Numerics in wind wave models [1]

Hendrik L. Tolman [2]

UCAR Project Scientist
NOAA, National Centers for Environmental Prediction
Camp Springs, Maryland, USA

Abstract

Numerical aspects of wind wave models are reviewed and discussed. The paper focusses on three main aspects: (i) Model resolution. (ii) Source terms integration. (iii) Numerical propagation schemes. Arguments for increasing spectral resolution rather than spatial resolution are presented. Alternatives for increasing spatial resolution are discussed. The discussion of source term integration schemes mostly focusses on the use of limiters. The discussion of propagation schemes mostly focusses on the so-called garden sprinkler effect, and its remedies.

1. Introduction

Almost a decade ago, a systematic assessment was made of sources of numerical errors in wave models (Tolman 1992). With several years of additional experience, it is now an appropriate time to revisit the issues raised in this paper.

The state-of-the-art in ocean wave modelling is now represented by the so-called third-generation wave models. Such models are based on a random-phase, spectral description of the ocean surface. The basic conservation equation used in such models is

$$\frac{DA}{Dt} = S \quad , \tag{1}$$

where A represents a wave action or energy spectrum, and S represents sources and sinks. In a third-generation model, a form of Eq. (1) is solved by directly parameterizing all processes involved, without assuming spectral shapes. The first such model was the WAM model (WAMDIG 1988, Komen et al. 1994). This model has been widely used for over a decade. More recently, the WAVEWATCH (Tolman 1991, 1999; Tolman and Chalikov 1996) and SWAN (Booij et al. 1999, Ris et al. 1999) models have become popular.

Typically, the spectrum A and source term S in Eq. (1) are described as a function of wave frequency f and direction θ, or as a function of the wavenumber k and direction θ. The exact equation and the parameters spanning spectral space differ from model to model, but are for the most part not relevant in the present context. An exception is the potential loss of spectral resolution in shallow water for a spectrum defined on k, as is discussed in detail in Tolman and Booij (1998). The actual parameterizations of the source terms S are also not expected to be important in the present discussion. Although details of source terms may differ drastically between models,

---

[1] OMB contribution 202.

[2] Hendrik.Tolman@NOAA.gov

their balance should be similar for models to give good results. This in principle implies that general numerical considerations can be expected to be valid for arbitrary models.

Tolman (1992) first addressed wave growth in homogeneous conditions, to assess the impact of numerical choices on the integration of source terms. Secondly, pure propagation tests and fetch-limited growth tests were performed to assess the impact of (errors in) propagation schemes on wave model results. Finally, all was brought together in some realistic test cases from the SWADE experiment (Weller et al., 1991).

Several years of experience with operational wave modelling at NCEP, including many interactions with the Weather Service Field Offices (WFO), have identified some additional points of interest. The original tests in Tolman (1992) do not focus well on swell propagation issues. Swell prediction is of major interest for the WFO's, particularly in the Pacific Ocean. Furthermore, it has become increasingly clear that requirements for a wave model as a research tool, or as an operational tool, are different, and in some ways conflicting. Also, the introduction of new super-computer paradigms, particularly that of parallel computing, have an impact on details of the model (e.g., Tolman 2001a). Parallel computing considerations, however, focus on general model designs rather than numerics, and will therefore not be addressed here.

In the present paper, several numerical issues will be discussed and revisited. We will first consider model resolution in general. Secondly, source term integration is addressed. Finally, wave propagation, particularly swell dispersion will be addressed.

2. Model resolution

For large-scale, deep ocean wind-wave models, the wave spectrum is typically discretized using 24 directions (15° resolution), and about 25 frequencies with a logarithmic distribution and a 10% increment in frequencies ($f_{i+1} = 1.1 f_i$, where i is the discrete grid counter in frequency space). Spatial resolutions range from the order of 1° in longitude and latitude to about 0.25° for operational models, and as small 1/12° or of the order of 10 km for regional operational and research applications. The trend over the last decade has been to increase the spatial resolution, while leaving the spectral resolution largely unchanged. There does not seem to have been much discussion on the appropriateness of this trend.

The need for additional spatial resolution is well known. To optimally use the information contents of the wind field, the wave model resolution should be comparable to the resolution of the driving wind field. This appears to have been the driving force behind increasing spatial model resolutions in the near past. Furthermore, increased spatial resolution makes it possible to realistically resolve more island chains. The importance of the latter is illustrated in, for instance, Bidlot et al. 1997 or Tolman et al. 2001.

As mentioned above, spectral resolutions have largely been left unchanged over the last decade. Most arguments about required spectral resolution focus on wave growth (e.g., Tolman 1992). Furthermore, the Discrete Interaction Approximation (DIA) to nonlinear interactions (Hasselmann et al. 1985), which still is the cornerstone of present operational state-of-the-art wave models, has been more or less developed for presently common spectral resolutions. Attempts to drastically increase spectral resolutions appear to result in the introduction of notable noise in the resulting spectra. Whereas this behavior has been mentioned by several sources, it has not yet been documented adequately (to the knowledge of the present author).

Two compelling reasons can be given for further increasing spectral resolutions. First, accurate description of exact nonlinear interactions suggest the need of higher spectral resolutions, of about 10° in direction and 7% in frequency or better (e.g., Van Vledder 2001). The second reason is that although the 15°, 10% resolution gives reasonable wave growth, this resolution is still too course to adequately describe a typical spectrum, like for instance the JONSWAP spectrum (Hasselmann et al. 1973). This seriously affects the capability of a wave model to adequately describe swell dispersion, as is illustrated in Figs. 16 and 25 through 27 of Tolman (1995).

At operational forecast centers, computational resources are bound to increase systematically. Considering the above, a conscious decision needs to be made where to use these additional resources. At NCEP, we are considering further increasing spectral resolutions first, mainly to increase our ability to accurately describe swell dispersion and arrival times of swell fronts. Such an increase will be predicate upon availability of alternatives to the DIA.

Limitations of the spatial resolution may well be alleviated without the use of additional computer resources. First, the commonly used regular longitude-latitude grid introduces two disadvantaged with respect to numerical resolution and resources required; The physical resolution is unevenly distributed over the grid for large-scale applications; Localized small grid increments near the poles result in unnecessary small time steps for most of the grid. One solution to this wasteful use of computer resources is to use an "equivalent" grid as suggested by Bidlot et al. 1997 (their Fig. 3). Whereas their grid appears elegant, a rather ad-hoc numerical approach has been used. More mathematically elegant solutions would be conformal mapping techniques, as have been suggested before for the PHIDIAS model (Van Vledder and Dee 1994; oral presentation by Van Vledder, 1996 WISE meeting in Venice, Italy). An interesting alternative is obtained when the globe is expanded to a cube, as suggested by Purser and Rancic (1998). Such a grid is the starting point for a newly developed ocean prediction system at NCEP (Chalikov et al., 2001), and might be considered for NCEP's ocean wave model in the near future. Increasing spatial resolution to resolve islands is also wasteful, because high resolution is required only near the islands considered. A more elegant yet economical way to deal with such islands it to treat them as sub-grid obstacles. A scheme that does this will be tested in the NCEP operational wave models this fall. A first report will be presented in Tolman (2001b).

3. Source term integration

During early development of third-generation wave models, time steps required for source term integration restricted the economical viability of such models. Particularly, time scales that may occur in the equilibrium range of the spectrum are generally in conflict with economically viable time steps for source term integration. To alleviate this problem, the integration is only performed up to a cut-off frequency, as will be discussed in some more detail below. Secondly, a semi-implicit time integration scheme is used to enhance numerical stability. Finally, a so-called "limiter" was introduced in the original WAM model (e.g., Hersbach and Janssen 1999), which limits the change in spectrum per time step. Using the frequency-direction spectrum $F(f,\theta)$ and its corresponding balance equation, the discrete change of spectral density per numerical time step ΔF was limited in WAM cycles 1 through 3 to be a fraction of a Phillips type spectrum

$$|\Delta F|_{\max} = 6.4x10^{-7} g^2 f^{-5} \tag{2}$$

Particularly the introduction of this limiter allowed for a major increase in feasible numerical time steps in third-generation wave models.

The use of this limiter, however, also has a negative effect. As shown by Tolman (1992), it introduces a significant sensitivity of resulting model growth characteristics to the numerical time step Δt. This is obviously undesirable. Tolman (1992) removed this sensitivity by using Eq. (2) to dynamically calculate the instantaneous and local maximum allowed time step Δt. In idealized conditions, this method reproduces the expected solution for the given source term parameterizations accurately and economically. This dynamic integration scheme has been used in the WAVEWATCH model ever since. Details of its present implementation include: (i) Parametric and relative criteria for calculating Δt. (ii) The option for a hybrid dynamic integration / limiter approach to avoid excessively small time steps. See Tolman 1999, section 3.5 for details. This integration method proved successful in the operational WAVEWATCH III implementation at the National Centers for Environmental Prediction (NCEP). By introducing the dynamically calculated time step, the average source term integration time for the global wave model increased from 20 min. to approximately 40 min.

The dynamical integration method for source terms cannot easily be introduced in WAM. The main reason for this is the basic design of the code. As a hold-over from its original design for a Cyber 205, the inner loops of the code deal with the spatial grid, even for source term integration and calculation. This implies that major source conversions are required to introduce dynamically adjusted time steps for individual spatial grid points.

In WAM cycle 4, the sensitivity of the source term integration was removed by making the limiter scale with the time step Δt (see Hersbach and Janssen 1999)

$$|\Delta F|_{\max} = 6.4x10^{-7}\, g^2\, f^{-5} \frac{\Delta t}{1200} \quad , \tag{3}$$

where Δt is expressed in seconds. This limiter, however, results in unrealistic short-fetch growth behavior (e.g., Tolman and Chalikov 1996, Fig. 7). This undesirable model behavior was remedied by Hersbach and Janssen (1999), by introducing an improved limiter

$$|\Delta F|_{\max} = 3.0x10^{-7}\, g\, \tilde{u}_*\, f^{-4}\, f_c\, \Delta t \quad , \tag{4}$$

where $\tilde{u}_*$ is a filtered friction velocity, and f_c is the cut-off frequency between the prognostic and diagnostic parts of the spectrum. As is shown in Hersbach and Janssen (1999), this limiter indeed removes most of the time step dependence from the solution, while reproducing good general growth behavior. For practical operational applications, this limiter therefore appears to be a good solution.

From a numerical or scientific perspective, however, the limiters (3) and (4) are less than ideal. The time step dependence of the solution is removed by scaling the limiter with the time step Δt itself. This implies that the relative impact of the limiter becomes independent of the time step, and that the limiter will remain an integral part of the solution, even for $\Delta t \rightarrow 0$. In numerical terms this implies that the integration scheme becomes non-convergent, that is, will never exactly produce the solution representative for the physics modelled. This obviously makes the limiters (3) and (4) less suitable for further research into the physics parameterizations in third-generation wave models. The fact that these limiters become an integral part of the solution also explains why several 'flavors' of the limiter (4) have been suggested (e.g., Luo and Sclavo 1997, Hargreaves and Annan 1998); An integration scheme with a non-convergent limiter by definition has to be sensitive to details of the limiter.

Hersbach and Janssen (1999) state that, based on their experience, the time step dependence of the source term integration can only be removed satisfactorily by using a non-convergent limiter of the form of Eq. (4). Some recent experiments with alternative limiters may prove this notion wrong.

Limiters of the form of Eqs. (2) through (4) put a hard limit on the maximum change of energy density in the spectrum. This limits both signatures of instabilities in the integration, and rapid but physically realistic growth behavior. The former effect of the limiter is desired, because it in effect stabilizes the integration. The latter effect is not desired, because it makes growth rates directly a function of the numerical time step. In principle, it may be possible to separate the physical signature of the numerical integration from the instability signature. Typically, instabilities show fluctuations at the resolution scale of the grid ($\Delta\theta$, Δf), whereas physical processes show more gradual fluctuations. This would then allow the instability to be limited as before, while relaxing limitations on physically realistic rapid growth behavior. This can be achieved by applying the limiter in an asymmetric fashion around the physically expected change of the spectrum

$$(\zeta - 1)L_0 < \Delta F < (\zeta + 1)L_0 \quad , \tag{5}$$

where ζ is the asymmetry, and L_0 is a conventional limiter form. For instance starting from the original convergent limiter in WAM cycles 1 through 3,

$$L_0 = 6.4x10^{-7}\, g^2\, f^{-5} \quad . \tag{6}$$

Assuming the existence of a scale separation as discussed above, the asymmetry ζ can be estimated from a normalized and averaged discrete spectral change

$$\zeta \approx \overline{\Delta F / L_0} \quad , \tag{7}$$

where the overbar indicates an average over a part of the spectral space small enough to be representative for the local physical change, yet large enough to average out oscillations related to numerical instabilities.

To illustrate the potential of such a limiter, it has been applied to the time-limited wave growth test of Hersbach and Janssen (1999), their Figs. 4 and 5. For simplicity, the physics and integration schemes of WAM cycle 3 have been used here. Details of these calculations will be presented elsewhere. Some preliminary results are presented in Fig. 1. Solid and dotted lines in the present Fig. 1 and in Fig. 4 of Hersbach and Janssen (1999) correspond to the convergent conventionally limited solutions for $\Delta t = 1200$ s. In spite of the different physics parameterizations used, the impact of the limiters are very similar. The dashed line in Fig. 1 is obtained with an asymmetric limiter with $\Delta t = 1200$ s. The deviation of this solution from the convergent solution (solid line) again is similar to the deviation found by Hersbach and Janssen (1999) for the new limiter (4) in WAM cycle 4 (their Fig. 5). Reduction of the time step for the convergent asymmetric limiter reproduces the convergent solution more closely as expected (chain and solid lines in Fig. 1). The non-convergent limiter (4) in Fig. 5 of Hersbach and Janssen (1999) is close to, but never reaches the convergent solution.

Obviously, the present test is not complete, as any new limiter will require rigorous testing in a wide range of idealized and real-world applications. The test does, however, indicate the potential of convergent limiters to be as successful as the non-convergent limiter (4), without sacrificing numerical convergence. In the same context, it should also be reiterated, that the limiter is not needed at all in the dynamic integration scheme used in WAVEWATCH.

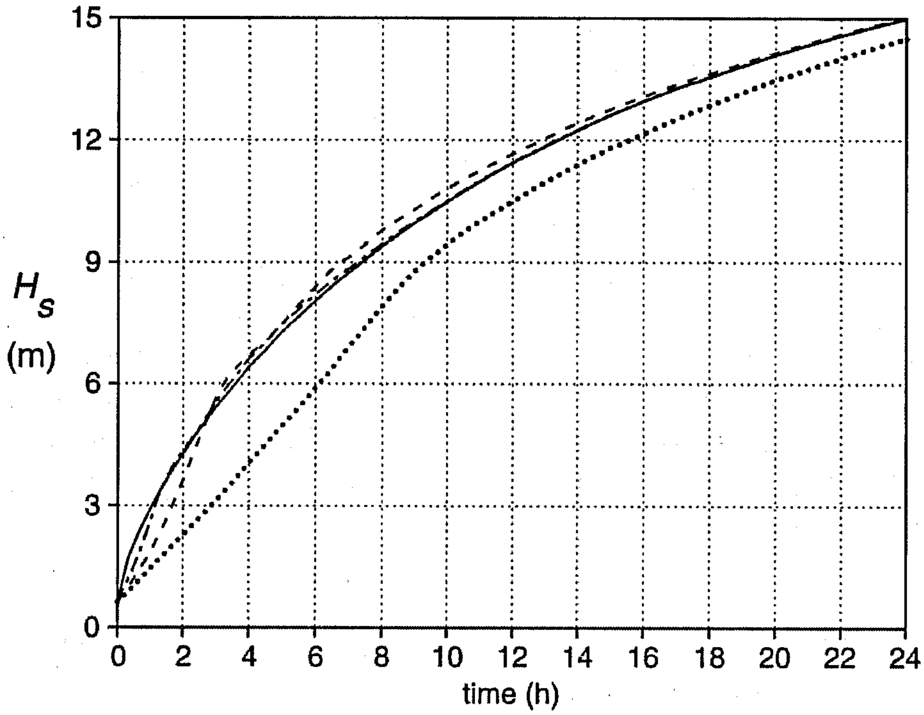

Figure 1. Time limited wave growth for a wind speed u_{10} = 25 m/s for WAM cycle 1-3 physics and different limiter configurations. Spectral discretization with 24 directions and 25 frequencies ranging from 0.042 to 0.42 Hz. Initial conditions consist of a JONSWAP spectrum with a peak frequency of 0.25 Hz. Solid line: convergent solution for original limiter (2). Dotted line: solution for limiter (2) with Δt = 1200 s. Dashed line: asymmetric limiter of Eqs. (5) through (7) with time step Δt = 1200 s. Chain line: like dashed line with Δt = 600 s.

So far, the focus in this section has almost entirely been on the limiter and its effects. Two additional aspects have been mentioned above, but not yet been discussed.

First, the basic time integration scheme as introduced in WAMDIG (1988) has been used without major modifications for over a decade in most third-generation wave models. The only modification that has been generally accepted in third-generation wave models is the change from a central-in-time to a forward-in-time version of this algorithm. The most convincing justification for this modification is given by Hargreaves and Annan (1998, also oral presentation at 1997 WISE meeting in San Francisco), who argue that for the equilibrium range, a root finding algorithm is more appropriate than a direct discretization of the equation. They then continue to show that the original scheme modified to be forward-in-time corresponds to a Newton-Raphson root-finding scheme.

Secondly, the use of a cut-off frequency in the calculations deserves some more attention This cut of frequency eliminates unnecessary small time scales in the equilibrium range of the spectrum from calculations, and furthermore assures proper scaling behavior of wave growth. Furthermore, physical arguments about the lack of knowledge of the physics in this spectral range justify the use of a cut-off frequency. Whereas the use of a cut-off frequency is based on solid considerations, the actual calculation of this frequency can be difficult. This is particularly true if wind seas and wells coexist with large separation in frequency space (e.g., Booij et al. 1999).

4. Swell propagation and dispersion

From a physical perspective, swell propagation can be described as a simple linear hyperbolic advection problem. However, because swell propagates over large distances, the numerical representation of swell propagation is difficult. Accurate propagation requires higher order schemes, which in turn introduce problems related to spurious oscillations and the generation of negative wave energy. Some resolution problems have already been discussed in a previous section. In addition, limited spectral resolution results in the disintegration of a continuous swell field into a set of discrete swell field. This phenomenon is known as the garden sprinkler effect, and is discussed in detail in Booij and Holthuijsen (1987). The garden sprinkler effect occurs because all spectral energy in a finite part of the spectral space [$\Delta f \times \Delta\theta$ around discrete grid points] is advected with the average propagation properties for the spectral grid point, ignoring additional dispersion occurring within the spectral band $\Delta f \times \Delta\theta$.

To illustrate the difference in behavior of different numerical approaches, a test similar to the garden sparkler test case of Booij and Holthuijsen (1987) is presented here in Fig. 2. An area of 4500 by 3500 km is considered. 500 km from the lower and left sides of the area, an initial wave field is defined with a wave height of 2.5 m, and a spatial distribution that is Gaussian with a width of 150 km. The spectrum has a mean frequency of 0.1 Hz, with a Gaussian distribution in frequency space with a width of 0.005 Hz. The mean spectral direction is 30° and the directional distribution is of the $\cos^2$ type. Model resolutions are 100 km in space, 10% in frequency, 15° in direction, and the time step $\Delta t = 3600$ s. This is fairly representative for large scale operational models. The wave field is propagated for five days. A semi-exact solution is obtained with the WAVEWATCH III third-order scheme and increased resolutions of 50 km, 4.9%, 2.5° and 1800 s. The test results have been obtained with an experimental version of WAVEWATCH III, which includes the option to use regular Cartesian grids. Test results are presented in Fig. 2. The upper left panel shows the initial conditions for all test cases. It also shows the near-exact solution. The middle left panel shows the classical garden sprinkler effect.

Virtually all WAM applications presently use a first order propagation scheme. The corresponding dispersion pattern for the present test case is presented in the upper right panel of Fig. 2. Apart from generally excessive diffusion as might be expected, the main draw back of this scheme is the directional anisotropy, with apparent preferred propagation directions along the axes of the spatial grid. As shown by Bidlot et al (1997), the occurrence of preferred directions can be greatly reduced by shifting the spectral directions by half the directional resolution $\Delta\theta$, thus avoiding that some spectral directions coincide with the axes of the spatial grid. Apart from in the preferred directions, the first order scheme does not clearly display a garden sprinkler effect, as it is partially masked by numerical diffusion. However, Fig. 2 of Bidlot et al (1997) clearly indicates that the garden sprinkler effect is notable in all spectral directions for a first order scheme in practical conditions.

Higher order schemes can readily be adopted from other fields in computational fluid mechanics. A third order scheme has been implemented in the Australian version of WAM cycle 3 (Bender 1996). Second order schemes were adopted in WAVEWATCH I and II (Tolman 1991, 1992), and a third-order scheme is used in WAVEWATCH III (Tolman 1999). With the adoption of such higher order schemes, the garden sprinkler effect becomes obvious. This is illustrated in the middle left panel of Fig. 2 for the third-order scheme used in WAVEWATCH III. Clearly, such a numerical solution has virtually no practical use.

As is show by Booij and Holthuijsen (1987), the garden sprinkler effect can be alleviated by adding a tensor-type diffusion to the spatial propagation equation (see their paper or Tolman 1995, 1999 for technical details), which accounts for dispersion within spectral bins. The resulting swell dispersion is shown in the middle right panel of

Fig. 2. The garden sprinkler effect has mostly disappeared. Note that there appear to be no clear preferred directions like in the first order scheme. Therefore, rotating the discrete spectrum does not appear to be necessary in combination with an accurate higher-order scheme (although it would not be detrimental either).

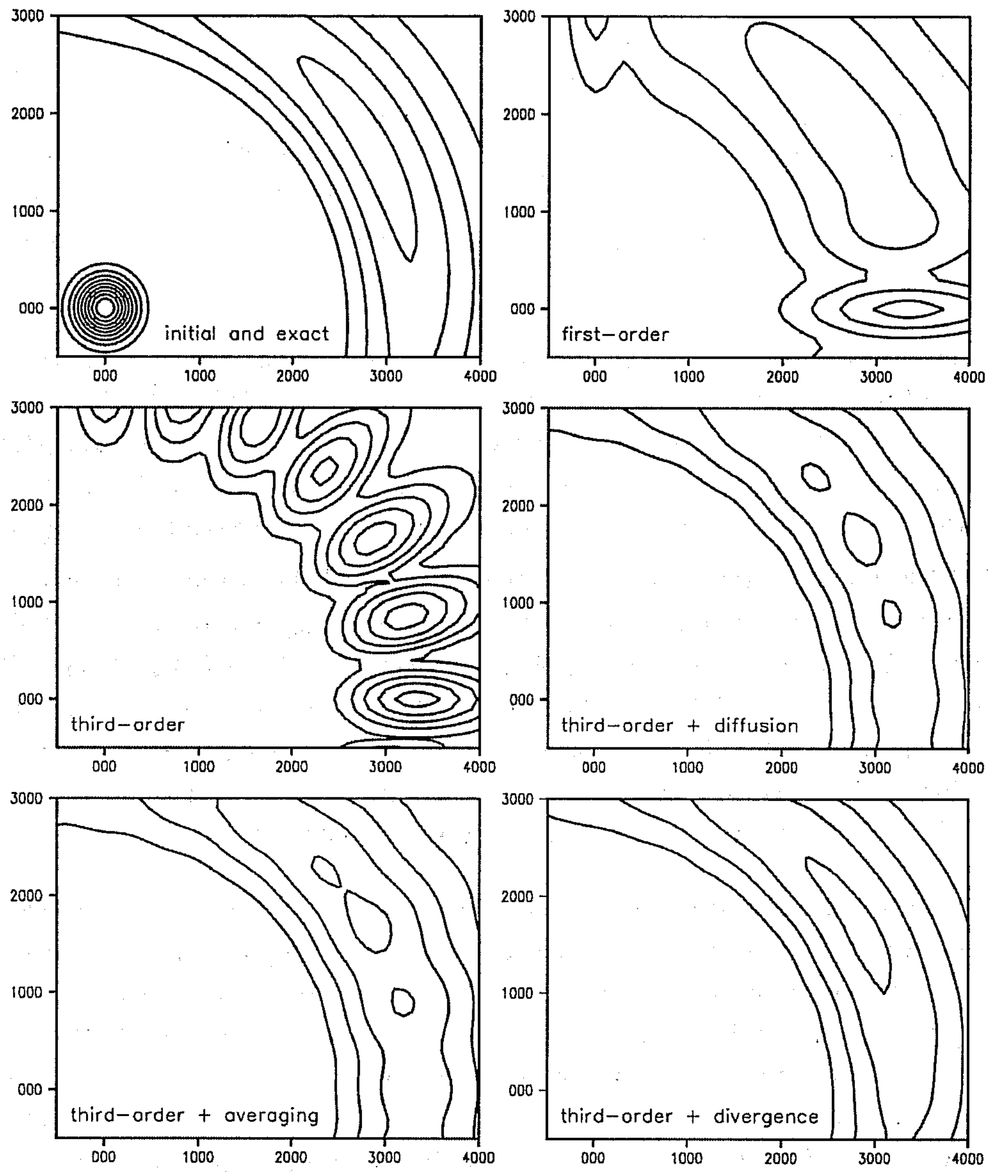

Figure 2. Long distance swell propagation tests. Wave heights in meters with contour intervals at 0.1 m (except for initial conditions with contour interval of 0.25 m). Initial wave height 2.5 m. Upper left panel: Initial conditions (lower left part of panel) and near-exact solution after 5 days. Upper right panel: Results of first order scheme. Middle left panel: results for third-order scheme in WAVEWATCH III. Middle right panel: Like middle left panel with Booij and Holthuijsen (1987) diffusive correction terms with a swell age of 4 days. Lower left panel: Like middle left panel with averaging technique. Lower right panel: Like middle left panel with divergent advection field.

An obvious disadvantage of this garden sprinkler solution is the added computational effort. This becomes particularly cumbersome for high-resolution models. Whereas the numerical time step for propagation, as determined by a CFL criterion, scales linearly with the grid resolution, typical implementations of the diffusion equation imply that the time step scales quadratically with the grid resolution. This implies that for higher spatial resolutions, the required time step for the diffusion equation suggested by Booij and Holthuijsen (1997) becomes more restrictive than the common CFL criterion for advection. For instance, for the global operational wave model at NCEP, which has a spatial resolution of approximately 1°, the time step is still dictated by the CFL criterion for advection. For the North Atlantic Hurricane wave model, however, which has a spatial resolution of 0.25°, advection allows for a time step of about 900 s, whereas the diffusion equation requires a time step as small as 300 s.

With this in mind, we have been working on two alternative solutions to remove the garden sprinkler effect. More details will be presented in Tolman (2001b). Here we will present some general considerations and initial results only.

The diffusion tensor introduced by Booij and Holthuijsen (1987) spreads energy in space corresponding to the natural dispersion for a given spectra bin size $\Delta f \times \Delta\theta$ and the corresponding average group velocity vector $\vec{c}_g$. This diffusion process is very similar to spatial averaging, where the size of the averaging area again scales with $\Delta f \times \Delta\theta$ and $\vec{c}_g$. This suggests that the mathematically elegant diffusion term of Booij and Holthuijsen (1987) can be replaced with a much more simple spatial averaging before or after the conventional propagation is performed. Results of such an approach are shown in the lower left panel of Fig. 2, and are virtually identical to the results of the original Booij and Holthuijsen (1987) approach. In principle, pre- or post-averaging of a wave field has no impact on the required time steps for propagation, and hence will result in much more economical high-resolution wave models. Moreover, the spatial extent of the averaging area scales with Δf, $\Delta\theta$, $\vec{c}_g$ and Δt in such a way, that satisfying a CFL criterion for propagation automatically implies that the averaging for a given spatial grid point should only include the directly neighboring points. This guarantees that a simple and efficient averaging algorithm is always sufficient.

A completely different way to add the proper dispersion to wave fields, is to add a small controlled divergence to the advection field $\vec{c}_g$. For a given spatial energy distribution for a single spectral bin (f,θ), the proper dispersion pattern can be modelled by shifting $\vec{c}_g$ from 0.5 $\Delta\theta$ to -0.5 $\Delta\theta$ from the far left to far right side of the spatial distribution (looking in the propagation direction), and similarly speeding-up or slowing-down $\vec{c}_g$ by $0.5\,\Delta\vec{c}_g$ to $-0.5\,\Delta\vec{c}_g$ in the propagation direction. The corresponding swell dispersion pattern is shown in the lower right panel of Fig. 2. This resulting dispersion pattern is very realistic, because the correction of $\vec{c}_g$ results in a dispersion pattern with proper spatial curvature for each individual spectral component. In contrast, the diffusion and averaging techniques only spread energy in the propagation and normal directions. This technique has so far only been applied to this simple test case. The challenge now is to make it efficient and robust for realistic conditions with multiple wave fields.

Dealing with proper swell dispersion and avoiding the garden sprinkler effect appear to be the main wave propagation issues, particularly for operational wave models. Note that the present test only deals with dispersion in a qualitative way. Additional assessment methods of dispersion characteristics for swell are presented in Tolman (1995). The above has furthermore ignored the impact of propagation schemes on fetch-limited wave growth. Tolman (1992) already established this impact to be generally small. The main problem here is the potential interaction between spurious oscillations in the spectrum due to propagation errors, and sensitive source terms (particularly nonlinear interactions). This proved to be a problem with the second order propagation schemes of WAVEWATCH I and II (Tolman 1991, 1992). It does not appear to be a problem with the more accurate third-order schemes presently used.

5. Discussion and conclusions

In the present paper, numerical aspects of third-generation wind wave models are discussed. The starting point is a similar study by Tolman (1992). The present paper mostly focusses on advances since then.

First, required resolutions are considered. Over the past decade spectral resolutions have remained more or less unchanged. Spatial resolutions have been systematically increased. Several arguments for increased spectral resolution have been presented: (i) Better description of the peak of the spectrum. (ii) Corresponding better prediction of swell fronts and swell arrival times. (iii) More accurate calculations of source terms, particularly of nonlinear interactions. The first two points are important for operational models. The last point is presently mostly important for research models, but may become relevant for operational models when alternatives to the DIA become available.

Problems arising from limited spatial resolution may be alleviated by other means than a straightforward increase of resolution. First, straightforward longitude-latitude grids as now commonly used are notoriously inefficient from a general resolution and numerical resources point of view. This may be remedied by using alternative grids, such as the "equivalent" grid of Bidlot et al. (1997), or by using conformal mapping techniques. Furthermore, a better description of coastlines, in particular of islands, may be obtained by introducing non-resolved features as sub-grid elements (Tolman 2001b). By nature, spatial resolution issues are particularly important for operational model applications, due to a continuous conflict between desired resolution and required model economy.

The main issue in source term integration is presently the need for limiters, and their form. The dynamical integration scheme of WAVEWATCH, as first introduced by Tolman (1992), avoids the need for limiters. Its operational implementation at NCEP has proven its economical viability for operational models, and its general characteristics make it an excellent tool for scientific research. The numerical integration scheme of WAM uses a limiter. With the most recent limiter formulations of Hersbach and Janssen (1999), this limiter-based approach has been proven to be a good numerical solution for operational wave models. However, the most recent limiter formulations like (4) attain some of their characteristics by abandoning numerical convergence. Hence, the limiter becomes an integral part of the solution. This makes such models and source term integration schemes less suitable for scientific research into the source terms. The present paper suggests an alternative asymmetric convergent limiter form, that promises integration behavior similar to that of the Hersbach and Janssen (1999) limiter, without sacrificing the numerical convergence of the integration scheme. However, such a limiter will require a significant research effort, before it can be considered for operational implementation.

An additional aspect of the source term integration is the underlying semi-implicit integration scheme. A small but important development in this scheme is the change from a central-in-time scheme, to forward-in-time scheme, as first advocated by Hargreaves and Annan.

Lastly, propagation schemes have been discussed. Accurate swell dispersion over large distances requires higher order schemes. Higher order schemes, in turn, require additional remedies to alleviate the garden sprinkler effect. The addition of a diffusive correction term to the propagation equation as suggested by Booij and Holthuijsen (1987), has been proven to be successful in operational implementations of the WAVEWATCH model. Such a solution, however, has a serious impact on required time steps in models with a high spatial resolution. Alternative solutions to the garden sprinkler effect, including spatial averaging, or adding divergence to the advection velocity, are presently being investigated (see Tolman 2001b).

References

Bender, L. C., 1996: Modification of the Physics and Numerics in a Third-Generation Ocean Wave Model, *J. Atmos. and Oceanic Techn.*, **13**, 726-750.

Bidlot, J. R., P. Janssen, B. Hansen and H. Günther, 1997: A modified set up of the advection scheme in the ECMWF wave model. *ECMWF Technical memorandum* No.**237**, 31 pp.

Booij, N. and L. H. Holthuijsen, 1987: Propagation of ocean waves in discrete spectral wave models. *J. Comput. Phys.*, **68**, 307-326.

Booij, N., R. C. Ris and L. H. Holthuijsen, 1999, A third-generation wave model for coastal regions, Part I, Model description and validation, *J. Geophys. Res.*, **104**, 7649-7666.

Chalikov, D., D. B. Rao, I. Rivin, V. Krasnopolsky and R. Grumbine, 2001: A unified ocean forecast system (UOFS): A three-dimensional real-time forecast system for applications to global and regional domains. *NOAA/NCEP/OMB technical note* **194**, In preparation.

Hargreaves, J. C. and J. D. Annan, 1998: Integration of source terms in WAM. *Proceedings of the 5th International Workshop on Wave Forecasting and Hindcasting*, 128-133.

Hasselmann, K., T. P. Barnett, E. Bouws, H. Carlson, D. E. Cartwright, K. Enke, J. A. Ewing, H. Gienapp, D. E. Hasselmann, P. Kruseman, A. Meerburg, P. Müller, D. J. Olbers, K. Richter, W. Sell and H. Walden, 1973: Measurements of wind-wave growth and swell decay during the Joint North Sea Wave Project (JONSWAP). *Ergänzungsheft zur Deutschen Hydrographischen Zeitschrift*, **Reihe A (8) Nr. 12**, 95 pp.

Hasselmann, S., K. Hasselmann, J. H. Allender and T. P. Barnett, 1985: Computations and parameterizations of the nonlinear energy transfer in a gravity-wave spectrum, Part II: Parameterizations of the nonlinear energy transfer for application in wave models. *J. Phys. Oceanogr.*, **15**, 1378-1391.

Hersbach, H., and P. A. E. M. Janssen, 1999: Improvement of the short fetch behavior in the WAM model. *J. Atmos. Oceanic Techn.*, **16**, 884-892

Komen, G. J., L. Cavaleri, M. Donelan, K. Hasselmann, S. Hasselmann and P.A .E.M. Janssen, 1994: *Dynamics and modelling of ocean waves*. Cambridge university press, 532 pp.

Luo, W., and M. Sclavo, 1997: Improvement of the third generation WAM model (cycle 4) for application in nearshore regions. *POL Internal Document*, **116.**

Purser, R. J., and M. Rancic, 1998: Smooth quasi-homogeneous gridding of the sphere. *Quart J. Meteor. Soc.* **124**, 637-647.

Ris, R. C., L. H. Holthuijsen and N. Booij, 1999, A third-generation wave model for coastal regions, Part II: Verification, *J. Geophys. Res.*, **104**, 7667-7681.

Tolman, H. L., 1991: A third-generation model for wind waves on slowly varying, unsteady and inhomogeneous depths and currents. *J. Phys. Oceanogr.*, **21**, 782-797.

Tolman, H. L., 1992: Effects of numerics on the physics in a third-generation wind wave model. *J. Phys. Ocean.*, **22**, 1095-1111.

Tolman, H. L., 1995: On the selection of propagation schemes for a spectral wind wave model. *NWS/NCEP Office Note* **411**, 30 pp. + figures.

Tolman, H. L., 1999: User manual and system documentation of WAVEWATCH III version 1.18. *NOAA/NWS/NCEP/OMB technical note* **166.**, 110 pp. Available from:

http://polar.wwb.noaa.gov/waves/wavewatch .

Tolman, H. L., 2001 a: Distributed memory concepts in the wave model WAVEWATCH III. *Parallel Computing*, Accepted.

Tolman, H. L., 2001 b: Improving propagation in ocean wave models. *Proceedings Waves 2001*, ASCE, San Francisco, In preparation.

Tolman, H. L., B. Balasubramaniyan, L. D. Burroughs, D. V. Chalikov, Y .Y. Chao, H. S. Chen, and V. M. Gerald, 2001: Development and implementation of wind generated ocean surface wave models at NCEP. Submitted.

Tolman, H. L., and N. Booij, 1998: Modeling wind waves using wavenumber-direction spectra and a variable wavenumber grid. *Global Atmosphere and Ocean System*, **6**, 295-309.

Tolman, H. L., and D. V. Chalikov, 1996: Source terms in a third-generation wind wave model. *J. Phys. Oceanogr.*, **26**, 2497-2518.

Van Vledder, G. Ph., 2001: Efficient algorithms for the computation of non-linear quadruplet wave/wave interactions in deep and shallow water. This volume.

Van Vledder, G. Ph., and D. Dee, 1994: Validation document of the third-generation wave prediction model PHIDIAS. *Delft Hydraulics Report* H1861

WAMDIG, 1988: The WAM model - a third generation ocean wave prediction model. *J. Phys. Oceanogr.*, **18,** 1775-1810.

Weller, R. A., M. A. Donelan, M. G. Brisco and N. E. Huang, 1991: Riding the crest of the wave: A tale of two wave experiments. *Bull. Amer. Meteor. Soc.*, **72**, 163-183.