

THE HADLEY CIRCULATION

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1. INTRODUCTION

This lecture is mainly a brief review of material developed over the last 13 years. Most, but not all, of this material is already in print. During this period there has been a notable return to the view that the thermally driven Hadley circulation is a central component of the general circulation. For several preceding decades there was a tendency to view the Hadley circulation as an eddy driven residual component of the flow. It is now recognized that the thermally driven Hadley circulation plays the major role in maintaining the angular momentum of the subtropics which, in turn, maintains the eddies of midlatitudes (Palmen and Newton, 1969, Wallace, 1978). The subtropical jet results from a balance between the advection of angular momentum by the thermally driven Hadley circulation and its 'dissipation' by the eddies. While these contentions are by no means rigorously established, it now appears that the thermally driven Hadley circulation may play a major role in determining the eddy heat transport; also, to the extent that the Hadley circulation helps determine the position of the subtropical jet, it may also play a major role in determining the response to planetary scale stationary forcing (Nigam and Lindzen, 1989). In this paper we will review the simple theory of the symmetric Hadley circulation, its crucial modification when heating is not centered over the equator, and the factors which determine the magnitude of the Hadley circulation.

2. REVIEW OF SIMPLE THEORY OF SYMMETRIC HADLEY CIRCULATION

This section is completely based on Schneider and Lindzen (1977), Schneider (1977), and Held and Hou (1980). All this material can also be found in Lindzen (1990). Only a cursory treatment will be presented here. Schneider and Lindzen (1977) examined whether, in the presence of forcing due to the observed annually and zonally averaged distribution of precipitation, the inclusion of cumulus friction could lead to a reasonable

simulation of the observed annually averaged Hadley circulation¹. Schneider (1977) examined whether in the limit of no friction, nonlinear angular momentum transport could lead to a reasonable simulation of the Hadley circulation. Both cases led to similarly good simulations. In both cases the Hadley circulation was weaker than observed. Since, in the presence of nonlinearity, the Hadley circulation was simulated regardless of the presence of cumulus friction, subsequent studies focussed on the nonlinear case. An example of Schneider's (1977) results is worth looking at. Such an example is shown in Figure 1. The most important features to note are i) the excessive amplitude of the subtropical jet, and ii) the finite extent of the Hadley circulation. The tropopause jump arises from the use of a radiative convective state with a tropopause around 250 mb, and the elevated tropical tropopause arises from the distribution of cumulus heating in the neighborhood of the equator, and the horizontal homogenization of temperature by the Hadley circulation. Schneider (1977) recognized certain features of the solution which enabled quick estimates in the absence of detailed computations. In particular he observed that (i) zonal angular momentum was approximately conserved on the upper, poleward branch of the Hadley circulation, and (ii) that the resulting zonal winds were in approximate cyclostrophic balance. In addition, the lapse rate was determined by a balance of cumulus heating and radiative cooling, and the intensity of the Hadley circulation was largely determined by the deviation of the final temperature from an equilibrium meridional distribution. These features were exploited in detail by Held and Hou (1980), who examined a Hadley circulation in a simplified Boussinesq geometry. Held and Hou (1980) not only carefully delineated the simplified 'equal-area' arguments of Schneider (1977), but were able to examine exactly how careful and complete nonlinear numerical solutions approached the simplified estimates in the limit of decreasing viscosity.

For purposes of discussion, it is necessary to recapitulate the simplified arguments. In addition to the two numbered items above, Held and Hou also utilized the following constraints: (iii) surface winds are small compared to upper level winds, and (iv) thermal

¹Actually our original motivation was the wish to explain the reversed meridional temperature gradient near the tropical tropopause.

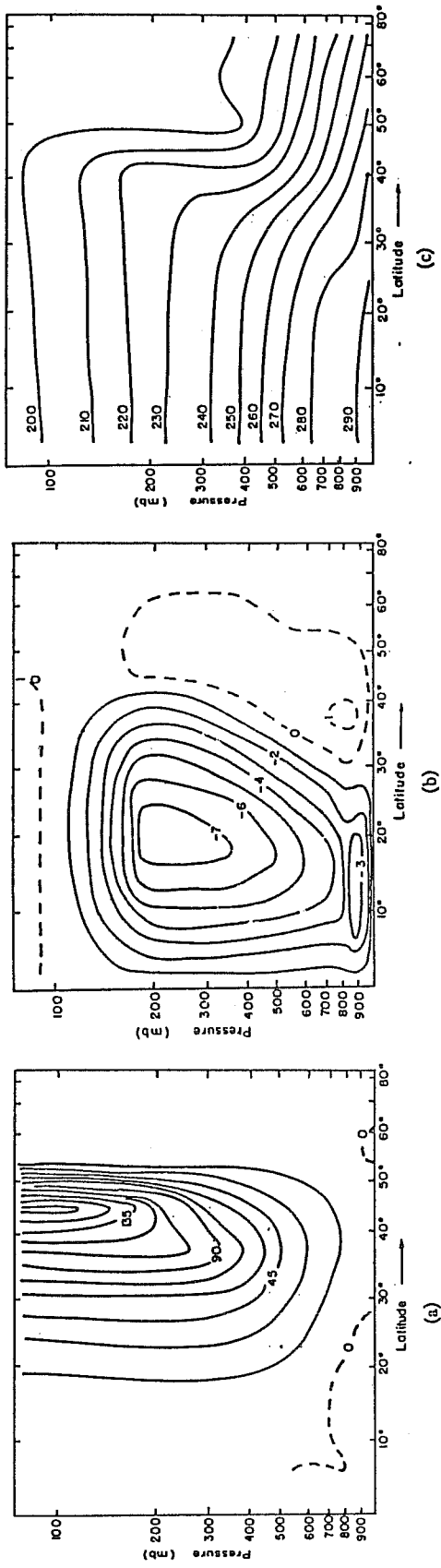


Figure 1. Example of an eddy-free symmetric circulation found by Schneider(1977). Panel (a) shows contours of zonal wind (contour intervals of 15ms^{-1}). Panel (b) shows streamfunction contours (contour interval of 10^{12}gs^{-1}). Panel (c) shows temperature contours (contour intervals of 10K).

diffusion is not of dominant importance in the heat equation. In their Boussinesq geometry, there was a well defined top to the atmosphere at $z=H$. Item (i) above implies

$$u(H,\phi) = u_M = \frac{\Omega a \sin^2 \phi}{\cos \phi} \quad (1)$$

Item (ii) implies

$$fu + \frac{u^2 \tan \phi}{a} = -\frac{1}{a} \frac{\partial \Phi}{\partial \phi} \quad (2)$$

Evaluating Equation 2 at $z=H$ and $z=0$ and subtracting the results, together with the use of an integrated version of the hydrostatic relation leads to a simplified 'thermal wind' relation

$$f[u(H) - u(0)] + \frac{\tan \phi}{a} [u^2(H) - u^2(0)] = -\frac{gH}{a\Theta_0} \frac{\partial \bar{\Theta}}{\partial \phi} \quad (3)$$

Substituting relation (iii) and Equation 1 into Equation 3 yields

$$2\Omega \sin \phi \frac{\Omega a \sin^2 \phi}{\cos \phi} + \frac{\tan \phi}{a} \frac{\Omega^2 a^2 \sin^4 \phi}{\cos^2 \phi} = -\frac{gH}{a\Theta_0} \frac{\partial \bar{\Theta}}{\partial \phi} \quad (4)$$

which can be integrated with respect to ϕ to obtain

$$\frac{\bar{\Theta}(0) - \bar{\Theta}(\phi)}{\Theta_0} = \frac{\Omega^2 a^2 \sin^4 \phi}{gH 2 \cos^2 \phi} \quad (5)$$

Held and Hou assume a linear heating law of the form

$$Q = \frac{\Theta_E - \Theta}{\tau} \quad (6)$$

For this heating law, one determines the temperature at the equator and the extent of the Hadley cell by assuming continuity of temperature at ϕ_H

$$\overline{\Theta}(\phi_H) = \overline{\Theta}_E(\phi_H) \quad (7)$$

and that there be no net diabatic heating over the extent of the Hadley circulation

$$\int_0^{\phi_H} \overline{\Theta} \cos \phi d\phi = \int_0^{\phi_H} \overline{\Theta}_E \cos \phi d\phi \quad (8)$$

Equations 7 and 8 simply tell us that the Hadley temperature distribution is obtained by shifting the solution in Equation 5 up and down until areas of heating and cooling are equal. This procedure is illustrated in Figure 2, and is known as the 'equal area' argument -- a name that has come to be attached to the whole simplified procedure. The vertically integrated flux of potential temperature is easily obtained

$$\frac{1}{H} \int_0^H \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (v \Theta \cos \phi) dz = \frac{\overline{\Theta}_E - \overline{\Theta}}{\tau} \quad (9)$$

and if one further assumes that static stability is unaffected by the circulation; i.e.,

$$\frac{\Theta(H) - \Theta(0)}{\Theta_0} \approx \Delta_V \quad (10)$$

then one can readily estimate the mass flux, V , in the upper and lower branches of the Hadley circulation

$$\frac{1}{\Theta_0} \int_0^H v \Theta dz \approx V \Delta_V \quad (11)$$

The flux of momentum in the upper branch is then approximately $V u_M$, and the divergence of this flux must be balanced by surface drag which then gives us an estimate of the component of the surface flow induced by the Hadley circulation.

One feature of the above argument should be emphasized for future reference. Equation 9 clearly shows that V may be increased by concentrating the maximum in Θ_E in a narrower meridional region. As it turned out, however, no amount of concentration was able to yield an intensity of the Hadley circulation comparable to the observed annual

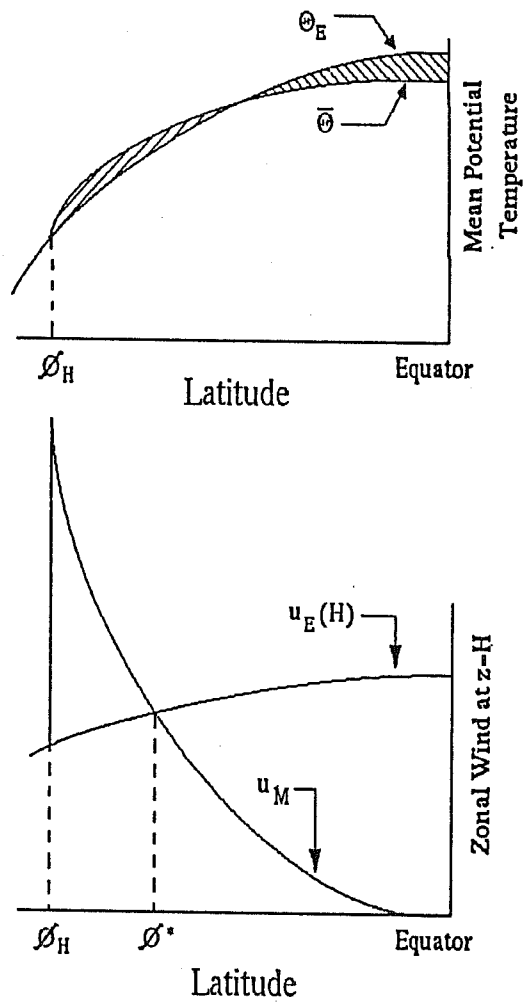


Figure 2. Schematic drawings of the vertical mean potential temperature distribution (upper figure) and the zonal wind distribution at the top of the Hadley circulation (lower panel). With Newtonian cooling (linear in Θ), conservation of potential temperature requires that the shaded areas be equal. Note that this idealized circulation increases the baroclinicity of the flow between ϕ^* (where $u_E = u_M$) and ϕ_H . From Lindzen (1990).

average for heating centered on the equator.

Figures 3-6 show how the numerical solutions for various quantities approach the results from the above procedure as the viscosity is reduced. In an important way, however, the angular momentum on the upper branch of the Hadley circulation forms a significant exception. Clearly, the angular momentum on the upper branch is not characterized by the angular momentum at the equator. What is going on is shown in Figure 7. A boundary layer always forms at the top where momentum from the entire upward region is mixed, and the upward moving region is never restricted to the equator.

3. HEATING CENTERED OFF THE EQUATOR

The Hadley circulation, when heating (Θ_E in the simplified treatment) is centered at the equator, is a highly degenerate system. Maximum heating, maximum updraft, and the separation between northern and southern cells all coincide at the equator. The situation changes dramatically when the maximum in Θ_E is moved off the equator. This new situation was dealt with in Lindzen and Hou (1988). Figure 8 schematically illustrates the new situation. The separating latitude between the northern (summer) and southern cells (ϕ_1) is now distinct from the latitude of maximum Θ_E (ϕ_0). Moreover, the updraft at ϕ_1 is now relatively weak; maximum updraft tends to be associated with ϕ_0 . Also, the extents of the summer and winter cells are different. In attempting to use the 'equal area' type arguments, one must now apply Equation 8 separately to each cell; continuity of temperature must now be applied not only at ϕ_{H+} and at ϕ_{H-} , but also at ϕ_1 ; symmetry no longer guarantees the last item. Figure 9 shows the dependence of ϕ_1 , ϕ_{H+} , and ϕ_{H-} on ϕ_0 ($\Delta_H=1/3$ corresponds to Θ_E having a pole to equator range of 96°C , while $\Delta_H=1/6$ corresponds to Θ_E having a pole to equator range of 48°C ; only the latter is relevant to the current discussion.). We see that ϕ_1 grows rapidly with small increases in ϕ_0 , and that ϕ_{H+} and ϕ_1 converge towards each other as ϕ_0 increases. Figure 9 also shows numerical results for ϕ_1 and ϕ_{H-} . Figure 10 shows 'equal area' results for Θ and Θ_E for $\phi_0=0$ and $\phi_0=6^\circ$. Recall that the intensity of the Hadley circulation is proportional to the difference between Θ and Θ_E . It is evident from Figure 10 that when $\phi_0=6^\circ$, the summer cell is virtually nonexistent. Also, comparing the scales for $\phi_0=0$ and $\phi_0=6^\circ$, we see that the intensity of the circulation (in the winter cell) is much greater than the intensity of the

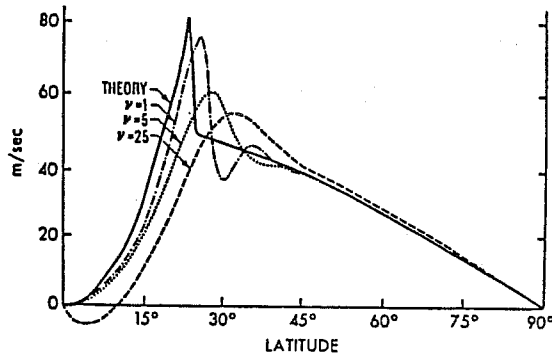


Figure 3. Zonal wind at $z=H$ for three values of ν , compared with the simple model for the limit $\nu \rightarrow 0$ (from Held and Hou, 1980).

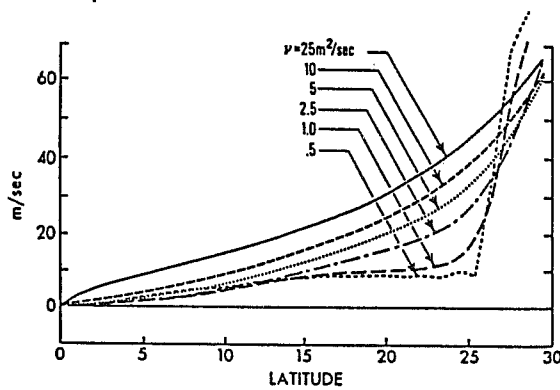


Figure 4. A measure of M (the angular momentum), namely, $((\Omega a^2 - M)/a)$, evaluated at $z=H$ as a function of ϕ for diminishing values of viscosity, ν . Note that zero corresponds to conservation of M (from Held and Hou, 1980).

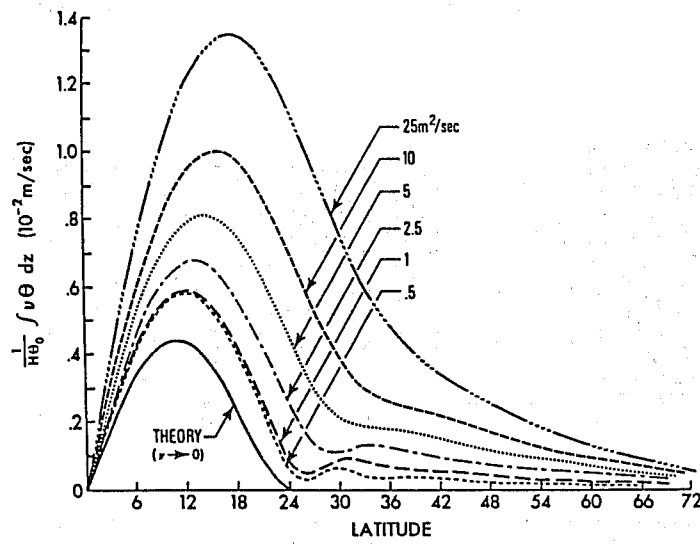


Figure 5. Meridional heat fluxes for various values of ν -- as well as the theoretical limit based on the simple calculations (from Held and Hou, 1980).

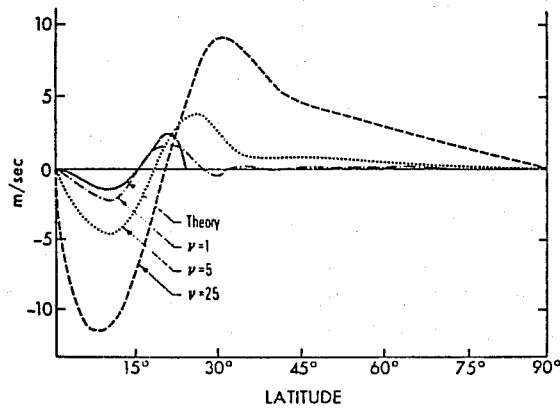


Figure 6. Surface wind for various values of ν , and the theoretical inviscid limit based on simple calculations (from Held and Hou, 1980).

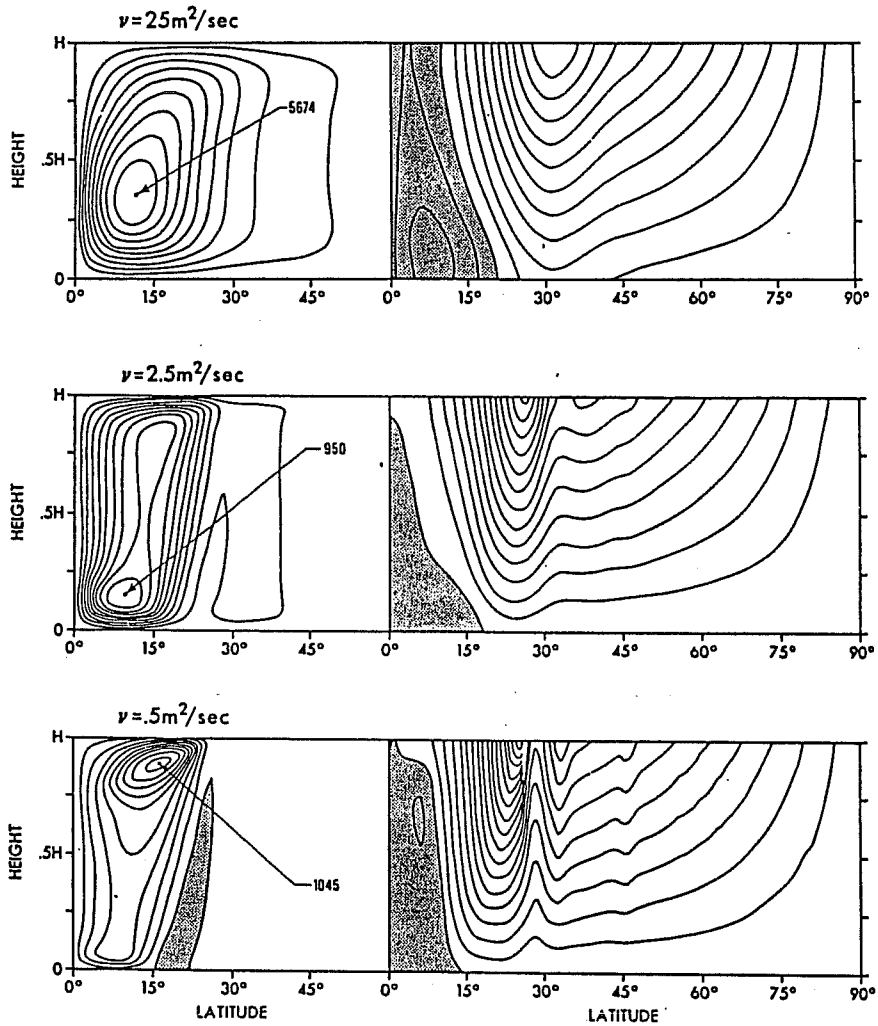


Figure 7. Meridional streamfunctions and zonal winds. In the left part of the figure, the streamfunction ψ is given for $\nu = 25, 2.5,$ and $0.5 \text{ m}^2\text{s}^{-1}$, with a contour interval of $0.1 \psi_{max}$. The value of ψ_{max} is marked by a pointer. The right part of each panel is the corresponding zonal wind field with contour intervals of 5 ms^{-1} . The shaded area indicates the region of easterlies (from Held and Hou, 1980).

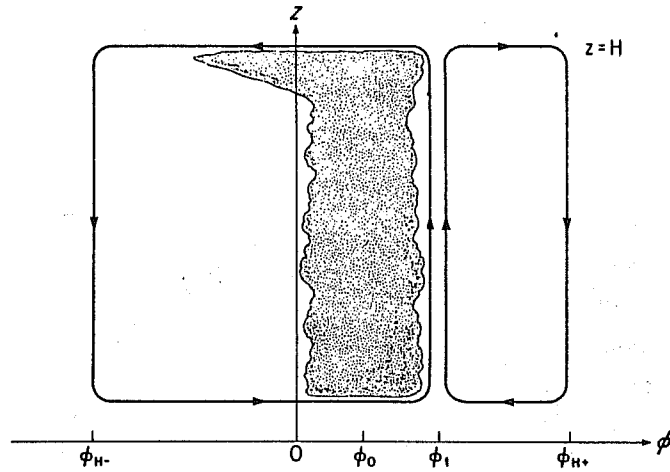


Figure 8. Schematic illustration of the Hadley circulation (from Lindzen and Hou, 1988).

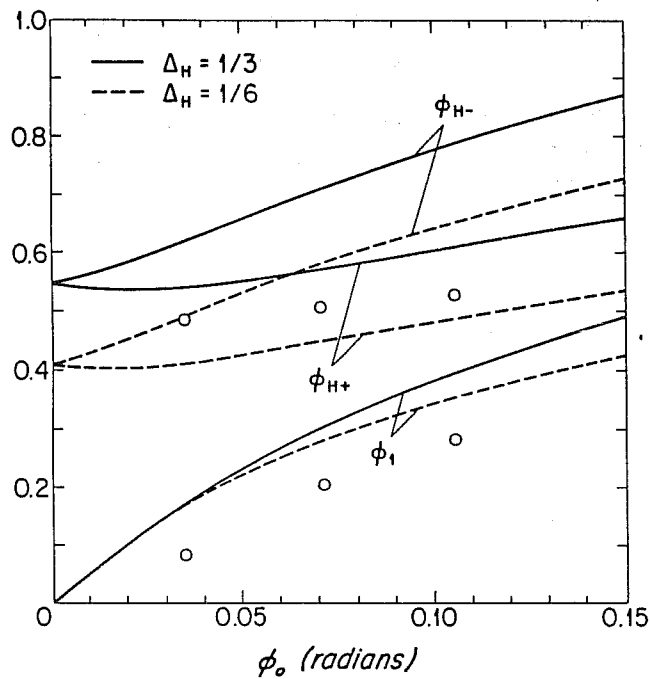


Figure 9. ϕ_1 , ϕ_{H+} , and ϕ_{H-} as functions of ϕ_0 . Open circles show results from numerical integration for ϕ_1 and ϕ_{H+} , when $\Delta_H = 1/6$ (note 1° of latitude ≈ 0.0175 radians.) (from Lindzen and Hou, 1988).

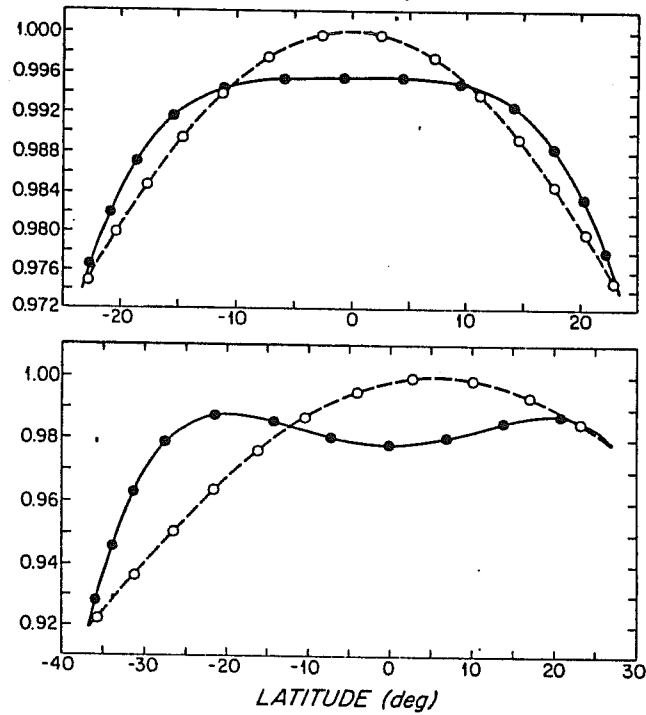


Figure 10. θ/θ_0 (solid line) and θ_E/θ_0 (dashed line) as functions of ϕ using the simple model. The upper panel corresponds to $\phi_0=0$, and the lower panel corresponds to $\phi_0=6^\circ$ (from Lindzen and Hou, 1988).

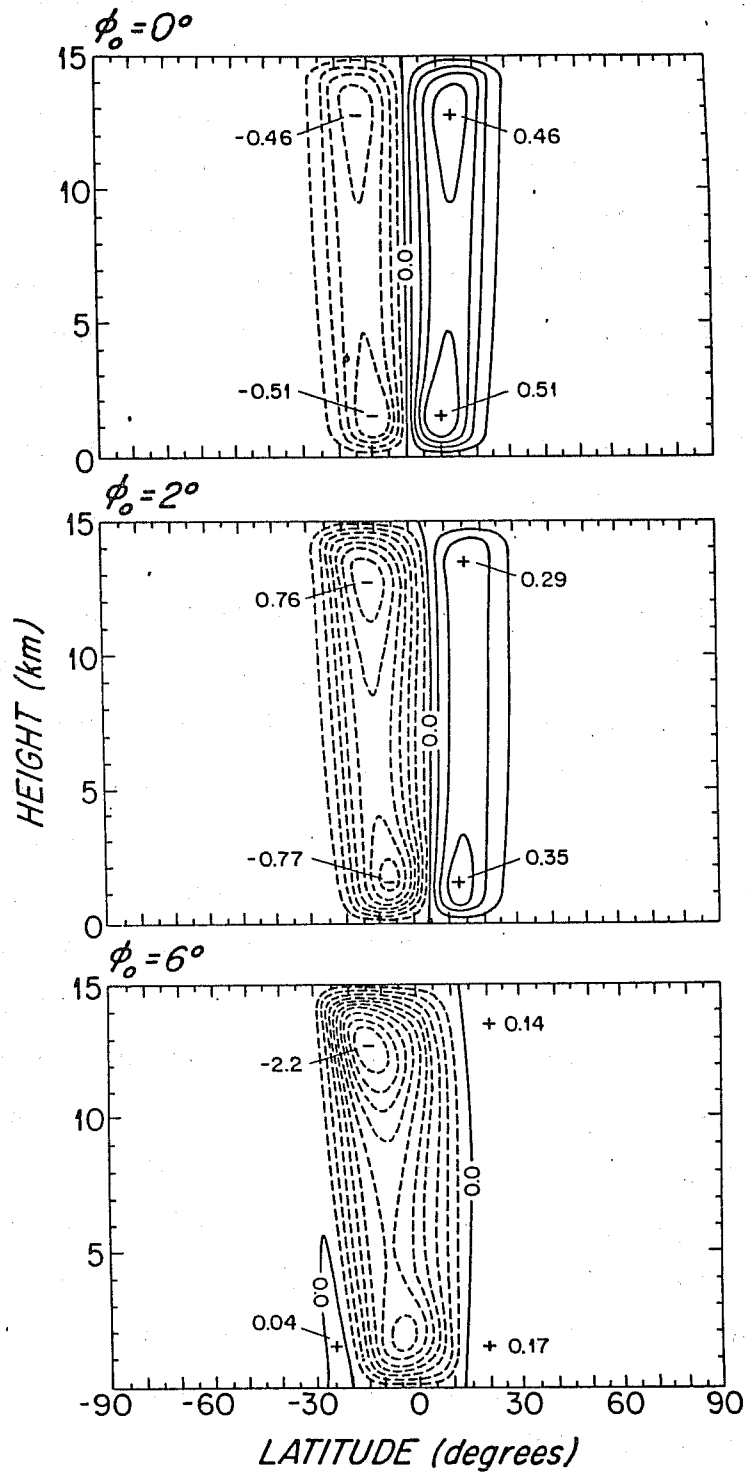


Figure 11. Numerical model results for ψ for (a) $\phi_0=0$, (b) $\phi_0=2^\circ$, and (c) $\phi_0=6^\circ$. Units are in $10^{10} \text{ kg s}^{-1}$ and the contour interval is $0.1 \times 10^{10} \text{ kg s}^{-1}$ for (a) and (b); twice this value for (c) (from Lindzen and Hou, 1988).

equinoctial circulation. Indeed, as LH point out, the annually averaged circulation is almost certainly the average of solstitial circulations and not the equinoctial circulation. Thus far, the results of 'equal area' calculations and numerical calculations are in approximate agreement. However, 'equal area' calculations become almost useless for calculating zonal winds. As noted in Section 2, the 'equal area' arguments assume the angular momentum on the upper branches of the Hadley circulation are characteristic of the 'rest' values at ϕ_1 , while, in reality, it is an average over the entire ascending region. When $\phi_1 = \phi_0 = 0$, the ascending region is at least centered around ϕ_1 , but when $\phi_0 = 6^\circ$, ascent is concentrated near ϕ_0 which is far removed from ϕ_1 . As a result, subsequent discussion will focus on numerical results.

Figure 11 meridional streamfunctions for different choices of ϕ_0 . We see that the least deviation of ϕ_0 from zero leads to an approximately unicellular Hadley circulation. This is consistent with the data analyses of Oort and Rasmussen (1970) which show that the Hadley circulation is almost always unicellular. Figure 12, from Oort (1983), shows average observed winter (DJF) and summer (JJA) Hadley meridional streamfunctions. Because of differences in definition, values in Figure 11 must be multiplied by 2π before comparing with values in Figure 12. The calculated values are still smaller (by a factor of about 2) than the observed values. Figure 13 shows the calculated zonal winds. In the summer hemisphere, we have the thermal wind in balance with the surface temperature gradient -- which is pretty much what is observed. The winter maximum occurs nearer the equator (as observed) and is much larger than observed. Figure 14 shows the surface winds induced by the Hadley circulation. We see that easterlies are concentrated on the winter side of the equator. However, as noted by Lindzen and Nigam (1987), the actual surface winds include significant portions which are directly forced by surface temperature gradients acting on the trade wind boundary layer; this boundary layer is not represented in the present calculations.

4. INCREASING THE INTENSITY OF THE HADLEY CIRCULATION

As we have just noted in Section 3, the intensity of the calculated solstitial Hadley circulation is somewhat weaker than the observed circulation. On the other hand, our forcing (in the form of Θ_E) is likely to be broader than the real forcing, which is likely to

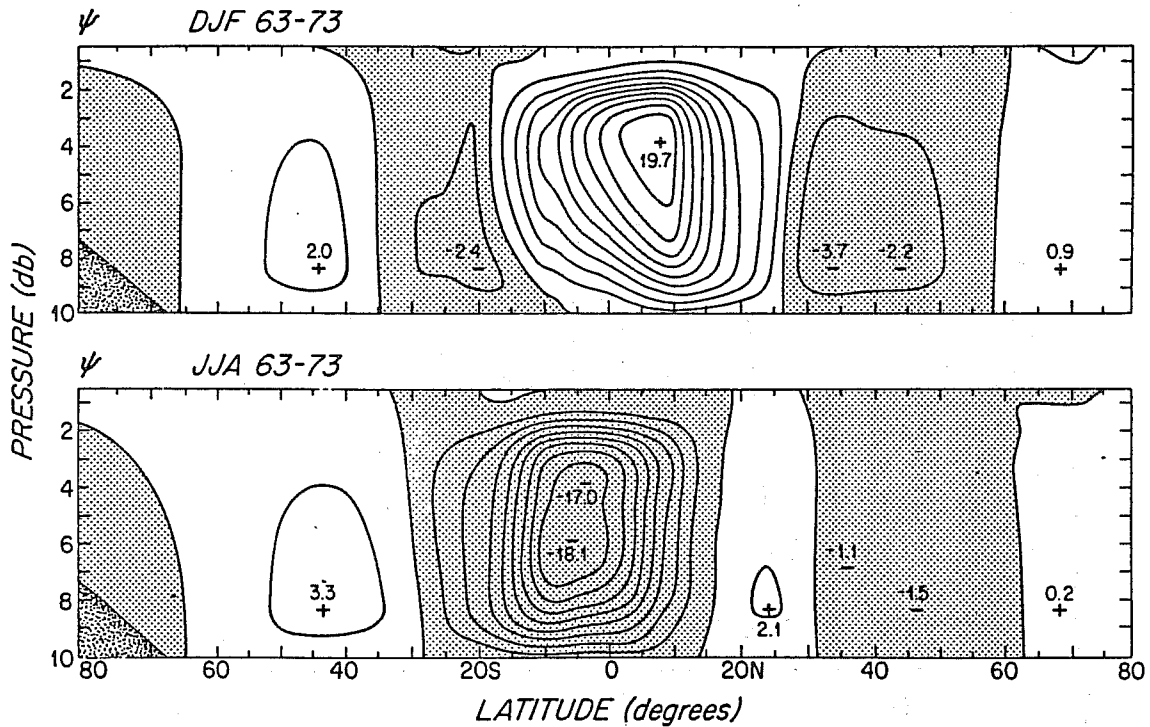


Figure 12. Time average meridional-height cross sections of the streamfunction for the mean meridional circulation. Units, 10^{13} gs^{-1} ; contour intervals, $0.2 \cdot 10^{13} \text{ gs}^{-1}$. December-February 1963-73 (upper panel) and June-August 1963-73 (lower panel) (from Oort, 1983).

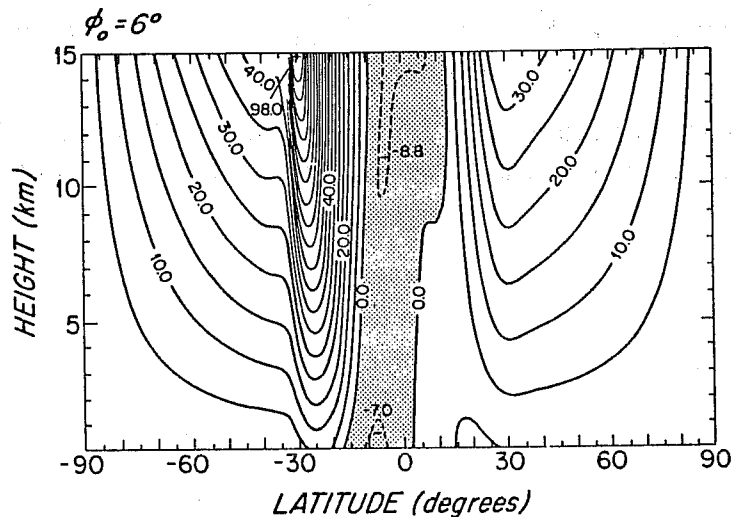


Figure 13. The calculated zonal wind for $\phi_0=6^\circ$ in ms^{-1} . Contour interval of 5 ms^{-1} (from Lindzen and Hou, 1988).

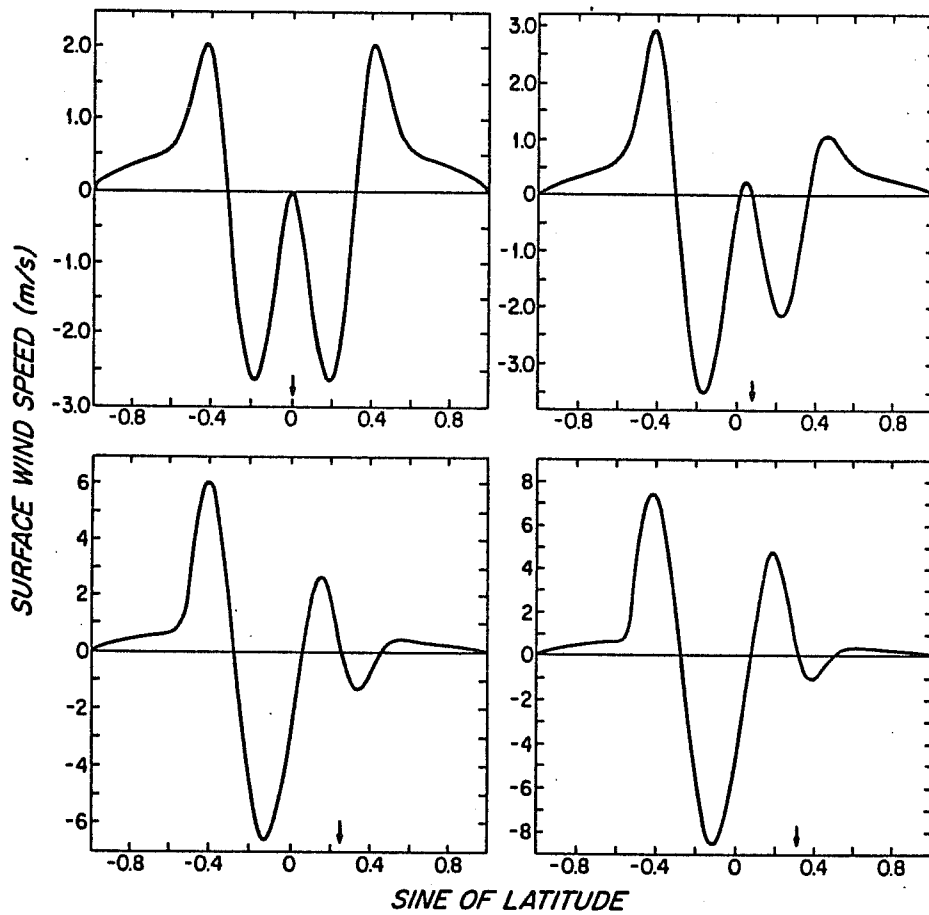


Figure 14. Distribution of surface winds in ms⁻¹ for (a) $\phi_0=0^\circ$, (b) $\phi_0=2^\circ$, (c) $\phi_0=6^\circ$, and (d) $\phi_0=8^\circ$. (N.b. $\sin \phi = 0.2$ corresponds to $\phi \approx 11.5^\circ$.) (From Lindzen and Hou, 1988.)

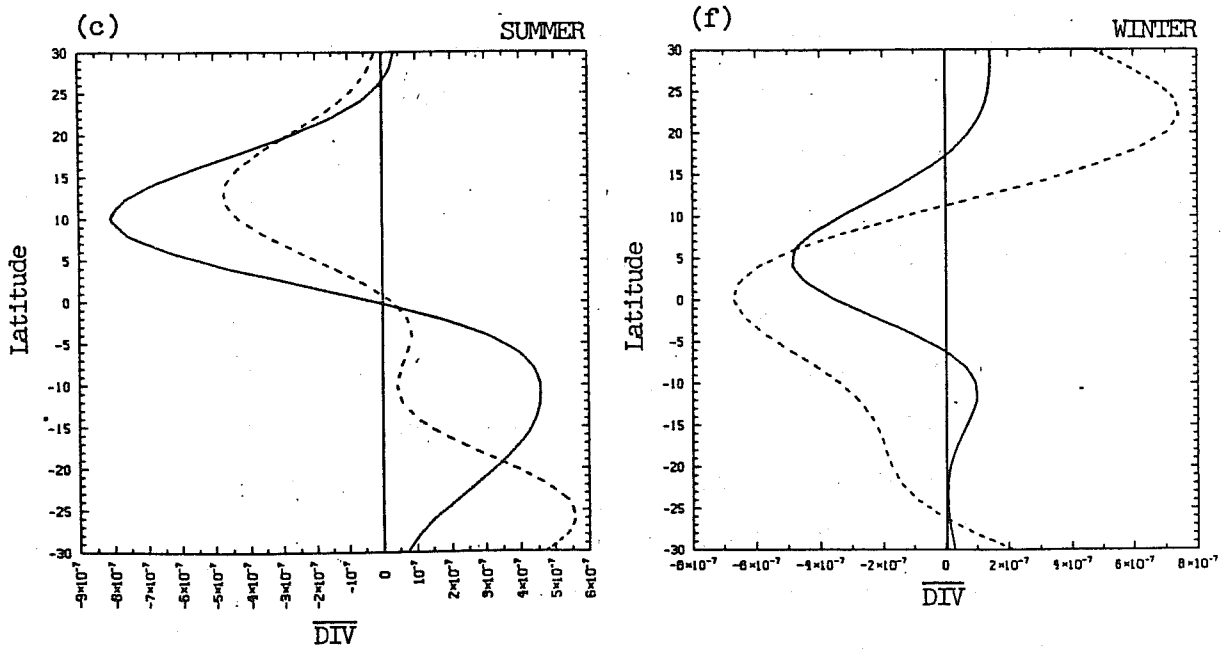


Figure 15. The calculated low-level zonally symmetric divergence forced by surface temperature gradients (solid lines) and a smoothed version of the ECMWF analyzed field (dashed lines) (from Lindzen and Nigam, 1987).

be concentrated at the ITCZ. As noted in Section 2, concentrating the heating will intensify the Hadley circulation. Arthur Hou and I are currently investigating this matter, and what follows are preliminary results.

It must first be noted that the intensification of the Hadley circulation that results from concentrating Θ_E , is not so trivial in the case when $\phi_0 \neq 0$ as compared to the case where $\phi_0 = 0$. Hou and I find, for example, that concentrating heating symmetrically about ϕ_0 , without reference to whether we are concentrating heat from the domains of the winter or summer cell leads to very little intensification of the winter cell. In order to intensify the winter cell, we must concentrate heat primarily from the winter side of ϕ_0 . Now, this concentration almost certainly arises from low-level convergence of water vapor which, in turn, concentrates the cumulonimbus convection and the accompanying release of latent heat. Moreover, as Lindzen and Nigam (1987) have noted, low-level convergence is largely driven by surface temperature gradients acting on the trade wind boundary layer; little convergence seems to be directly forced by the upper level Hadley circulation. *A priori*, therefore, there is little reason to suppose that the convergence will draw primarily from the region of the winter cell. However, as Figure 15 shows, the convergence is drawn primarily from the winter cell (both on the basis of explicit calculation and from ECMWF analyses). We, therefore, do expect some measure of intensification. The nature of the concentrations we examined is shown in Figure 16 ($\phi_0 = 6^\circ$). Figures 17-19 show the numerically calculated streamline distributions for various degrees of concentration. While concentrations on the order of those associated with the ITCZ lead to almost an order of magnitude intensification, what is actually needed is achieved with much less intensification (about what Lorenz (1967) shows for the zonally averaged concentration of rainfall; viz Figure 20).

5. SOME REMARKS

The results in Section 4 confirm what may, in fact, be obvious. Namely, the zonally averaged Hadley circulation responds to the zonally averaged concentration of rainfall rather than the local concentration associated with the ITCZ. A number of factors lead to the broadening of the zonal average: easterly waves, monsoons, etc. In the absence of such features, we might reasonably expect a much stronger Hadley circulation. In view

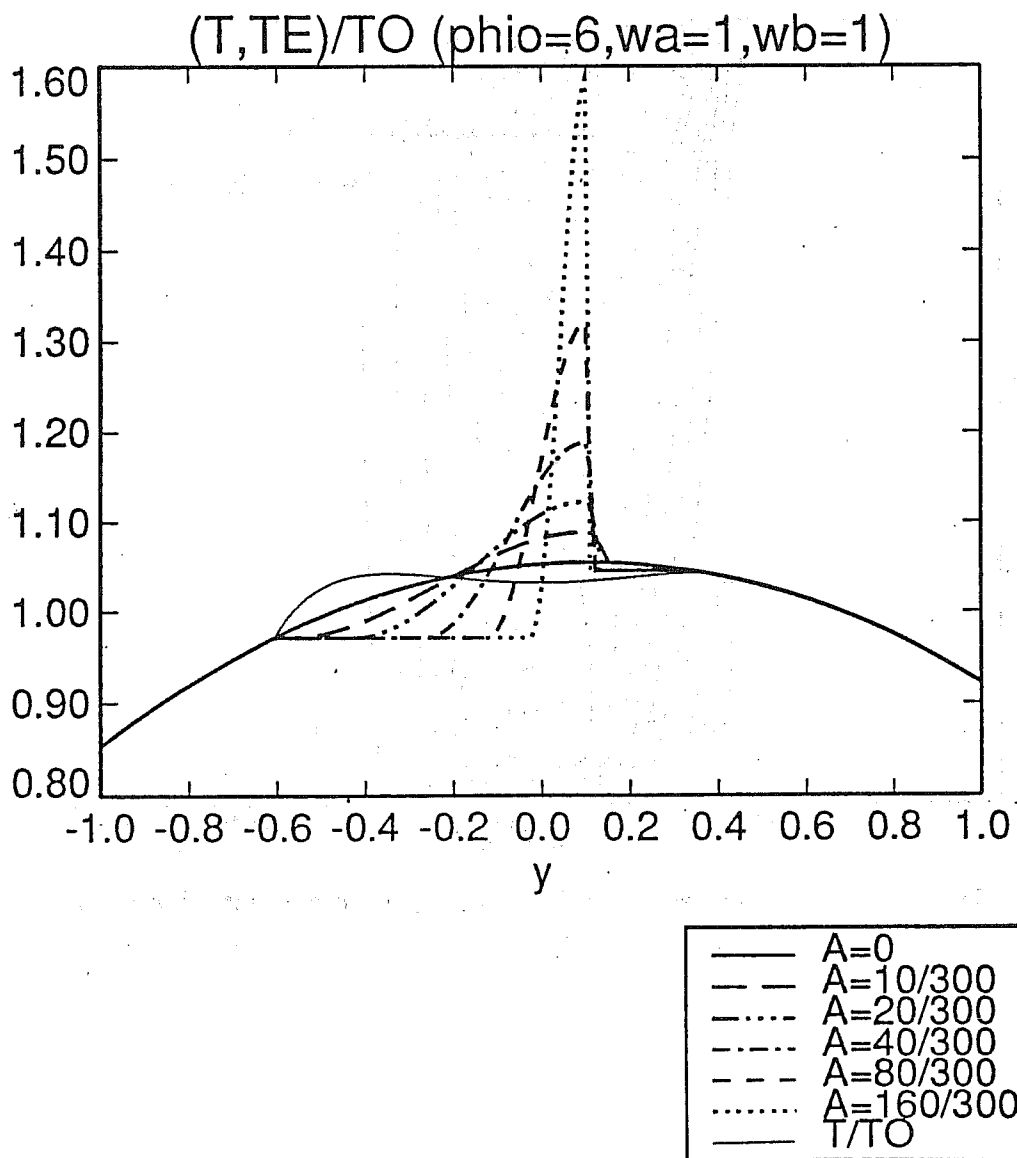


Figure 16. Θ_E/Θ_0 for $\phi_0=6^\circ$, and various degrees of concentration.

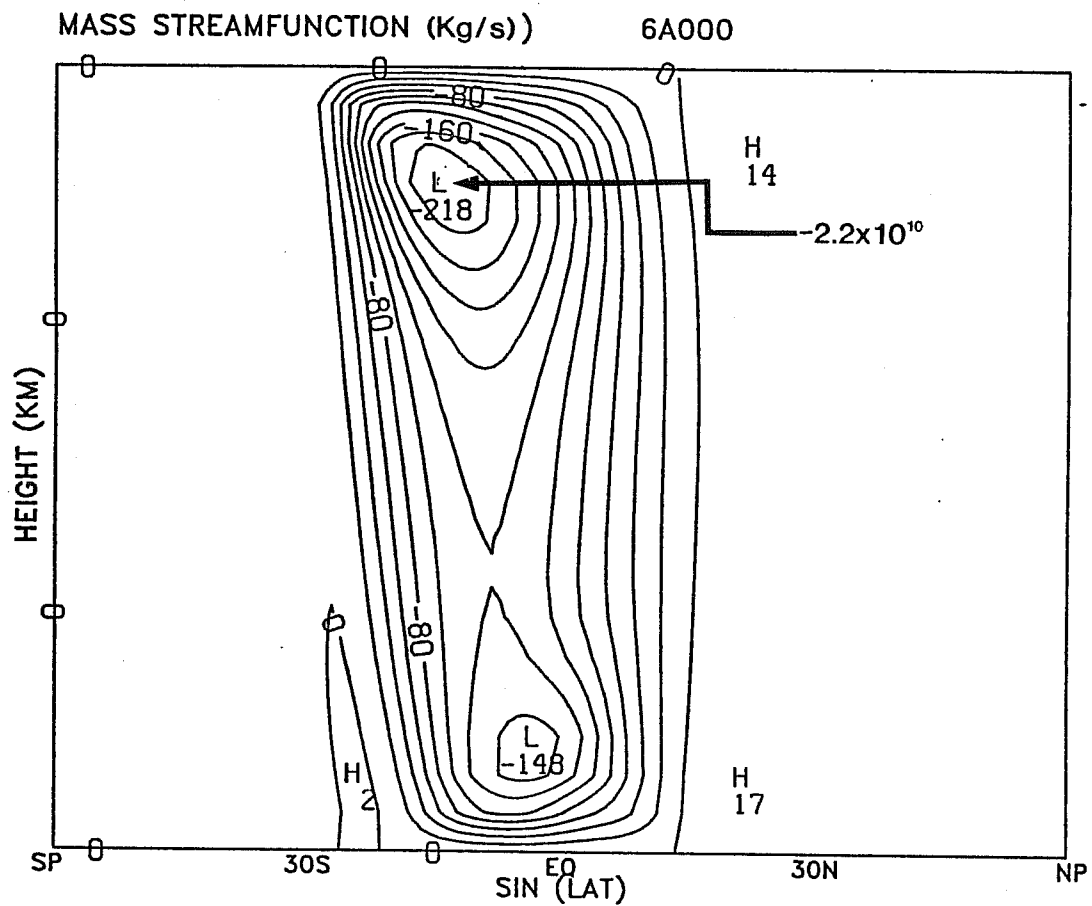


Figure 17. Mass streamfunction for $\phi_0=6^\circ$, and no concentration of Θ_E .

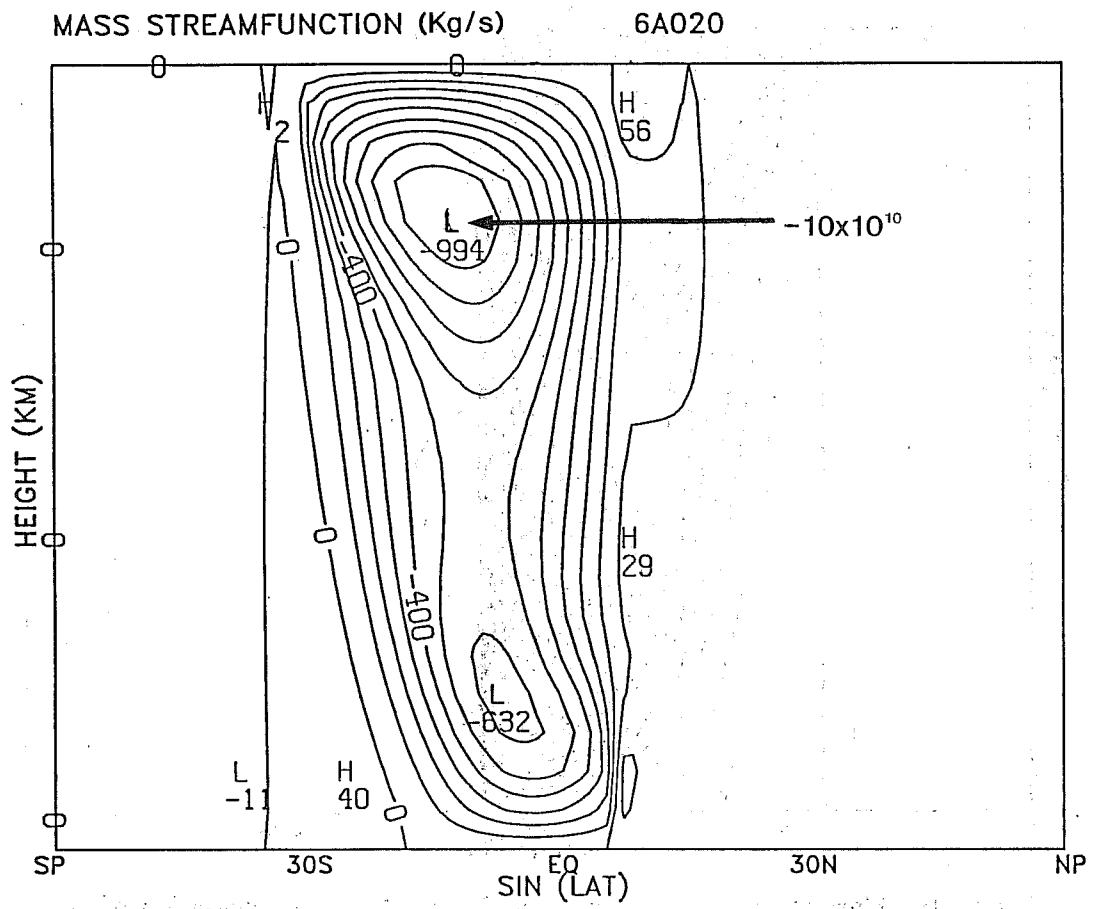


Figure 18. Mass streamfunction for $\phi_0=6^\circ$, and modest concentration of Θ_E .

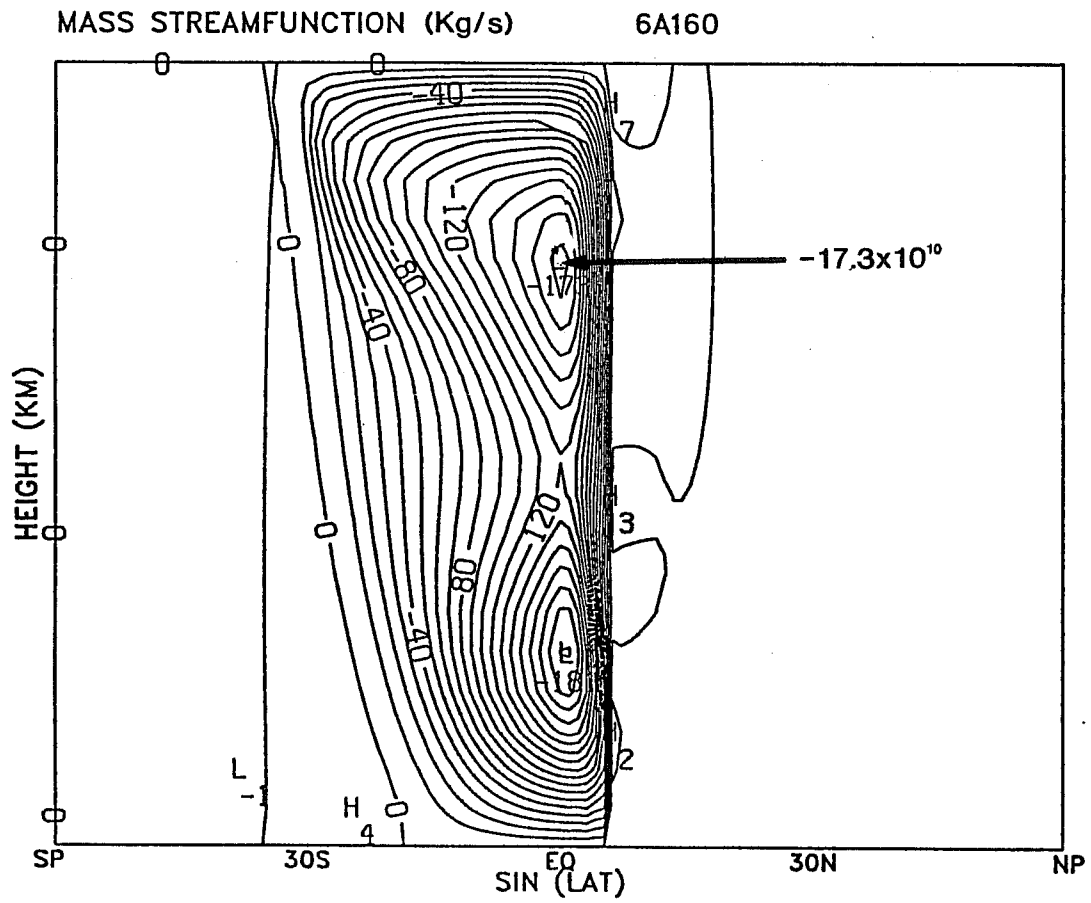


Figure 19. Mass streamfunction for $\phi_0=6^\circ$, and strong concentration of Θ_E .

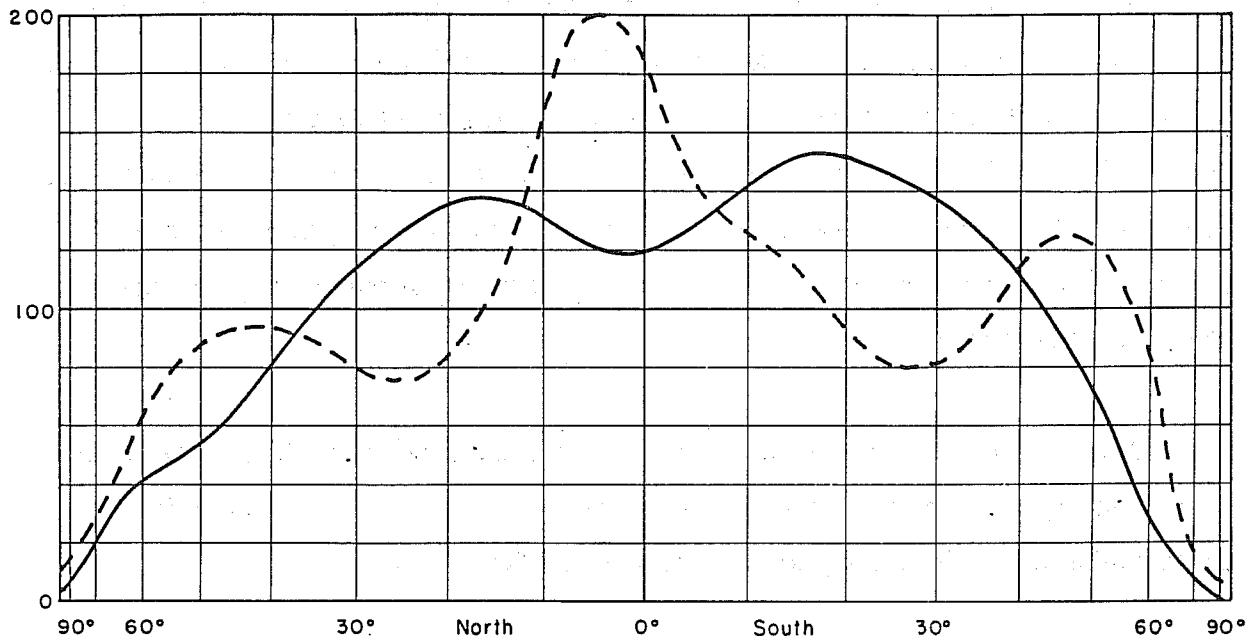


Figure 20. Average annual evaporation (solid curve) and precipitation (dashed curve) per unit area as given by Sellers (1966). Values are in centimeters of water per year, or $\text{g cm}^{-2} \text{ year}^{-1}$ (scale on left).

of our introductory remarks, an intensified Hadley circulation might be associated with more intense eddy heat flux in middle latitudes and excessive winter temperatures. It may also be expected to modify the position of the subtropical jet. Some idea of this effect may be gleaned from a comparison of winter and summer in the Northern Hemisphere. In summer, when there is effectively no Hadley flux, the jet is at 40°; in winter when the Hadley fluxes are strong, the jet is at 30°. According to Nigam and Lindzen (1989), the position of the jet relative to the Himalayas is a major determinant of stationary wave amplitudes in middle and high latitudes. When the jet is over or south of the Himalayas, the stationary waves are free to reach higher latitudes; when the jet is north of the Himalayas, the waves are deflected. The difference between the summer and winter positions of the jet appear to be sufficient to account for the greatly reduced stationary wave amplitudes in summer, and position changes of only a couple of degrees have significant effects.

It is also possible that a model, with an inadequate parameterization of the trade wind boundary layer and its coupling to the surface temperature, might underestimate the concentration of rainfall leading to a reduction of Hadley fluxes.

At least for climatic purposes, it appears that a proper treatment of the seasonality of cross equatorial gradients and of the concentration of zonally averaged precipitation may be essential to the proper modeling of eddy activity, stationary waves and storm paths in middle and high latitudes.

Acknowledgements

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