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The development and testing of a new two-time-level semi-Lagrangian scheme (SETTLS) in the ECMWF forecast model

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ABSTRACT

A new treatment of the two-time-level semi-Lagrangian scheme is presented which avoids extrapolation in time of the velocities used for the computation of the trajectories and for the non-linear terms of the evolution equations.

Extrapolation in time is used in the original two-time-level scheme in order to make it centered in time and therefore second-order accurate. This time extrapolation can lead to instabilities. In the new scheme the second order accuracy is achieved by means of a Taylor series expansion to second order around the departure point of the semi-Lagrangian trajectory thus avoiding extrapolation of the first-order term. The estimate of the second order term in this expansion is obtained by means of a stable time extrapolation from the previous time step and therefore the scheme is named "Stable Extrapolation Two-Time-Level Scheme" or SETTLS.

We diagnosed some cases of noise in forecast fields as arising from an instability of the time extrapolation in the version of the two-time-level semi-Lagrangian scheme used operationally at ECMWF between December 1996 and April 1998 and show that the noise can be removed by the use of the SETTLS scheme.

KEYWORDS: Two-time-level Semi-Lagrangian Gaussian grid Extrapolation

1. Introduction

Since the original demonstration of the efficiency advantage of the semi-Lagrangian semi-implicit time integration method for atmospheric forecast models by *Robert* (1981), this scheme is being used in an increasing number of atmospheric models.

The three-time-level version of the method, described by *Ritchie et al.*(1995), was implemented operationally at ECMWF in September 1991.

In December 1996 a two-time-level semi-Lagrangian scheme was made operational at ECMWF with a resolution of T213L31 (triangular spectral truncation to 213 waves in the horizontal and 31 hybrid levels in the vertical {*Simmons & Burridge* (1981)}) using a time step of 1800 sec (*Temperton et al.* 1999).We call this version of the two-time-level scheme the "1997" version since it was used in operations from December 1996 to April 1998. Despite extensive pre-operational testing, signs of a weak instability in the form of noise in several forecast fields became apparent after operational implementation, an example of which is shown in Figure 1:(b). The noise seen in this forecast does not show in the corresponding forecast made with the three-time-level scheme (Figure 1:(a)). This problem led to the decision to reduce the size of the time step in operations to 900 sec. from the 21st of January 1997 thus giving up much of the efficiency advantage of the two-time-level over the three-time-level scheme. The noise was reduced but not totally eliminated with this change.

Some further problems of the "1997" scheme were also found in research mode. For example many applications of the forecast model utilize the "linear" Gaussian grid (see paragraph 2.1). Integrations using the linear Gaussian grid are more sensitive to noise produced by the time-integration scheme for reasons which will be explained later. Forecast tests using the linear grid (Figure 1:(d)) confirmed the existence of an instability of the "1997" version of the model. The "1997" two-time-level scheme has nevertheless been used successfully with the linear grid for the Ensemble Prediction System (EPS) since 1996 with a resolution of T_L159 and a time step of 2700 sec (suffix L means it uses the linear grid). Also a 50-level version of the model resolving the atmosphere up to 0.1 hPa (~65 Km) is more sensitive to instabilities in the numerical scheme than the 31 levels version. This is probably due to the fact that the 50-level version unlike the 31-level version resolves the whole stratosphere with its strong polar night jet. An example is shown in the forecast zonal wind at model level 10 (~5hPa) with the "1997" model in Figure 2:(a). Figure 2:(b) shows the corresponding three-time-level forecast for the same time. The contrast between these two integrations shows that there is a problem with the "1997" version of the two-time-level scheme.

The paper is organized as follows: in section 2 the configurations of the forecast model (linear grid and 50-level version) most sensitive to instability problems are briefly reviewed. The cause of the noise problems seen in the "1997" version of the two-time-level semi-Lagrangian advection is analysed in section 3 and the solution (the SETTLS scheme) is presented in section 4. Section 5 presents some of the many tests through which the performance of the new approach was tested, including the preoperational test-suite results, and finally some conclusions are drawn in section 6.

2. The linear Gaussian grid and the 50-level version of the model

2.1 The linear Gaussian grid

When using the spectral method for the solution of the atmospheric evolution equations, the Gaussian grid originally proposed by *Eliassen et al* (1970) is often used. *Orszag* (1970) and *Machenhauer* (1979) show that the computation of quadratic terms is alias-free if the number of points "NF" of the grid along each Gaussian latitude satisfies $NF \geq 3N+1$ and the number of latitudes "NG" satisfies $NG \geq (3N+1)/2$, N being the spectral truncation. Alias-free computation of quadratic terms is essential in the Eulerian method because the Eulerian advection terms are large and quadratic. This grid will be called here the "quadratic" grid.

Côté and Staniforth (1988) introduced a "reduced-resolution" Gaussian grid in which $NF \geq 2N+1$ and $NG \geq (2N+1)/2$. This grid has the smallest number of points compatible with exact spectral transforms of a field (or a linear combination of fields). That means that if we start with a field in spectral space, transform it to grid-point space using the "reduced-resolution" Gaussian grid, and transform it back to spectral space we recover exactly the original field. For this reason this grid has recently received the name of "linear" Gaussian grid. In the semi-Lagrangian treatment of the evolution equations the Eulerian advection terms do not appear and the linear Gaussian grid should be suitable for their integration.The advantage of the linear grid over the quadratic grid is that, given a spectral resolution, the number of grid-points is smaller in the former by a factor of $(\sim 2/3)^2$. As most of the computations in a semi-Lagrangian model are performed in grid-point space, the use of the linear Gaussian grid should give an efficiency increase of about 9/4=2.25 over the quadratic grid at the same spectral resolution.

The relationship between the linear and the quadratic Gaussian grids can be viewed from a different point of view. Assume that we have a certain distribution of grid-points corresponding to the quadratic Gaussian grid of a spectral resolution N, that is (3N+1)/2 latitudes with 3N+1 points each. When computing the direct spectral transform of a field represented on that set of points, the maximum number of spectral coefficients we can compute corresponds to a spectral truncation of N'≅3N/2. Quadratic terms computed on that grid lead to aliassing for the wavenumbers larger than 2N'/3≅N and this is the reason why discarding the wavenumbers from N to N' avoids quadratic aliassing. The linear Gaussian grid corresponding to the spectral resolution N' is precisely the distribution of grid-points from which we started.

Given then a Gaussian grid, this grid can be viewed as the quadratic grid for the spectral resolution N or as the linear grid corresponding to a larger spectral resolution N', the difference between the two cases is that we keep all the spectral coefficients computed when it is considered as the linear grid of resolution N' and we discard the larger wavenumbers when it is considered as the quadratic grid of resolution N. Integrations using the linear Gaussian grid are more sensitive to noise produced by the time-integration scheme as the noise tends to concentrate in the shortest wavelengths which are eliminated using the quadratic grid but retained in the linear grid.

An example of the larger sensitivity to noise of the linear grid as compared with the quadratic grid can be seen in Figure 1:(d). This panel shows the level-9 temperature 5-day forecast from 1997-01-04 using the spectral resolution T_L319 (triangular truncation at wavenumber 319 and the linear Gaussian grid), whose linear grid coincides with the quadratic grid of resolution T213. Figure 1:(b) forecast was produced by the operational model at T213 resolution using the quadratic grid. Both runs used the same time step of Δt=1800s.

2.2 The 50-level version of the forecast model

Since September 1991, a 31-level resolution has been used operationally at ECMWF (*Ritchie et al.*(1995)), with levels distributed as shown in the left-hand portion of Figure 3:. The top four levels are located at pressures of exactly 10, 30, 50 and 70 hPa. *Untch et a*l (1998) describe the 50-level version of the forecast model which became operationa in March 1999.

The 50-level version of the model, illustrated in the right-hand portion of Figure 3: has a distribution of levels identical to that of the 31-level version below 150 hPa, and levels between 60 and 5 hPa are close to equally-distributed in height with a spacing of 1.5 Km. The spacing increases above the 5 hPa level and the top level is at 0.1 hPa (~65 Km).

3. The problems caused by extrapolation in time in the “1997” scheme

The general form of each evolution equation in the framework of the semi-Lagrangian method can be written as

$$\frac{dX}{dt} = [rhs] \equiv N + L$$

where the right hand side (r.h.s.) is split into a linearized part L and the remainder or non-linear term N. In the “1997” scheme this equation is discretized as

$$\frac{X_A^{t+\Delta t} - X_D^t}{\Delta t} = \frac{1}{2}\left[N_D^{t+\frac{\Delta t}{2}} + N_A^{t+\frac{\Delta t}{2}}\right] + \frac{1}{2}[L_A^{t+\Delta t} + L_D^t]$$

Here subindex A indicates a quantity evaluated at the arrival point of the semi-Lagrangian trajectory (one of the points of the Gaussian grid), subindex D indicates a quantity interpolated to the departure point of the trajectory and the superindex refers to the instant in time. The values at time t+Δt are the unknowns and the value of the non-linear term at time t+Δt/2 is found by means of linear extrapolation in time from the values at time t and at time t-Δt.

The computation of the semi-Lagrangian trajectory is done by solving the equation

$$\frac{d}{dt}\vec{R} = \vec{V} \tag{1}$$

where $\vec{R}$ is the position vector of a parcel of air and $\vec{V}$ is the (three-dimensional) velocity of that parcel.

In the "1997" version of the two-time-level scheme, this equation is discretized by means of the second-order accurate scheme centered in time and space:

$$\vec{R}_A^{t+\Delta t} \approx \vec{R}_D^t + \Delta t \cdot \vec{V}_M^{t+\frac{\Delta t}{2}} \tag{2}$$

The positions $\vec{R}_A^{t+\Delta t}$ are the Gaussian (arrival) grid points, where the parcels of air will arrive at time t+Δt, the points $\vec{R}_D^t$ are the departure points at the present time, t, and $\vec{V}_M^{t+\frac{\Delta t}{2}}$ are the velocities at the middle of the trajectory, estimated at an instant midway between the present and the future time steps. This estimation is done by linear extrapolation in time exactly the same as with the non-linear terms of the evolution equations.

The cause of the noise problem seen in the T213L31 forecast shown in Figure 1:(b) was traced to the time extrapolation of the vertical velocity used in the computation of the semi-Lagrangian trajectories. In making a linear extrapolation in time of the velocity of an air parcel one implicitly assumes that $\frac{\partial}{\partial t}\vec{V}$ is constant. This procedure is unstable for CFL numbers larger than one if the quantity $\vec{V}$ depends on the unknown $\vec{R}$, according to linear stability analysis performed by *Durran* (personal communication). According to his analysis, an extrapolation "along the semi-Lagrangian trajectory" (which assumes $\frac{d}{dt}\vec{V}$ to be constant) is stable. This suggestion was tested in the ECMWF model and indeed the noise shown in the forecast temperature at level 9 in Figure 1:(b) disappears. Unfortunately the verification scores were much worse than with the "1997" scheme. This is probably due to large errors which can be introduced when estimating the wind at the middle of the trajectory by using the departure point and the extension of the trajectory one time step backwards, particularly in areas with large gradients in horizontal velocity such as near orography.

A further problem is that the 'extrapolation along the trajectory' procedure cannot be applied to the non-linear terms of the momentum equations due to the semi-implicit treatment of these equations. The incompatibility of the extrapolation along the trajectory with the semi-implicit scheme comes from the requirement that the orographic part of the geopotential gradient term has to be in a very good equilibrium with the surface pressure part and this is only possible if the linear terms are evaluated at the same points as the non-linear terms in the momentum equations. This is also the reason why in both the "1997" two-time-level and in the three-time-level schemes the non-linear terms of the momentum equations are averaged along the semi-Lagrangian trajectory rather that interpolated to the middle point of the trajectory.

4. Stable Extrapolation Two-Time-Level Scheme (SETTLS)

An alternative second-order accurate scheme to solve Eq. (1) can be derived by expanding the unknown quantity $\vec{R}$ in a Taylor series around the departure point of the semi-Lagrangian trajectory:

$$\vec{R}_A^{t+\Delta t} \approx \vec{R}_D^{t} + \Delta t \cdot \left[\frac{d}{dt}\vec{R}\right]_D^t + \frac{\Delta t^2}{2} \cdot \left[\frac{d^2}{dt^2}\vec{R}\right]_{AV}$$

Here subscript AV indicates some average value along the semi-Lagrangian trajectory.

Substituting from Eq. (1) we find

$$\vec{R}_A^{t+\Delta t} \approx \vec{R}_D^{t} + \Delta t \cdot \vec{V}_D^t + \frac{\Delta t^2}{2} \cdot \left[\frac{d}{dt}\vec{V}\right]_{AV} \tag{3}$$

This equation describes an uniformly accelerated movement. The trajectories can no longer be considered as straight lines on a plane or as arcs of a great circle in spherical geometry as is traditionally done in semi-Lagrangian schemes and the position of the middle point of the trajectory is no longer an average between the departure and the arrival points.

To proceed, one has to estimate the quantity

$$\left[\frac{d}{dt}\vec{V}\right]_{AV} \tag{4}$$

To estimate Eq. (4) the first possibility explored was to use an average along the trajectory of the explicit estimate of the r.h.s. of the momentum equations as the horizontal part and the expression

$$\left[\frac{dW}{dt}\right]_{AV} \approx \frac{W_A^t - W_{D^-}^{t-\Delta t}}{\Delta t} \tag{5}$$

evaluated at the previous time step as the vertical part. Here D^- means the position at time t-Δt of the parcel of air arriving at gridpoint A at time t. Expression Eq. (5) is also not applicable to the non-linear terms of the momentum equations since it uses information from the departure point of the previous time step trajectory which is not exactly the same as the point to which the linear terms are interpolated. Several tests with this version showed that it eliminated the noise in the T213L31 forecast. However experiments including data assimilation failed to produce forecasts of a similar quality as the "1997" two-time-level version.

After exploring many other possibilities, the following estimate was adopted:

$$\left[\frac{d}{dt}\vec{V}\right]_{AV} \approx \left[\frac{d}{dt}\vec{V}\right]^{t-\frac{\Delta t}{2}} \approx \frac{\vec{V}_A^{t} - \vec{V}_D^{t-\Delta t}}{\Delta t} \tag{6}$$

using the departure point of the semi-Lagrangian trajectory corresponding to the present time step instead of the departure point of the trajectory corresponding to the previous time step as in Eq. (5). Here D means the position at time t of the parcel of air which will arrive to gridpoint A at time t+Δt.

This estimate assumes that the total time derivative of the velocity is constant with time, following *Durran's* suggestion of "extrapolating along the trajectory", but the estimate uses only the arrival and departure points of the present trajectory and is therefore compatible with the semi-implicit treatment of the evolution equations. This scheme should therefore be also stable according to linear stability analysis and has accordingly been named "Stable Extrapolation Two-Time-Level Scheme" or SETTLS.

Substituting Eq.(6) into Eq.(3) we obtain:

$$\vec{R}_A^{t+\Delta t} = \vec{R}_D^{t} + \frac{\Delta t}{2} \cdot \left(\left[2\vec{V}^{t} - \vec{V}^{t-\Delta t}\right]_D + \vec{V}_A^{t}\right) \tag{7}$$

and a similar expression can be used in every evolution equation to treat the non-linear terms of the r.h.s.

This expression is formally similar to the treatment of the non-linear terms of the evolution equations proposed by *Bates et al* (1993). Their scheme uses Eq. (2) with the value at the middle point estimated as an average between the value at the present time step at the departure point and the value at the future time step at the arrival point. The value at the future time step is estimated by means of linear extrapolation in time. The result is

$$\vec{R}_A^{t+\Delta t} = \vec{R}_D^{t} + \frac{\Delta t}{2} \cdot \left(\vec{V}_D^{t} + \left[2\vec{V}^{t} - \vec{V}^{t-\Delta t}\right]_A\right)$$

as Eq. (7) but with the arrival and departure points interchanged. The *Bates et al* scheme has been tested in the 50-level version of the model and the results are noisier than with the "1997" scheme (not shown).

5. Experimental results

The case of the forecast starting from analysis at 12Z the 4th January 1997 which was particularly noisy in operations using the T213L31 resolution and the "1997" scheme (Figure 1 (b)) was rerun with the SETTLS scheme. The resulting forecast level 9 (~200hPa) temperature is shown in Figure 1 (c). The field looks almost identical to that obtained with the three-time-level scheme shown in Figure 1 (a) and no sign of instability is apparent.

The forecast using the linear Gaussian grid at T_L319L31 resolution shown in Figure 1 (d) was rerun with the SETTLS scheme, producing the forecast shown in Figure 1 (e). The SETTLS scheme avoids completely the noise seen in the "1997" scheme. The few wiggles left correspond with those seen in the T213 three-time-level forecast and are a bit sharper as expected from a higher resolution forecast. They are even present in an Eulerian T213 forecast (not shown) and therefore they are not sign of semi-Lagrangian instability.

5.1 More tests at T213L31

A series of 12 forecasts was run with the SETTLS scheme from the 15th of each month starting from 1996-05-15 and the results compared with the "1997" scheme and with the three-time-level scheme, all of them using a time step of 900s. Anomaly correlation (AC) and root mean square (RMS) scores at 500 hPa over the Northern Hemisphere (NH) the Southern Hemisphere (SH) and Europe together with 850 hPa temperature AC and RMS over the tropics are shown in Figure 4 . The scores of the SETTLS runs (full line) are almost identical to the three-time-level runs (dotted line) and neutral with respect to the "1997" scheme (dash line) up to day 7 into the forecast when the anomaly correlation over the NH line crosses over the 60% line. There are no large differences in forecast quality between the SETTLS scheme and the other two schemes because the time step used is quite short and the "1997" scheme is not degraded too much by noise

The mean eddy kinetic energy of the forecasts with the SETTLS scheme are compared with analysis, the "1997" scheme and the three-time-level scheme in Figure 5:. Panel (a) is the Northern Hemisphere, panel (b) the Southern Hemisphere and panel (c) the tropical region. The "1997" scheme shows an excessive increase of eddy kinetic energy in the second half of the forecast range over the Southern Hemisphere which is not shown by the three-time-level scheme. The SETTLS scheme cures to some extent that problem.

5.2 Tests covering the stratosphere (resolution T_L159L50: triangular truncation at wavenumber 159 using the linear grid and 50 levels in the vertical)

In the upper stratosphere the model is very sensitive to instabilities in the numerical scheme. Several 10-day forecasts were run with the 50-level version of the model at resolution T_L159 to test the performance of the different two-time-level schemes in the stratosphere. The time step used in these runs, unless otherwise stated, was 3600s. Figure 2 shows the 5-day forecast zonal wind in the SH at level 10 (~5hPa) from initial data of 1996-08-15. The forecasts were made with: a) the "1997" scheme, b) the three-time-level scheme (time step of 1800s), and c) the SETTLS scheme. The "1997" scheme produced an extremely noisy field (Figure 2 (a)) whereas the field computed with the three-time-level scheme is smooth (Figure 2 (b)). Almost all L50-forecasts run with the "1997" scheme look noisy in the upper stratosphere and some fail because of noise within 10 days of integration, (*Untch* personal communication). The run with the SETTLS scheme (Figure 2 (c)) produces a forecast which is almost identical to the field obtained with the three-time-level scheme (Figure 2 (b)). There is no sign of the instability shown by the "1997" scheme.

Many more experiments have been performed at different horizontal resolutions using the SETTLS scheme and 50 levels in the vertical. The successful results will be reported by *Untch et al* elsewhere.

5.3 Seasonal runs

Two ensembles (of three elements each) of seasonal forecast experiments (125 forecast days each at T63L31 resolution using a time-step of Δt=3600s) have been run with the SETTLS scheme and compared with the corresponding controls using the "1997" scheme. The first ensemble corresponded to the Northern Hemisphere summer and the second one to the Northern Hemisphere winter.

No significant differences have been found in the mean rainfall rate, the cloud cover or the radiation fields. The relative mass conservation was of the same order of magnitude in both schemes of up to a maximum relative deviation from the initial mass of 2.10^{-4} (~0.2 hPa) but with a typical relative deviation of about 5.10^{-5} (~0.05 hPa) either loss or gain during the whole 125 day period of each forecast.

5.4 Tests with Data Assimilation (4D-Var) at T_L319L31

The results of the tests described here, together with a multitude of other tests which showed there were no problems connected with the SETTLS scheme, led to the decision to change the operational ECMWF forecast model to this new scheme. At the same time there was an increase in horizontal resolution to triangular truncation at wavenumber 319 using the linear grid (notice that this grid coincides with the quadratic T213 grid and therefore the computational cost was kept almost the same). Correspondingly a parallel analysis/ forecast suite was started on February 9th 1998 and run until the operational implementation of the new scheme on April 1st 1998.

The increased horizontal spectral resolution gives a better description of small-scale synoptic features, mainly those related with the orography such as Mediterranean cyclogenesis produced by the interaction of the air flow with the Alps (*Hortal* 1998), even when the physical parameterizations have not changed as the distribution of grid points is identical at both resolutions.

The operational change included also the introduction of an orography field computed from a new global dataset (DTM5 by GETECH). Separate tests using this new orography showed a negligible impact on forecasts scores as compared with the forecasts scores using the orography computed from the US Navy dataset. As the instability problem which led to the reduction of the size of the time step to 900s in January 1997 was solved with the SETTLS scheme, the size of the time step could have been reverted to 1800s. In order not to degrade too much the accuracy of the physical parameterizations it was conservatively set at 1200s.

Mean scores of the 41 first cases (those which can be verified up to day 10 against the parallel suite analysis) out of these 51 are shown in Figure 6 . Panels (a) and (b) display AC and RMS of the 500 hPa geopotential in the NH and panels (e) and (f) display the same scores for the SH. They show a neutral impact of the new setup in spite of the longer time-step used with the SETTLS scheme and the implied smaller horizontal diffusion used with the higher resolution (the e-folding time at the shortest wavelength is set to similar values in both cases). On the other hand the mean RMS over Europe (panel (d)) shows a more positive impact than in AC (panel (c)) and the RMS of temperature and wind at 850 hPa over the tropics shown in panels (g) and (h) give a clear advantage of the new setup over the old one.

5.5 Further tests

Several runs using the dynamical core configuration (*Jablonowski* personal communication) as proposed by *Held and Suarez* (1994) have been performed at a resolution of T63L31 (Δt= 3600s), producing forecasts to 1200 days. The mass conservation in these runs using the SETTLS scheme was (~14 hPa total gain) slightly better than using the "1997" scheme (~18 hPa total gain). This change in total mass is larger than with the full model (para 5.3) for reasons not yet fully understood. It can be eliminated with the use of a mass fixer available in the ECMWF model. Diagnostics from these runs did not show any dramatic difference between the SETTLS and the "1997" schemes. Some runs were also made at T63L50 (Δt= 3600s) using a modification of the *Held and Suarez* forcing as proposed by *Williamson et al* (1998). The tests were run up to 3600 days as the stratosphere does not reach an equilibrium state as the troposphere does. The total mass in the SETTLS runs increased by ~22 hPa as compared with an increase of ~42 hPa in the "1997" runs. These results will be reported by *Jablonowski et al* elsewhere.

The SETTLS scheme has also been used at different resolutions for the testing of different treatments of the Coriolis term of the momentum equations. These tests led to the improvement of the advective treatment of the Coriolis parameter by updating the rotation matrix used in spherical geometry in connection with the interpolation to the departure point of the semi-Lagrangian trajectories. These results will be reported by *Temperton et al* elsewhere shortly.

Several assimilation and forecast experiments have been performed at T_L639L31 (Δt=900s), T_L639L50 (Δt=900s), and T_L511L50 resolutions (Δt=1200s). No sign of noise produced by instability in the scheme has been found in any of these runs. The experimentation at these resolutions is ongoing and the results will be reported in due curse.

6. Conclusions and remaining issues

The "1997" two-time-level semi-Lagrangian scheme had instability problems arising from the time extrapolation of the velocities used to compute the semi-Lagrangian trajectories and from the time extrapolation of the non-linear terms of the r.h.s. of the evolution equations. These instabilities appear as noise in some of the forecast fields, mainly when using a long time step. The noise is more prominent with the linear grid than with the quadratic grid. This problem can be eliminated by the use of the SETTLS scheme described in the present paper.

In the 50-level version of the forecast model noise problems near the polar night winter jet exist in all runs made with the "1997" scheme. The SETTLS scheme makes these forecasts stable and the noise problem disappears.

The solution of the instability problems allows one to take full advantage of the efficiency of the two-time-level scheme and of the linear Gaussian grid, which increases the resolution of the model with negligible increase in computational cost. Within the Eulerian framework the increase of resolution from T213 to T319 would have resulted in an increased cost of the model by a factor $(3/2)^3 \cong 3.4$. Instead, the operational changes described in this paper have made the model cheaper by a factor of 0.75. If we had used a time step of 1800s with the two-time-level scheme (a time step at which the SETTLS scheme does not show any sign of instability), we could have reduced the cost of the ten-day forecasts by a factor of 0.5.

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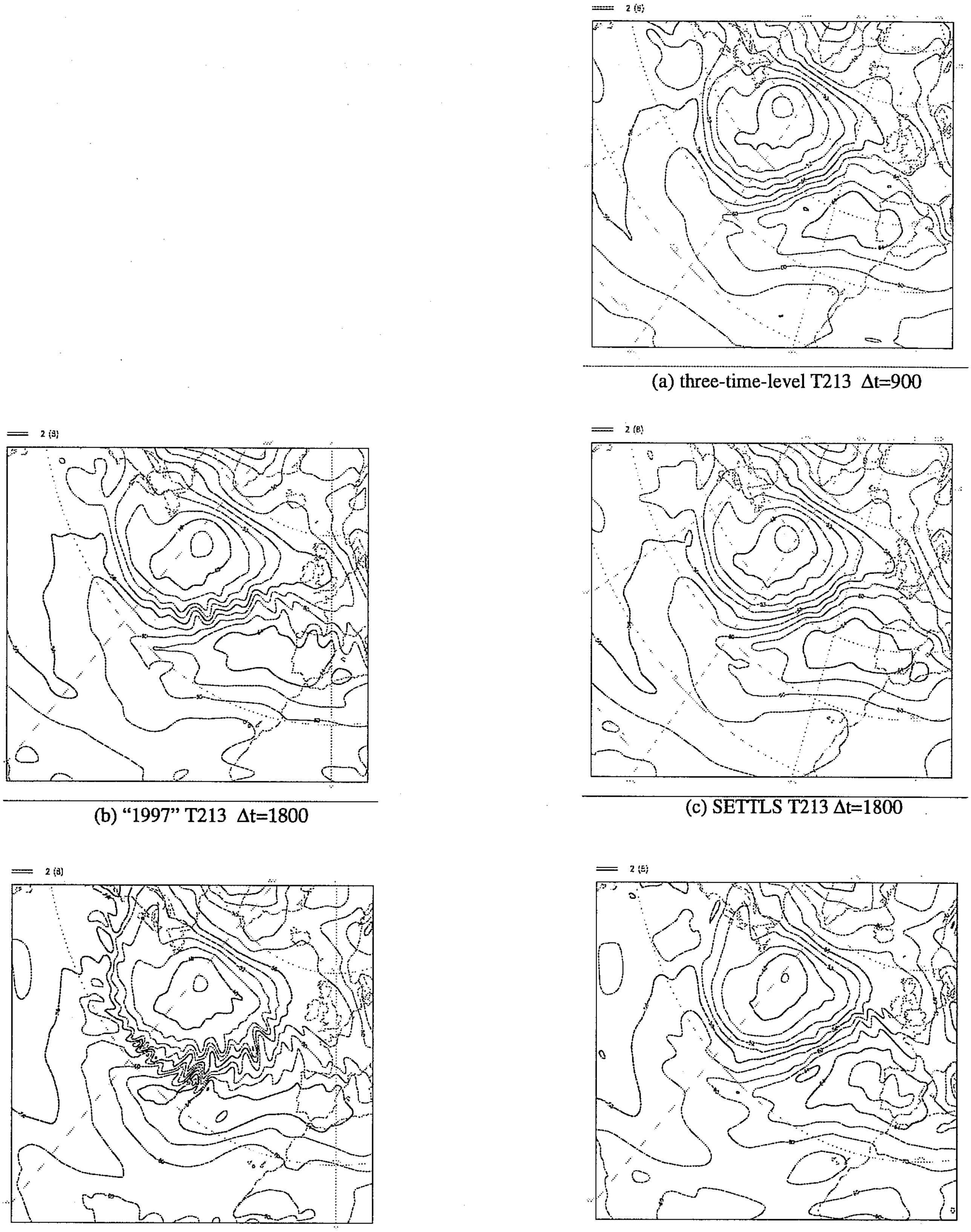


(a) three-time-level T213 Δt=900

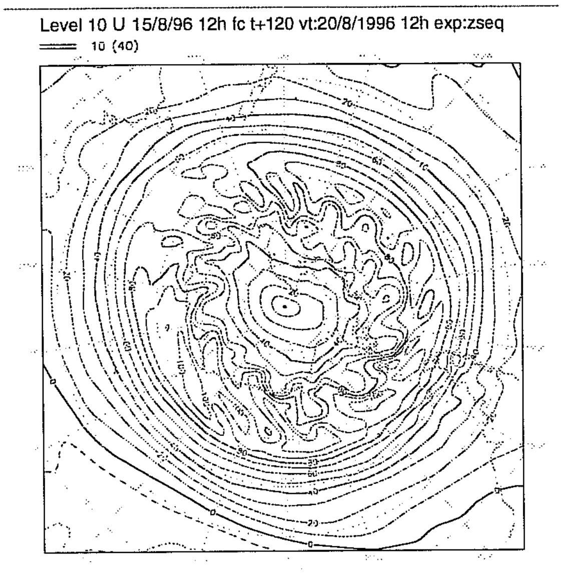
(b) "1997" T213 Δt=1800

(c) SETTLS T213 Δt=1800

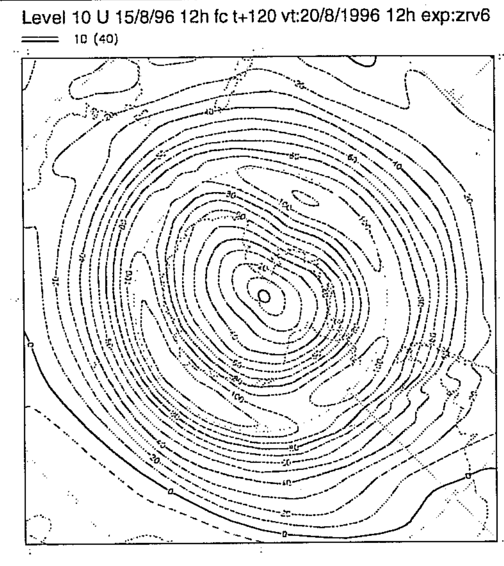
(d) "1997" T_L319 Δt=1800

(e) SETTLS T_L319 Δt=1800

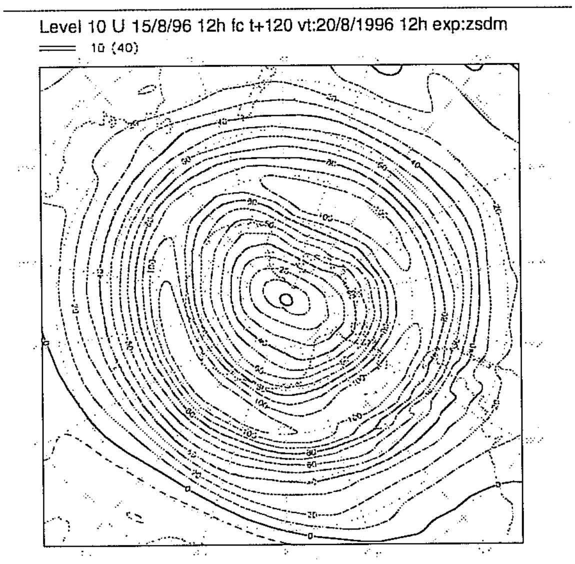
Figure 1: Forecast level 9 (~200 hPa) temperature from model runs at high resolution using: a) The three-time-level scheme at T213L31 with Δt=900s, b) The "1997" semi-Lagrangian scheme at T213L31 with Δt=1800s, c) The SETTLS scheme at T213L31 with Δt=1800s, d) The "1997" scheme at T_L319L31 with Δt=1800s, and e) The SETTLS scheme at T_L319L31 with Δt=1800s.



(a) "1997"Δt=3600s



(b) three-time-level Δt=1800s



(c) SETTLS Δt=3600s

Figure 2: Forecast level 10 (~5hPa) zonal wind over the SH in a run started from initial data of 1996-08-15 at T_L159L50 resolution a): "1997" scheme with Δt=3600s b) three-time-level scheme with Δt=1800s c) SETTLS scheme with Δt=3600s.

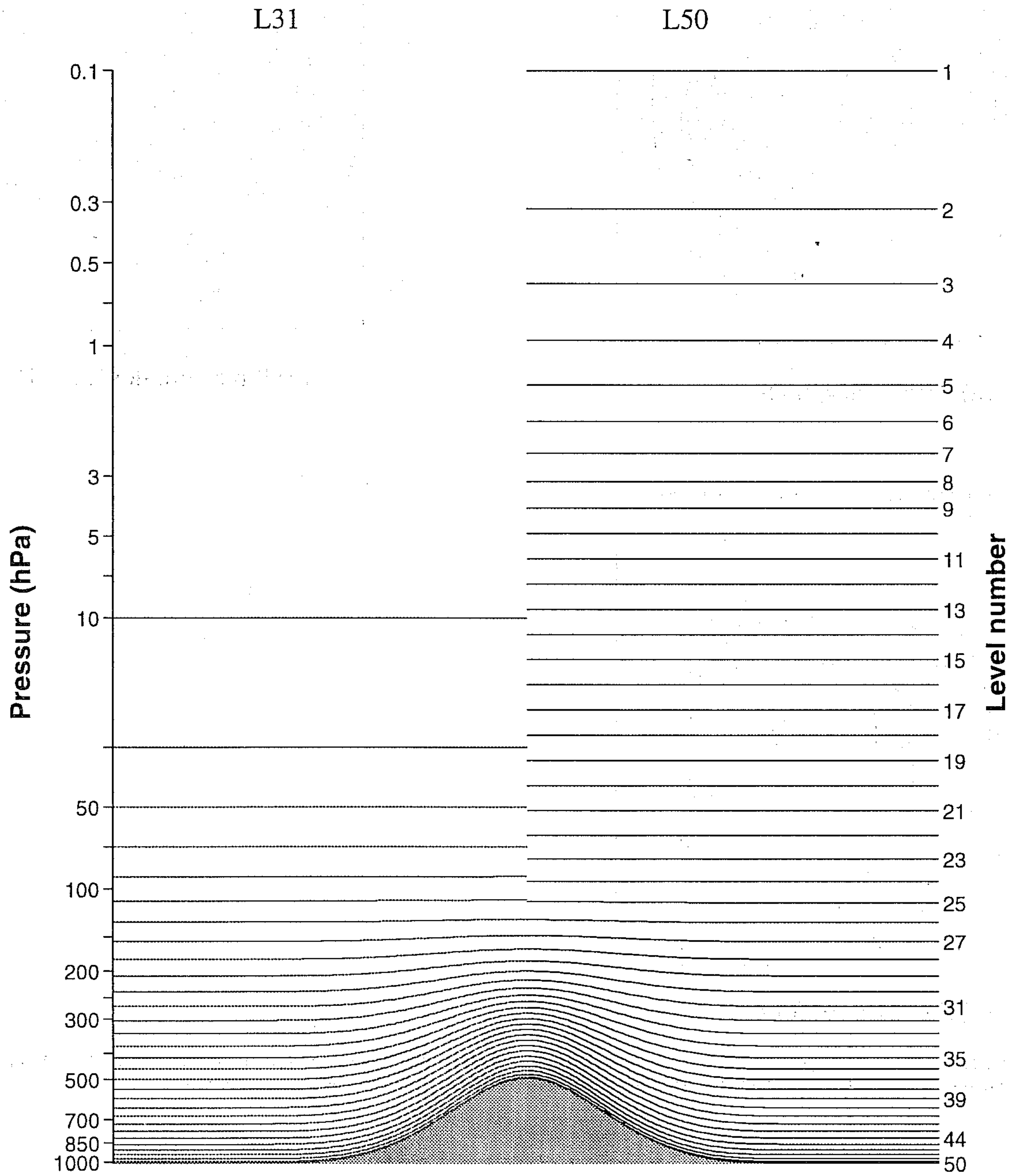


Figure 3: The distribution of the full model levels for the 31-level (left) and the 50-level (right) vertical resolutions, plotted for surface pressures which vary from 1013.25 to 500 hPa. (after Untch et al 1998).

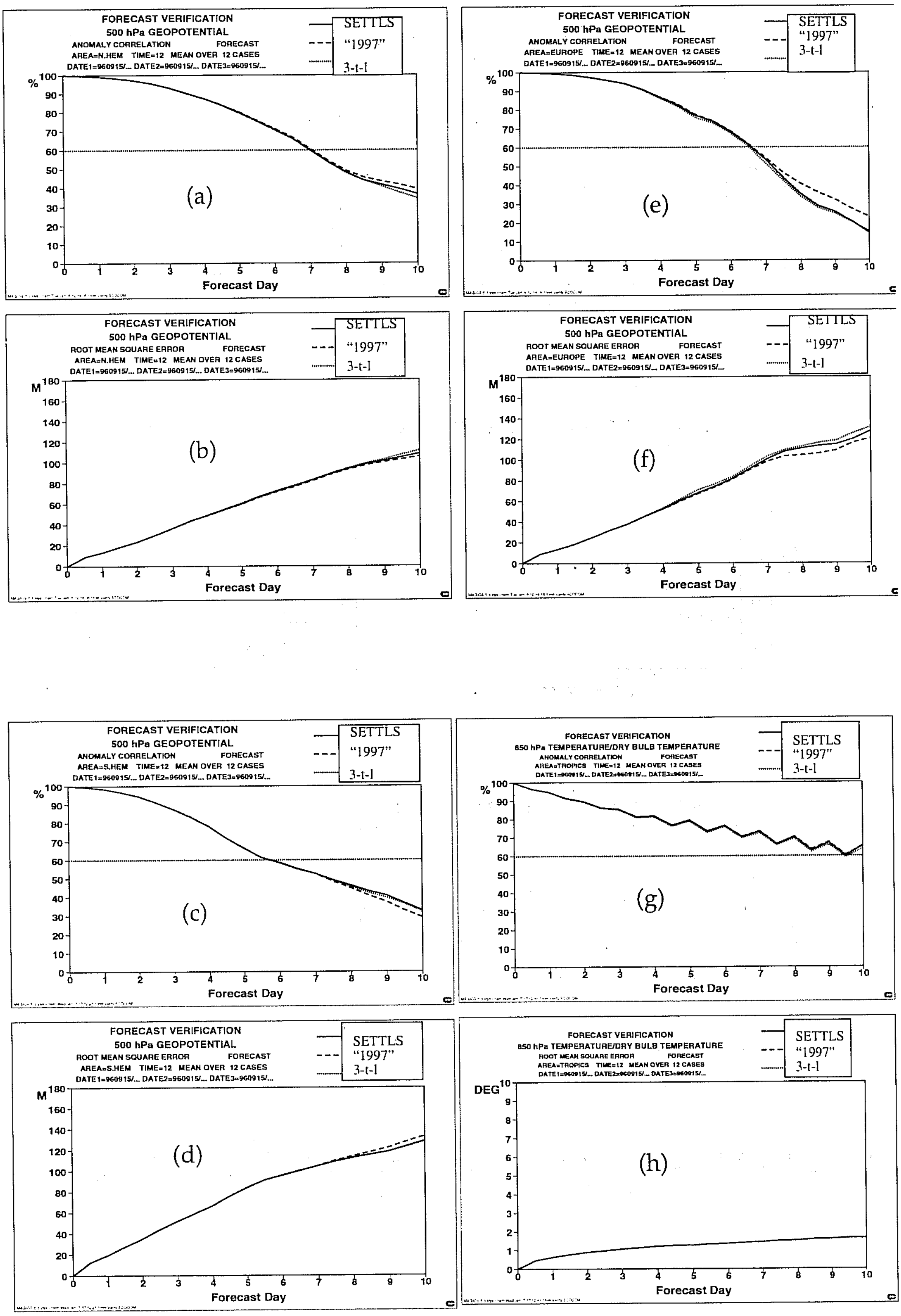
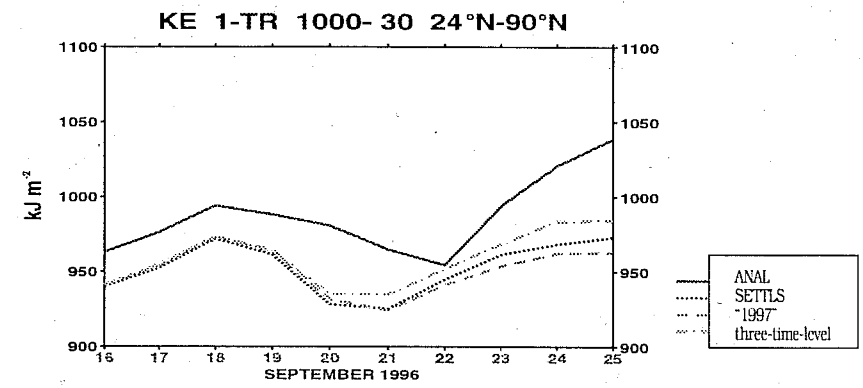
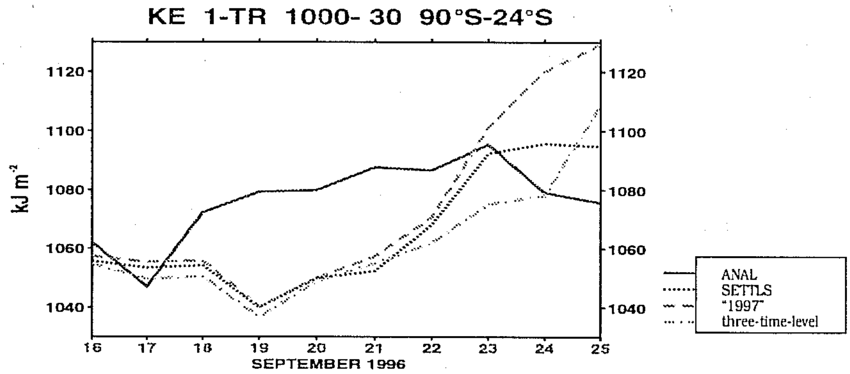


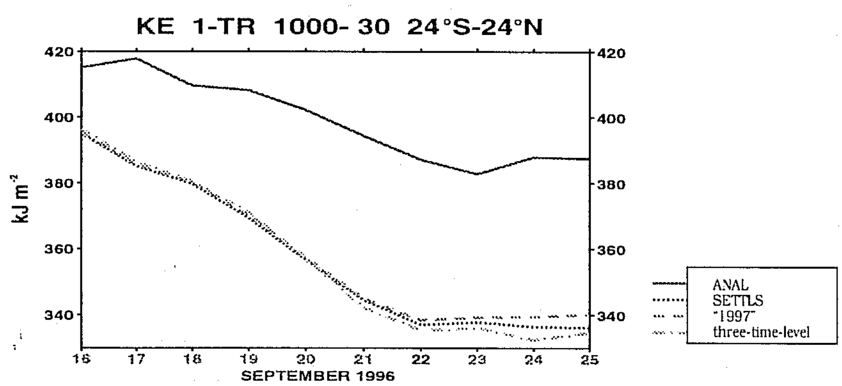
Figure 4: Some mean scores from three 12-element series of T213L31 forecasts. Full line is the SETTLS scheme, dash line is the "1997" scheme and dotted line is the three-time-level scheme, all of them using Δt=900s. (a) AC of 500 hPa geopotential over the NH, (b) RMS of 500 hPa geopotential over the NH, (c) AC of 500 hPa geopotential over the SH (d) RMS of 500 hPa geopotential over the SH, (e) AC of 500 hPa geopotential over Europe, (f) RMS of 500 hPa geopotential over Europe, (g) AC of 850 hPa temperature over the tropics and (h) RMS of 850 hPa temperature over the tropics



(a)



(b)



(c)

Figure 5: Mean eddy kinetic energy for the same three series of 12 elements each of T213L31 forecasts as Figure 4 , compared with the verifying analyses (full line): dotted line with the SETTLS scheme, dash line with the "1997" scheme, dot-dash line with the three-time-level scheme. Panel a) Northern Hemisphere, b) Southern Hemisphere, c) Tropical belt

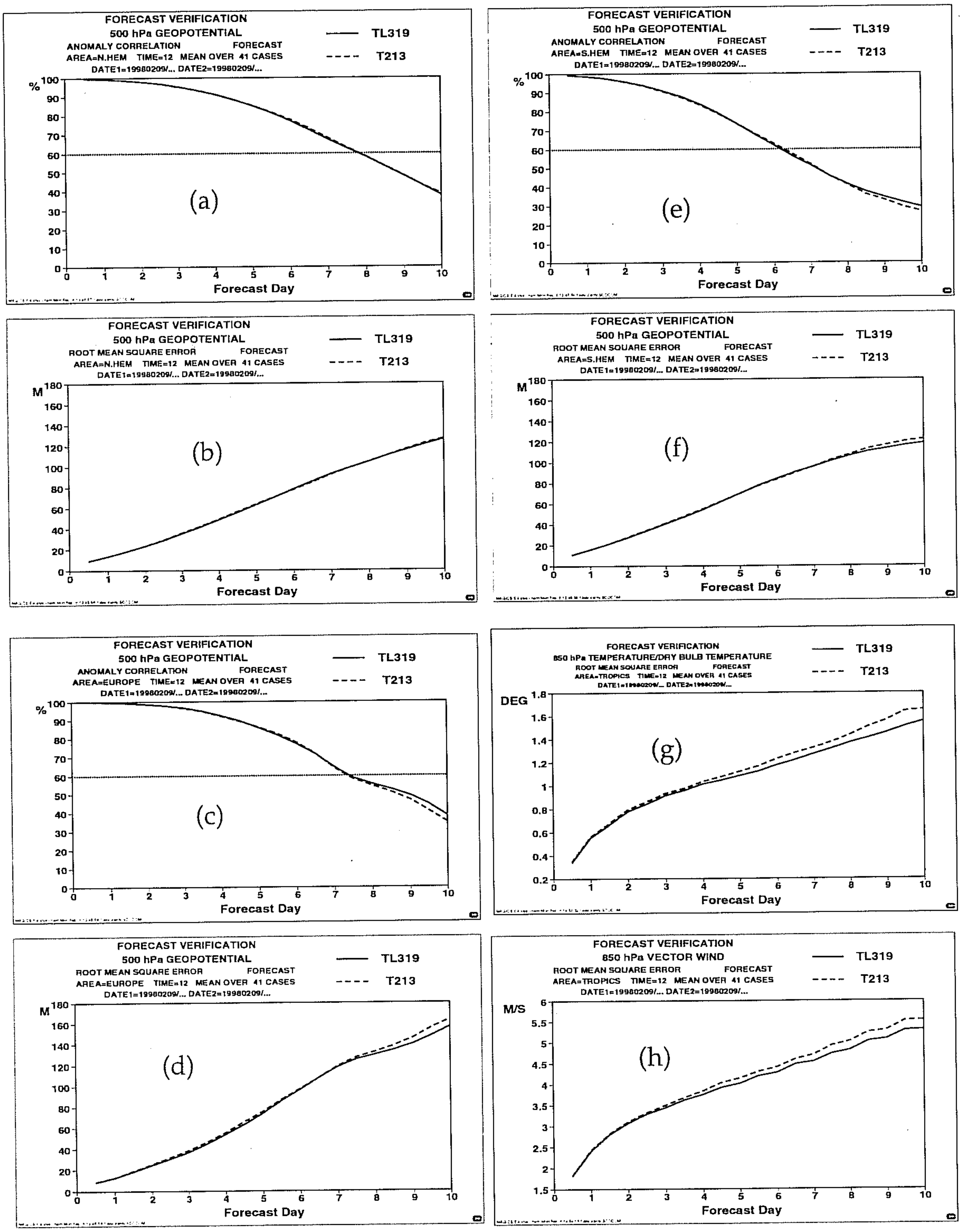


Figure 6: Verification scores of the 41 forecasts of the parallel suite at $T_L319L31$ which could be verified against its own analyses up to day 10. The parallel suite used the SETTLS scheme with a time step of 1200s and a new orography (full line), compared with the operational forecasts produced with the "1997" scheme, a time step of 900s and the old orography (dash line). (a) AC of 500 hPa geopotential over the NH, (b) RMS of 500 hPa geopotential over the NH, (c) AC of 500 hPa geopotential over Europe, (d) RMS of 500 hPa geopotential over Europe, (e) AC of 500 hPa geopotential over the SH, (f) RMS of 500 hPa geopotential over the SH, (g) RMS of 850 hPa temperature over the tropics, and (h) RMS of 850 hPa vector wind over the tropics.