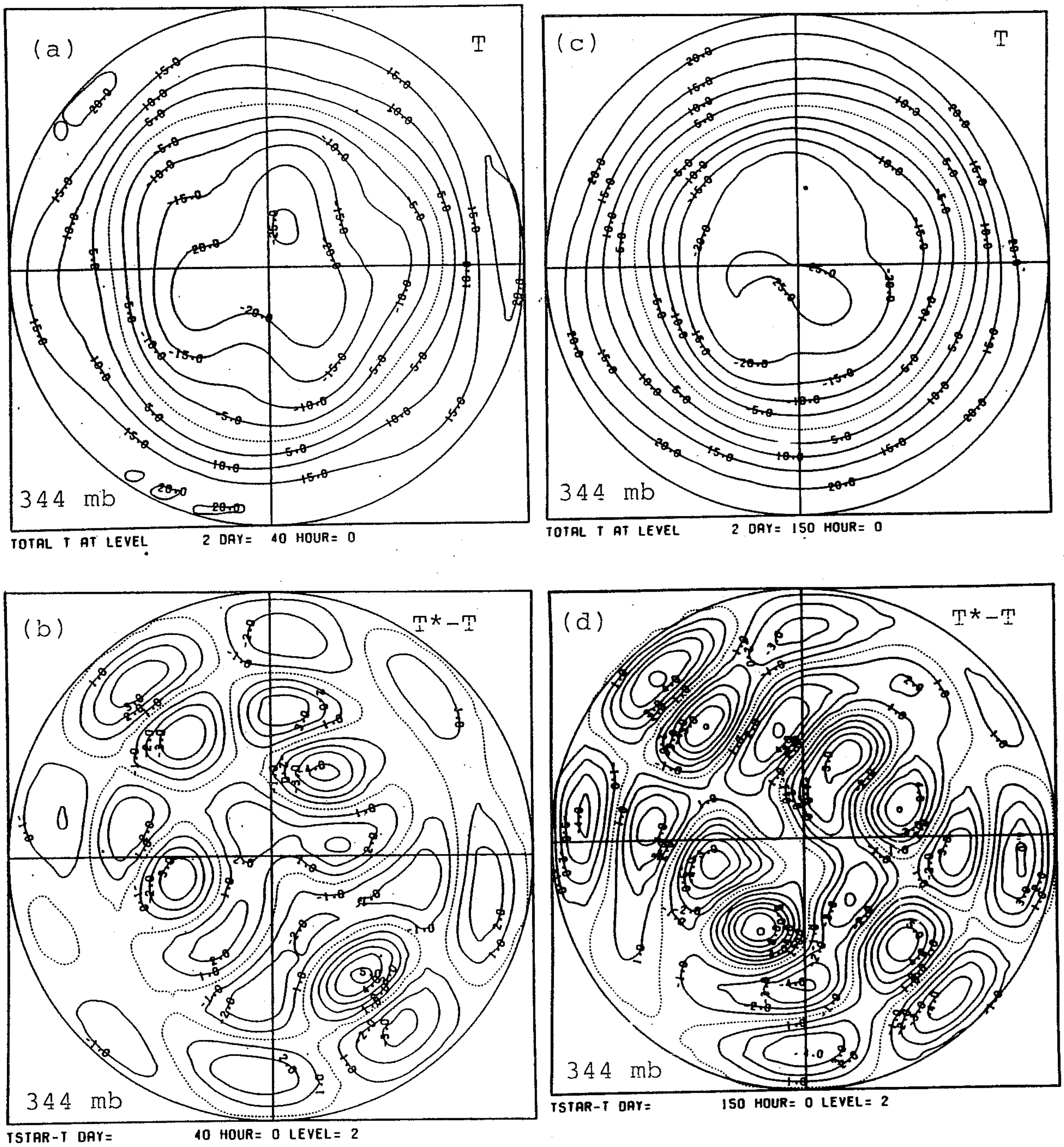


Fig 2 (a) T at 344mb near equilibration on day 40. Contour interval 5 K.
(b) T^*-T at 344mb near equilibration on day 40. Contour interval 1 K.
(c),(d) as (a) and (b) but for the off-resonant response on day 150.

a quasi-steady state close to the quasi-free mode, i.e a near equilibrated response is attained. Fig.2a shows T at 344mb and fig.2b ($T^* - T$) at 344mb. The essential characteristics of the free mode of fig.1 are reproduced: domination by wave number 3 and 5 at high and middle-latitudes respectively, the pattern of ridges and troughs, nearly vertical phase lines, a weak surface pressure pattern (the maximum amplitude is 3mb) and low diabatic heating rates (on average less than 0.5 K/day). The phase-shift of T relative to T^* is at its maximum only 12 degrees of longitude. Scatter plots show a good functional relationship between q and ψ at level 2 and 3 but with some bending of the curves; nearer the surface at level 1 there is somewhat more scatter. It is noteworthy that the thermal equilibration mechanism can operate successfully under these less-than-ideal conditions: indeed it is tempting to speculate that the bending of the q / ψ plots blurs the resonance condition enabling it to be satisfied more easily.

ii) The forced, off-resonant response

The non-equilibrated response can be illustrated by strengthening the zonal wind (decreasing n_z) beyond that which can support the stationary, free Rossby wave whilst keeping the wavy components of T^* unchanged (n_w=8). Accordingly we set $n_z = 7$; the corresponding zonal flow consists of P^o_2 and P^o_4 with a jet at 30°N of strength 6, 18, 32 m/s at level 1 to 3, 2.2, 6.6 and 7.9 m/s stronger, respectively, than those of the quasi-free mode. The steady response is shown in figs.2c and 2d. The strong zonal flow has indeed blown the waves downstream: troughs in T now appear downstream of troughs in T^* rather than in phase with them. Furthermore, the wave amplitude has markedly decreased and the heating rate increased relative to the equilibrated response. In fact the flow now has all the hall-marks of a thermally forced response with a strong surface pressure pattern, and the planetary waves tilting westwards with height.

It is worth noting in passing that $T^* - T$, fig. 2d, which could also be thought of as an anomaly field, might be mistaken for a

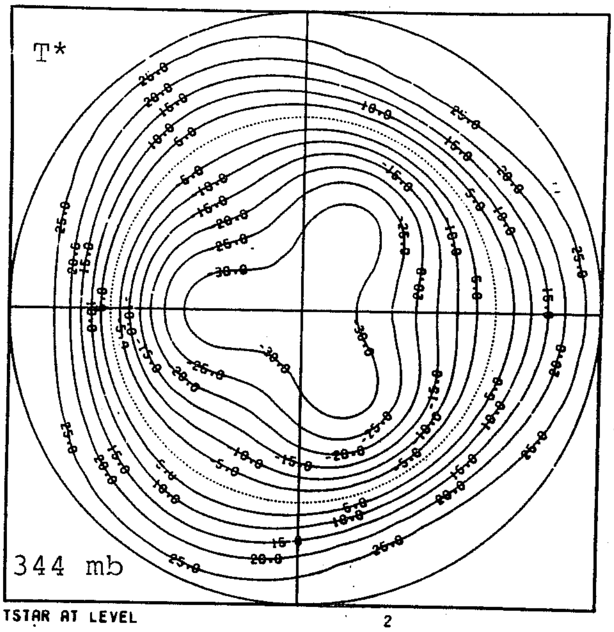


Fig.3 T^* of the quasi-free mode at 344mb for n=6. Contour inteval 5 K.

wave-train propagating from the tropics. This pattern is not, of course, associated in any way in our model with wave propagation but is merely the difference between the forced and equilibrated responses.

iii) Oscillation of wave-number three

The numerical experiments described above illustrate the two extremes in the spectrum of thermal response. It can be envisaged that the thermally forced component of the planetary wave pattern will exhibit both responses, but the degree to which either limit is achieved will depend on the extent to which T^* projects onto stationary, free modes. This spectrum of response is well illustrated in the following experiment in which wavenumber three is thermally excited and subsequently proceeds to spontaneously oscillate between equilibrated and forced responses, the period of the oscillation being set by the radiative - convective relaxation time-scale. The zonal flow commensurate with $n_z=6$ is strengthened by 3 m/s at each model level to give a jet at 30^oN of strength 8, 24, and 42 m/s. The model is initialised with T set equal to a T^* given by $P^3{}_6$ (shown in fig.3) corresponding to $n_w=6$. The parameters of the model are as before except that now the thermal relaxation timescale is 10 days.

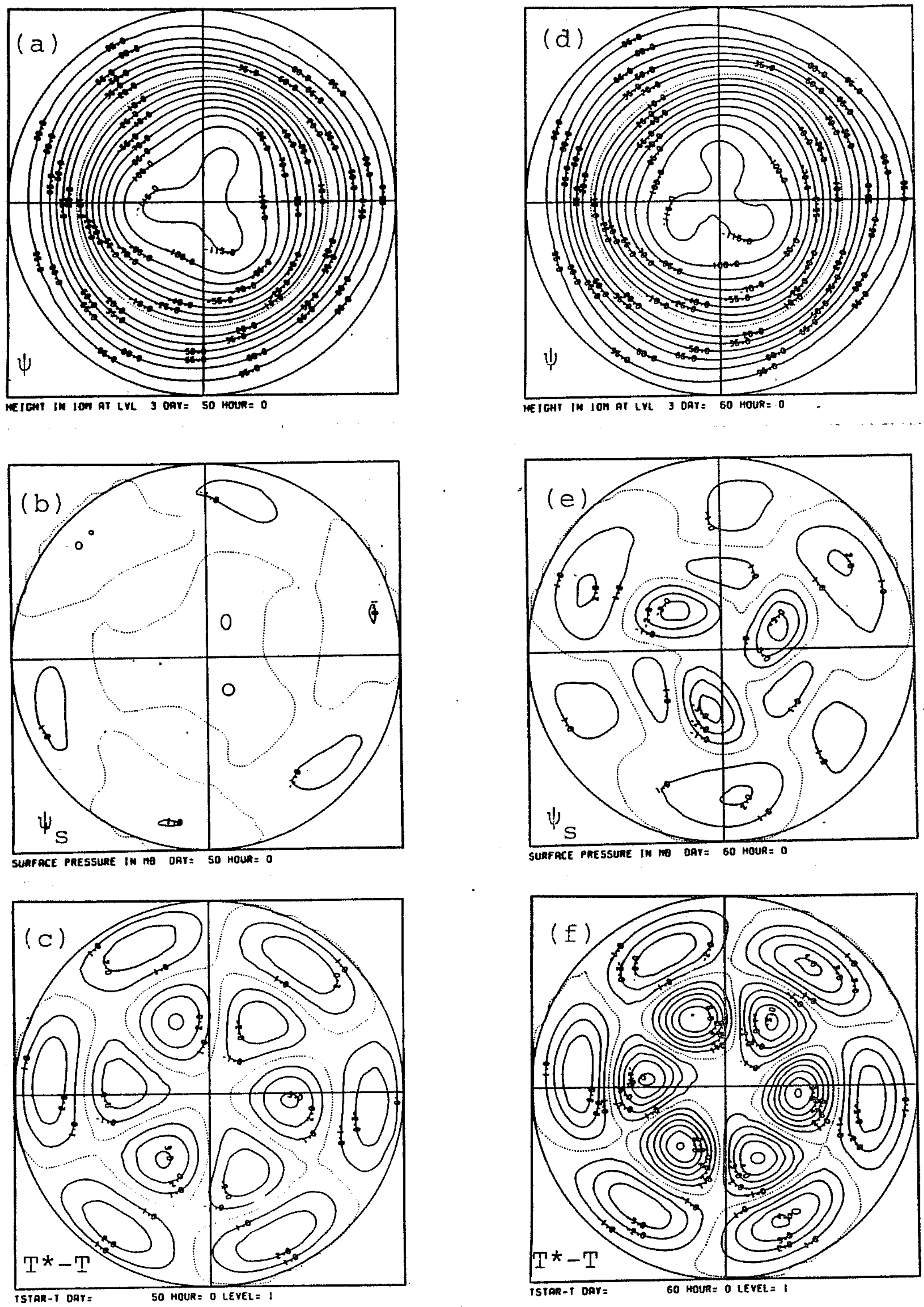


Fig 4 The equilibrated (day 50, a, b and c) and off-resonant (day 60, d,e and f) responses of wave number three with n=6.

(a) and (d): Streamfunction at 264mb. Contour interval 15 decametres.

(b) and (e): Surface pressure pattern. Contour interval 1mb.

(c) and (f): T^{*}-T at 587mb. Contour inteval 1 K.

After some initial adjustment wavenumber 3 begins to oscillate between forced and equilibrated states. By day 50 it is close to equilibration; from figs.4 a) to c) it can be seen that the surface pressure pattern is featureless, the phase lines vertical, T almost in phase with T^* and the diabatic heating rates very low (in general less than 0.2K/day). However, 10 days later, figs.4 d) to f) the wave has evolved into a more familiar state: T is now downstream of T^*, there is a strong surface pressure pattern, large phase shifts in the vertical (nearly 180° at high latitudes) and heating rates more than twice that at day 50. Wavenumber three continues to evolve until by day 75 it is near to equilibration again. Evidently a steady state cannot be achieved because the T^* towards which the waves are relaxing does not project sufficiently onto the free modes of the model.

We have here, then, a thermal mechanism capable of producing internal variability on radiative - convective time scales: the radiative 'spring' pulls T back toward T^* on γ^{-1} time-scales only for it to be blown downstream again by the strong westerlies.

5. Discussion

We have studied the thermal equilibration mechanism in a three-level hemispheric model driven by a Newtonian heating in an extension to a sphere of the β-plane channel studies of Mitchell and Derome (1983). If a T^* is chosen to be compatible with a superposition of finite amplitude Rossby waves which are stationary on the hemisphere, then resonance and equilibration toward T^* is readily achieved provided a constraint on the surface wind is applied: i.e. that it, too, equilibrates to zero. A large amplitude wave can be maintained close to T^* with vanishingly small diabatic heating rates. Longitudinally confined solutions can also be grown by exciting the appropriate spherical harmonics - see also Haines and Marshall (1987) for a discussion of spatially isolated free modes (modons) as a prototype of atmospheric blocking.

The more familiar thermal response can be illustrated by, say, changing the zonal wind or the form of T^* so that it is no longer compatible with a free solution. In this case a thermal trough appears downstream of the region of maximum cooling with the planetary waves tilting westwards (or simply changing sign in the vertical), strong surface pressure patterns and large diabatic heating rates.

Our numerical experiments show that the planetary waves can switch from one extreme to the other on radiative - convective relaxation time scales and suggest that this may be an important mechanism modulating low frequency variability in middle latitudes.

There may be some merit in considering the anomalous weather regimes, ATL(±) and PAC(±) identified by Dole(1986), as opposite extremes of a thermal response typified by thermally equilibrated and forced solutions, respectively. For example the PAC(+) pattern (equivalent to the negative phase of the PNA) is characterized by a large amplitude ridge over the eastern Pacific, the trough over the east-coast of North America is upstream of its normal position and the Aleutian low is markedly weakened. These are all hall-marks of an equilibrated response. We also note that systematic errors in medium and long-range forecasts are a strong function of circulation regime - see, for example, Arpe and Klinker (1986) - and seem to be largest when the planetary wave amplitude is a maximum corresponding to ridging over the oceans. Since equilibration toward free mode form relies on the simultaneous 'switching off' of the boundary layer physics and internal diabatic heating, great demands on the physical parametrizations are made and so errors in the 'physics' are likely to compromise the ability of the model to sustain these states. Conversely, the more zonal off-resonant response, with strong surface winds and fluxes and large diabatic heating rates, may not be so sensitive to such errors.

Finally it is fascinating to observe that the difference fields between equilibrated and forced responses computed from our model are very reminiscent of wave trains propagating along great circles!-see, for example, fig.2d. Indeed the anomaly patterns over the Pacific documented by Dole(1986) are the two phases of the ubiquitous 'PNA' pattern. Could it be that the 'PNA' pattern is in part a consequence of the planetary waves locally switching between these two thermal states? Furthermore, errors in the forecast model tend to have the appearance of wave trains and are often attributed to Rossby wave propagation triggered by errors in the tropical heating field. The present study provides an additional mechanism; the anomaly fields between equilibrated regimes and the climatology have a similar characteristic signature.

Appendix

We use standard quasi-geostrophic equations expressed in spherical polar geometry for a dry atmosphere using $h=\ln(p_o/p)$ as a height co-ordinate where p_o is the constant average surface pressure. The non-dimensional potential vorticity equation is

$$\frac{\partial q}{\partial t} + J(\psi,q) = \frac{G_0}{P_*}\frac{\partial}{\partial h}\left(\frac{P_* Q}{S}\right) \qquad \text{(A1)}$$

where the time and length scales are Ω^{-1} and radius of the earth, a, respectively,

$$q = \nabla^2\psi + 2\sin\Theta + \frac{G_0}{P_*}\frac{\partial}{\partial h}\left(\frac{P_*}{S}\frac{\partial\psi}{\partial h}\right)$$

is the potential vorticity and ψ is the streamfunction,

$$P_* = P/P_0 \quad ; \qquad G_0 = \sin {}^{\pi}/_{4}$$

$$S = R\left(\frac{\partial T_0}{\partial h} + \varkappa T_0\right)/(f_0\Omega a^2)\text{, the static stability}$$

$$Q = \left(\begin{array}{c}\text{rate of heating}\\ \text{in K/sec}\end{array}\right) \times R/(f_0\Omega^2 a^2)$$

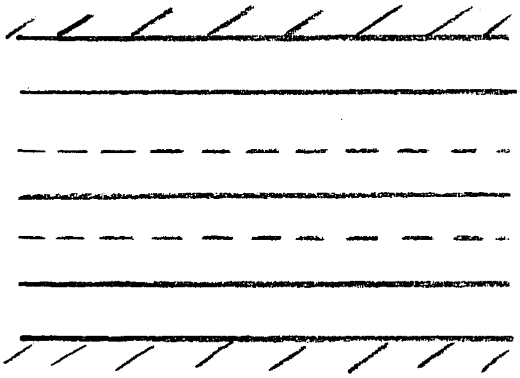
and f_o , T , R , $\varkappa$, have their usual meanings.

In Eq.A1 the Jacobian of ψ and q is written in spherical co-ordinates thus

$$J(\psi,q) = \frac{\partial}{\partial\mu}(Vq) + \frac{1}{1-\mu^2}\frac{\partial}{\partial\lambda}(Uq) \quad ; \quad \begin{array}{l} V = v\cos\theta \\ U = u\cos\theta \end{array}$$

where μ = sin(latitude) and λ is the longitude.

A conventional level discretization is used with, in our case, two thermodynamic levels mid-way between three kinematic levels as shown in the diagram



$\omega = 0$ at 202 mb
ψ_3 at 264 mb
T_2 at 344 mb
ψ_2 at 450 mb
T_1 at 587 mb
ψ_1 at 766 mb
ψ_s at 1000 mb

The horizontal streamfunction fields at each level of the model are expanded into spherical harmonics of odd parity for the wave fields and even parity for the zonal-mean fields.

$$\psi_0 = \sum_{\substack{n=2 \\ n\ \text{even}}}^{14} \psi^0{}_n(h,t)\,\bar{P}_n^0(\mu)$$

$$\psi' = \sum_{m=1}^{14} \sum_{\substack{n=m+1 \\ n-m\ \text{odd}}}^{15} \left(\psi^m_{cn}(h,t)\cos m\lambda + \psi^m_{sn}(h,t)\sin m\lambda\right)\bar{P}^m_n(\mu)$$

where $\bar{P}^m{}_n$ are the normalised associated Legendre functions. This mixed parity formulation is required in this hemispheric model because it automatically ensures that the equator acts as a 'wall'.

Evaluation of divergence terms is accomplished by the grid-transform technique.

The model was built in the Atmospheric Physics Group of the Department of Physics, Imperial College, by Drs. Mansbridge, Shutts and White. For further details see Shutts (1983).

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