

# Linearized Physics (LP): Progress and Issues

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# Outline

- Why do we need LP in DA?
- Sources of sub-optimality in 4D-Var
- The non-linear model issue
- The “0-rain” issue
- Asymmetry of analysis increments
- Tangent-linear approximation and resolution
- Summary and recommendations

## Why do we need LP in DA?

In 4D-Var, the analysis is obtained by **minimizing** the following **cost function** which is a measure of the distance of the model state to a set of available observations and to the background state:

$$J = \underbrace{\frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_b)}_{J_b} + \underbrace{\frac{1}{2} \sum_{i=0}^{nsteps} (H_i M_i[\mathbf{x}_0] - \mathbf{y}_{oi})^T \mathbf{R}_i^{-1} (H_i M_i[\mathbf{x}_0] - \mathbf{y}_{oi})}_{J_o}$$

**B** = background error covariance matrix,

**R** = observation error covariance matrix (instrumental + interpolation + observation operator errors),

**H** = **observation operator** (model space  $\rightarrow$  observation space),

**M** = **forward nonlinear forecast model** (time evolution of the model state).

LP

$$\min J \Leftrightarrow \nabla_{\mathbf{x}} J = \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_b) + \sum_{i=0}^{nsteps} \mathbf{M}'^T [t_i, t_0] \mathbf{H}_i'^T \mathbf{R}_i^{-1} (H_i M_i[\mathbf{x}_0] - \mathbf{y}_{oi}) = 0$$

**H'** = **tangent-linear** of observation operator, **H'<sup>T</sup>** = **adjoint** of observation operator,

**M'** = tangent-linear of forecast model, **M'<sup>T</sup>** = adjoint of forecast model.

## LP in 4D- Var

- Building successful parameterizations for variational data assimilation implies to achieve the best compromise between:
  - **Linearity** (central assumption in 4D-Var; “regularization” needed).
  - **Simplicity** (cost and memory limitations + coding comfort).
  - **Realism** (reasonable fit to reality and to full non-linear model).

## Linearized physics in operational centres

Non-exhaustive list of linearized physical packages currently used in operational global assimilation systems:

	Environ. Canada	ECMWF	JMA	Météo- France	Met Office	NCEP	NRL
Method	4D-Var	4D-Var	4D-Var	4D-Var	4D-Var	3D-Var	4D-Var
Radiation		<b>X</b>	<b>X</b>			NR	
Vertical diffusion	<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>	NR	<b>X</b>
Gravity wave drag	<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>		NR	<b>X</b>
Convection		<b>X</b>	<b>X</b>		<b>X</b>	NR	
Large-scale condens.	<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>	NR	
<b>Control moist variable(s)</b>	$\ln(q_v)$	$\frac{\delta RH}{\sigma(RH_b)}$	$\ln(q_v)$	$\frac{\delta q_v}{\sigma(q_v^b)}$	$q_t = q_v + q_c$	$q_v/q_{sat}, q_l$	$q_v/q_{sat}(T_b)$

## Sources of sub-optimality in 4D-Var

- Tangent-linear and adjoint parameterizations in minimizations are only simplified linearized versions of the full non-linear schemes used in trajectories.
- Existence of strong nonlinearities in the forward model (thresholds, switches, discontinuities → especially true for moist processes, convection in particular).
- Minimizations are run at coarser horizontal resolutions than trajectories:

e.g. in ECMWF operations: T159 / T255 / T255 minimizations, but T1279 trajectories.

$$\Rightarrow \boxed{HM(\mathbf{x}_b)} + \boxed{\mathbf{H}'\mathbf{M}'\delta\mathbf{x}} \neq \boxed{HM(\mathbf{x}_b + \delta\mathbf{x})}$$

High-resolution  
background  
trajectory  
(NL model)

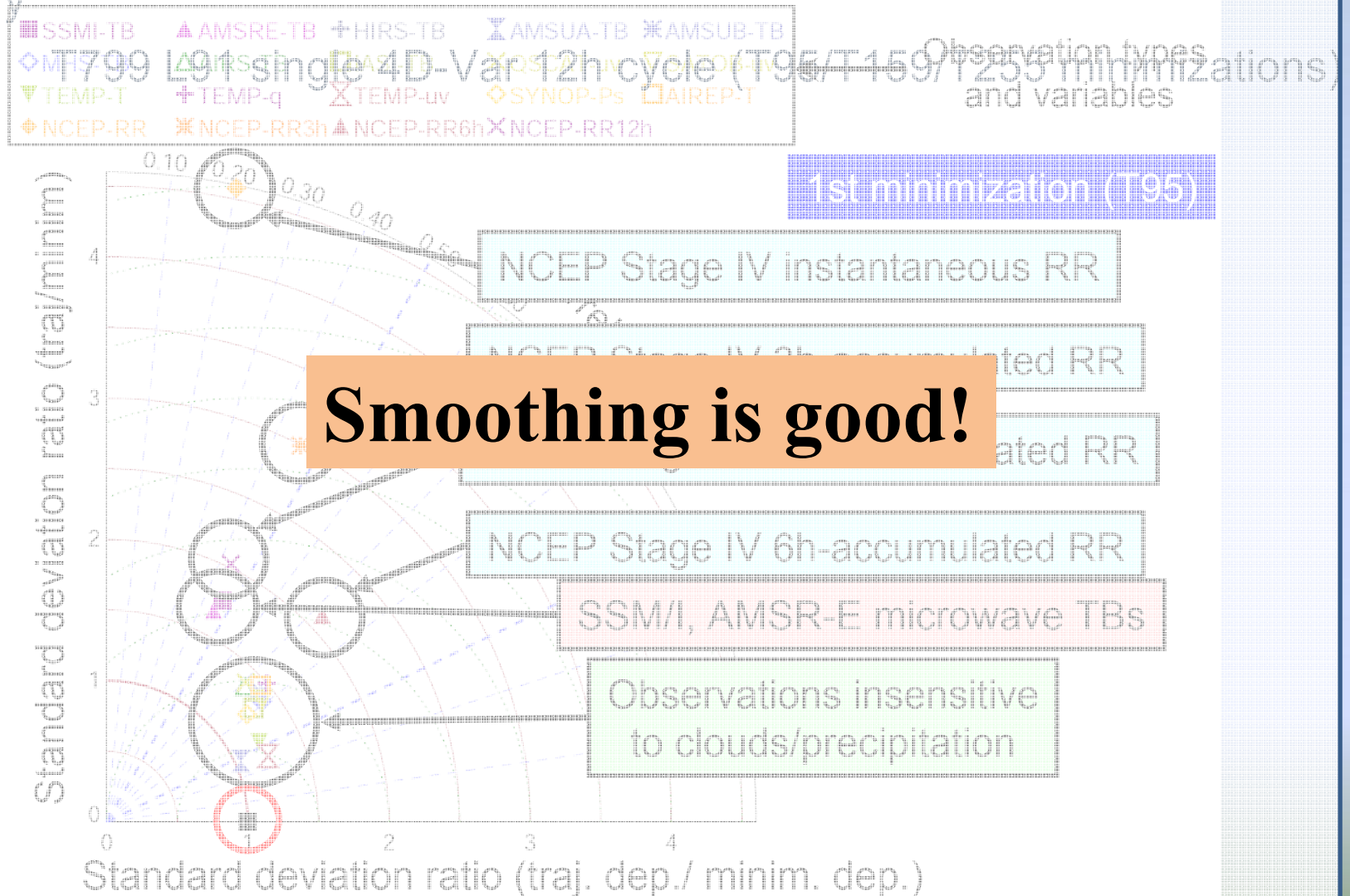
Low-resolution  
increment from  
minimization  
(TL model)

High-resolution  
updated  
trajectory  
(NL model)

# Trajectory / Minimization (mis)match in 4D-Var (in obs space)

Low-resolution “linear” departures:  $y_o - (HM(x_b) + H'M'\delta x)$  from minimization

High-resolution nonlinear departures:  $y_o - HM(x_b + \delta x)$  from trajectory



## The non-linear model issue (1)

- Recent experimentation has evidenced some **strong amplification of small-amplitude initial perturbations** in ECMWF's **non-linear model**, even over a single time-step.
- **Invalid tangent-linear approximation** (even though TL model is OK).
- This problem appears when activating either **vertical diffusion, large-scale condensation** or **convection** in the non-linear model.

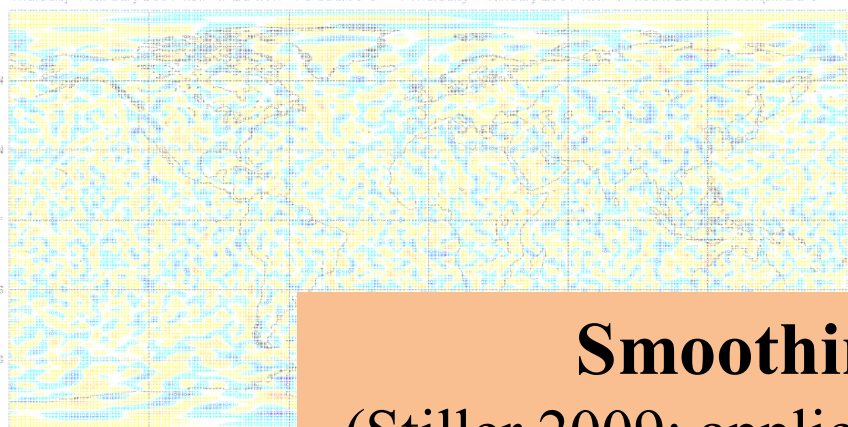


# Evolution of a small size initial perturbation with non-linear (NL) and tangent-linear (TL) model (dynamics + full physics)

Initial perturbation = white noise with max amplitude  $\approx 10^{-5}$  K and  $\text{m s}^{-1}$

TL evolution (1 step)

Thursday 4 January 2001 12UTC ECMWF Forecast t+0 VT; Thursday 4 January 2001 12UTC Model Level 60 Temperature



Difference between two NL runs (1 step)

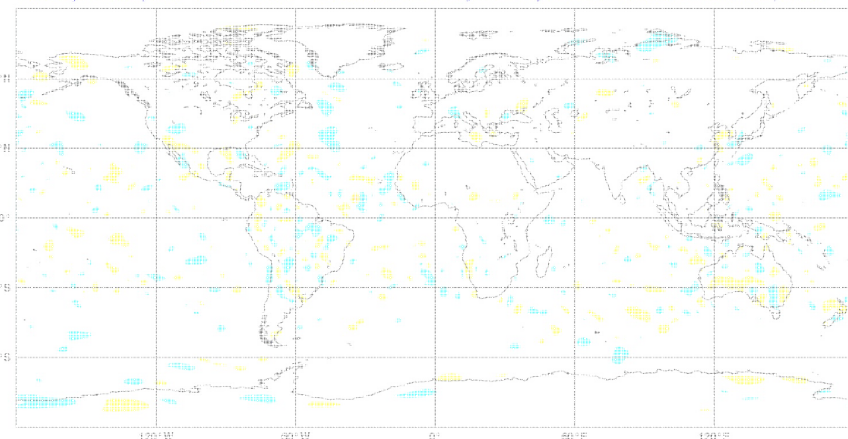
Thursday 4 January 2001 12UTC ECMWF Forecast t+0 VT; Thursday 4 January 2001 12UTC Model Level 60 Temperature



**Smoothing might help**  
(Stiller 2009: applied to convective tendencies in NL trajectory)

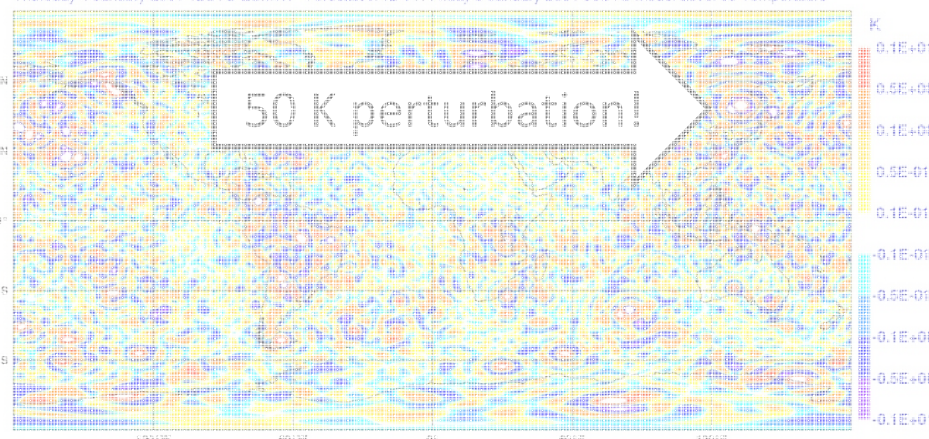
TL evolution (24 steps)

Thursday 4 January 2001 12UTC ECMWF Forecast t+12 VT; Friday 5 January 2001 00UTC Model Level 60 Temperature



Difference between two NL runs (24 steps)

Thursday 4 January 2001 12UTC ECMWF Forecast t+12 VT; Friday 5 January 2001 00UTC Model Level 60 Temperature

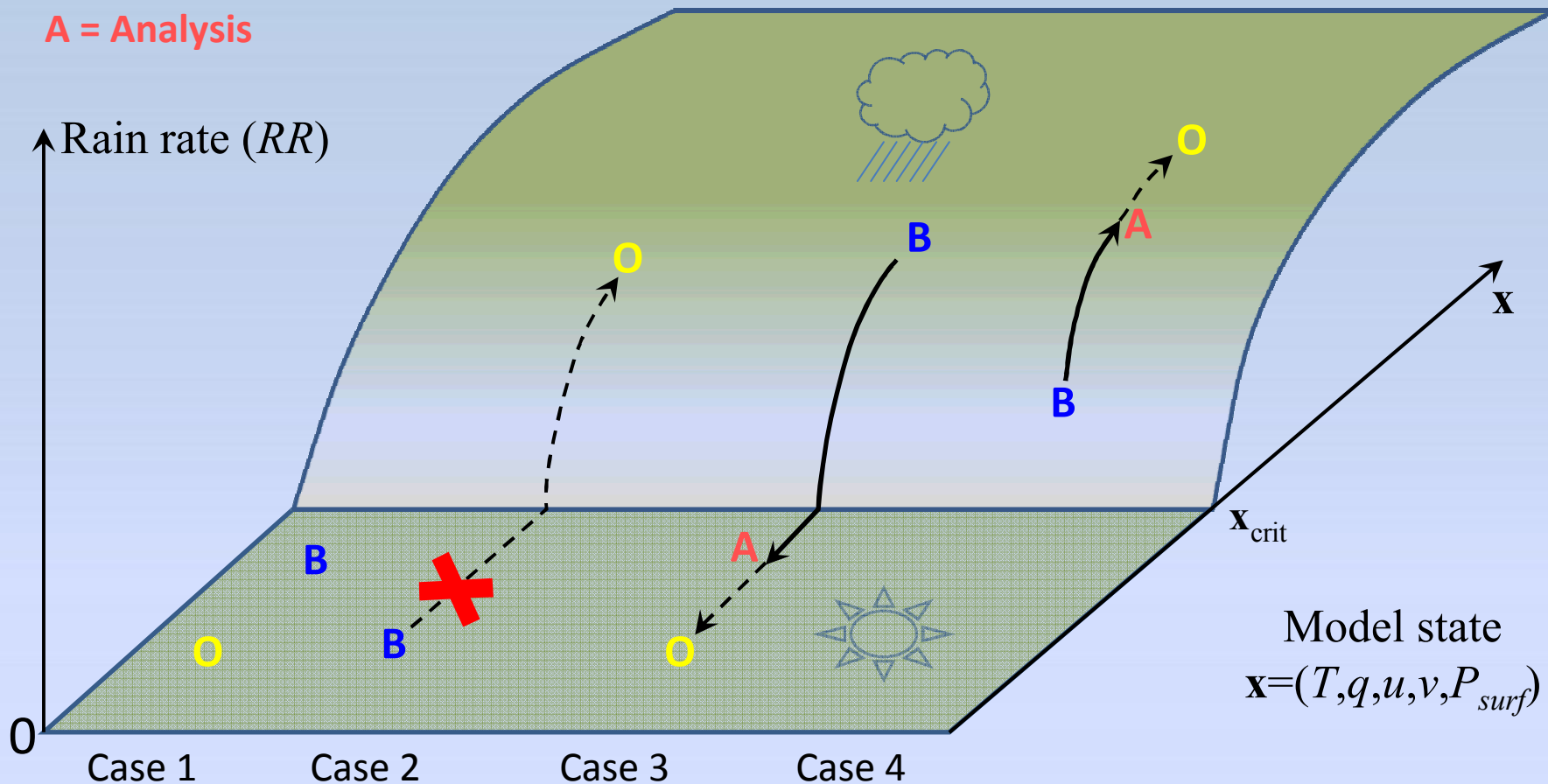


# The "0-rain" issue (1)

B = Background

O = Observation

A = Analysis



$$\frac{\partial RR_b}{\partial \mathbf{x}} = 0$$

Case 1 is irrelevant ( $J_o=0$ ).

Case 2 does not work (no sensitivity in physics).

Case 3 is ambiguous (unless other obs are available).

Case 4 is the only case that can be safely treated.

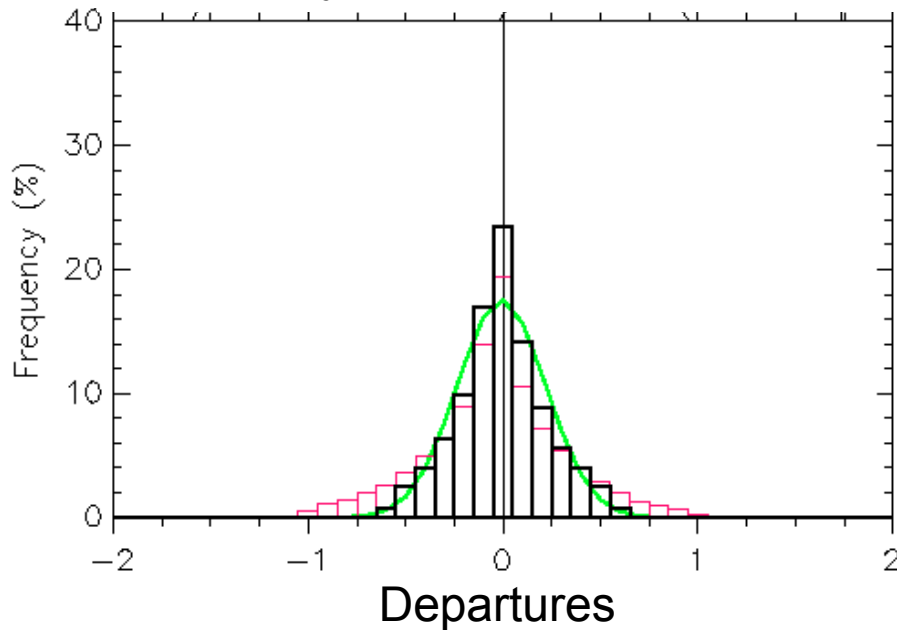
## The “0-rain” issue (2)

- If Case2 is discarded, then Case 3 should also be, to avoid creating a bias in the analysis, but this:
  - reduces the number of used observations,
  - makes the moving of misplaced weather systems difficult.
- **Solution:** to use a first-guess modified from the background to produce clouds & precipitation (e.g. Caumont *et al.* 2010, in 1D+3D-Var approach).
  - works OK at kilometric resolution (assuming saturation if precipitation is observed).
- **Question:** Can a similar procedure be implemented in 4D-Var at lower resolution?

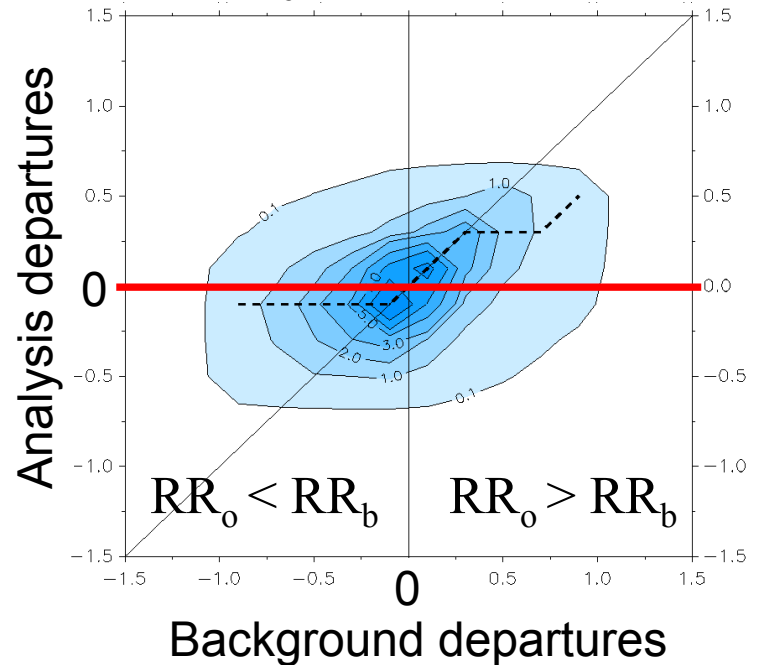
# Asymmetry of analysis increments

Statistics of direct 4D-Var assimilation of NCEP Stage IV ground-based radar & gauge precipitation data over Eastern U.S.A. in April-May 2009 (T511 L91).  
Observable =  $\ln(RR_{6h} + 1)$  with  $RR_{6h}$  in  $\text{mm h}^{-1}$ .

PDF of background (red) and analysis (black) departures



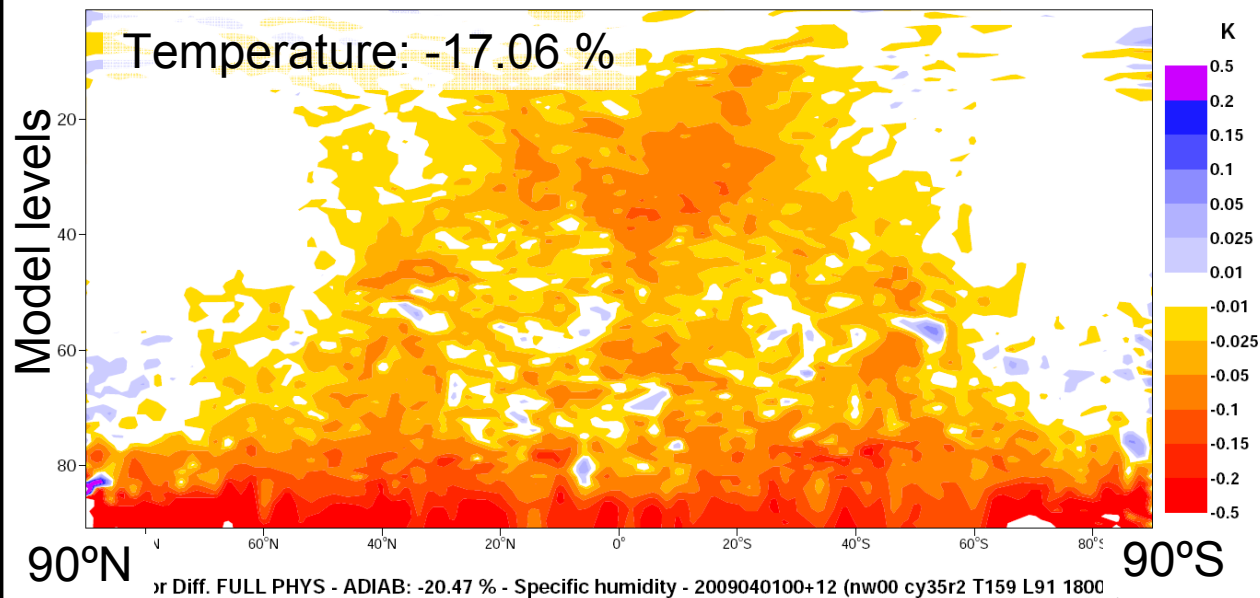
2D PDF of background and analysis departures



Always **easier to reduce precipitation than to increase it** during assimilation.  
← Limiting effect of saturation (capping of RH increments + asymmetry in moist physics sensitivities to input T and q).

# TL approximation and resolution (1)

TL Error Diff. FULL PHYS - ADIAB: -17.06 % - Temperature - 2009040100+12 (nw00 cy35r2 T159 L91 1800s)



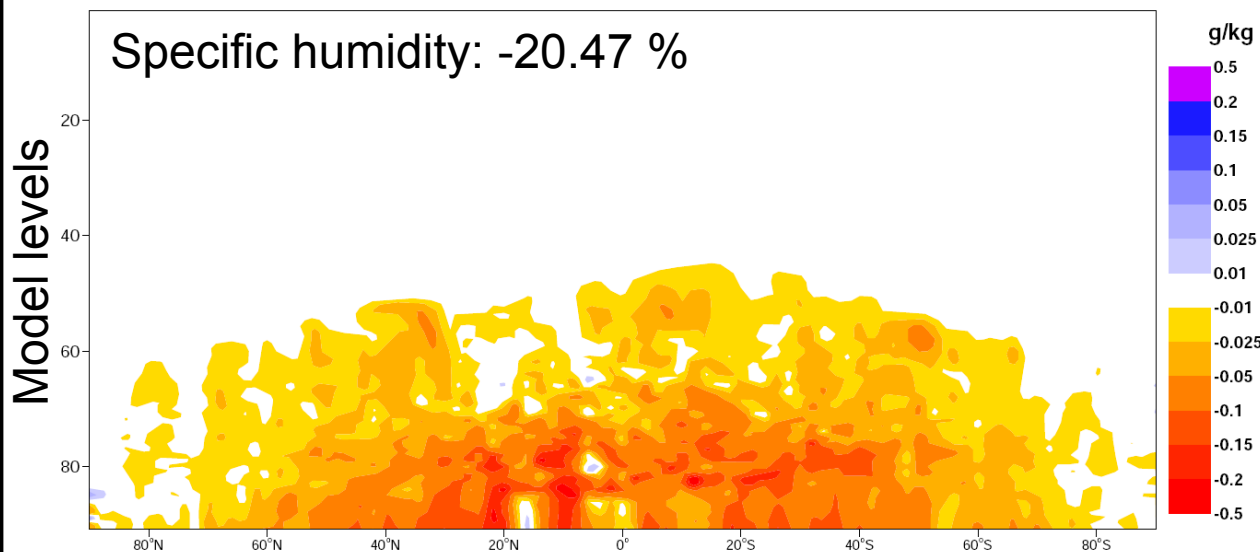
Zonal mean change in TL approx. error brought by the use of full linearized physics w.r.t. adiabatic TL

red → good

blue → bad

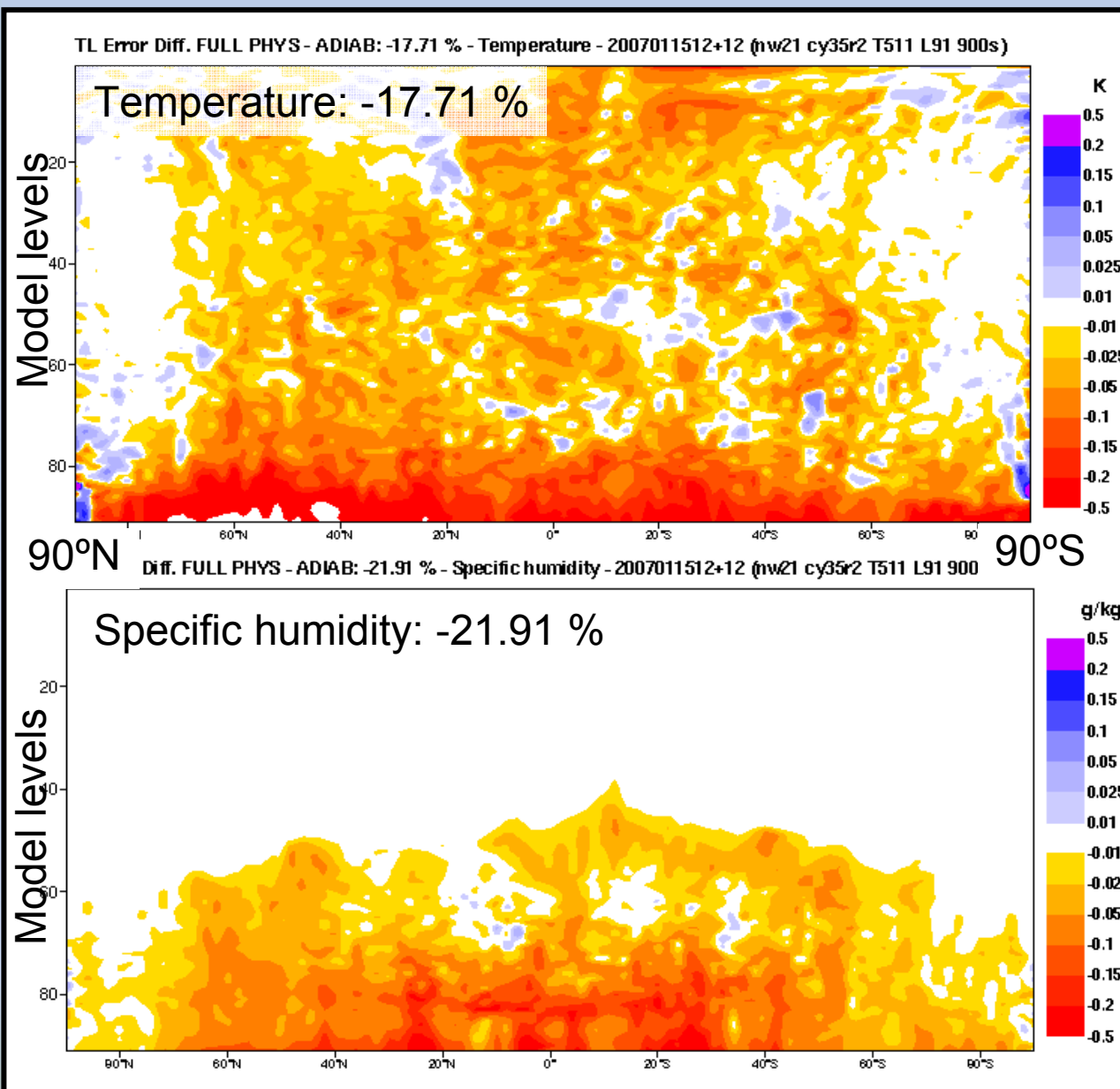
**T159 (~130 km) resol.**  
**91 vertical levels**  
**12-hour integration**

from 1 April 2009 00Z  
 ECMWF model



TL error defined as  
 $|M(\mathbf{x}+\delta\mathbf{x})-M(\mathbf{x})-M'\delta\mathbf{x}|$

## TL approximation and resolution (2)



Zonal mean change in  
TL approx. error  
brought by the use of  
full linearized physics  
w.r.t. adiabatic TL

red → good  
blue → bad

**T511 (~40 km) resol.**  
**91 vertical levels**  
**12-hour integration**

from 15 Jan 2007 12Z  
ECMWF model

→ Still OK in current operations, but what happens at much higher resolution?

## Summary (1)

- LP has become a **crucial component of 4D-Var** data assimilation, even more so since the advent of cloud and precipitation observations (new observation operators and time evolution in 4D-Var minimizations).
- The amount of work needed for maintenance and adjustments of LP (to follow upgrades in the NL forecast model) and for brand new developments should not be overlooked / overestimated → **more people** would help!
- Assimilation of cloud/precipitation is currently affected by **minimization/trajectory discrepancies** (nonlinearities, simplifications in TL, resolution differences).
- **Smoothing** in time (and space) can improve the match between minimization and trajectory.  
It can be applied to:
  - observations (e.g. 6h rain accumulations),
  - NL trajectory (e.g. smoothing of moist physics tendencies)

## Summary (2)

- The assimilation of cloud or precipitation retrievals or reflectivities (but not TBs) in 4D-Var is currently hampered by the **“0-rain” issue**.
- The assimilation of all cloud/precipitation affected observations suffers from an **asymmetry in analysis increments**.
- Some indications that cloud/precipitation assimilation might be more efficient if some **hydrometeor variable(s)** are included in the DA control vector.



## Recommendations

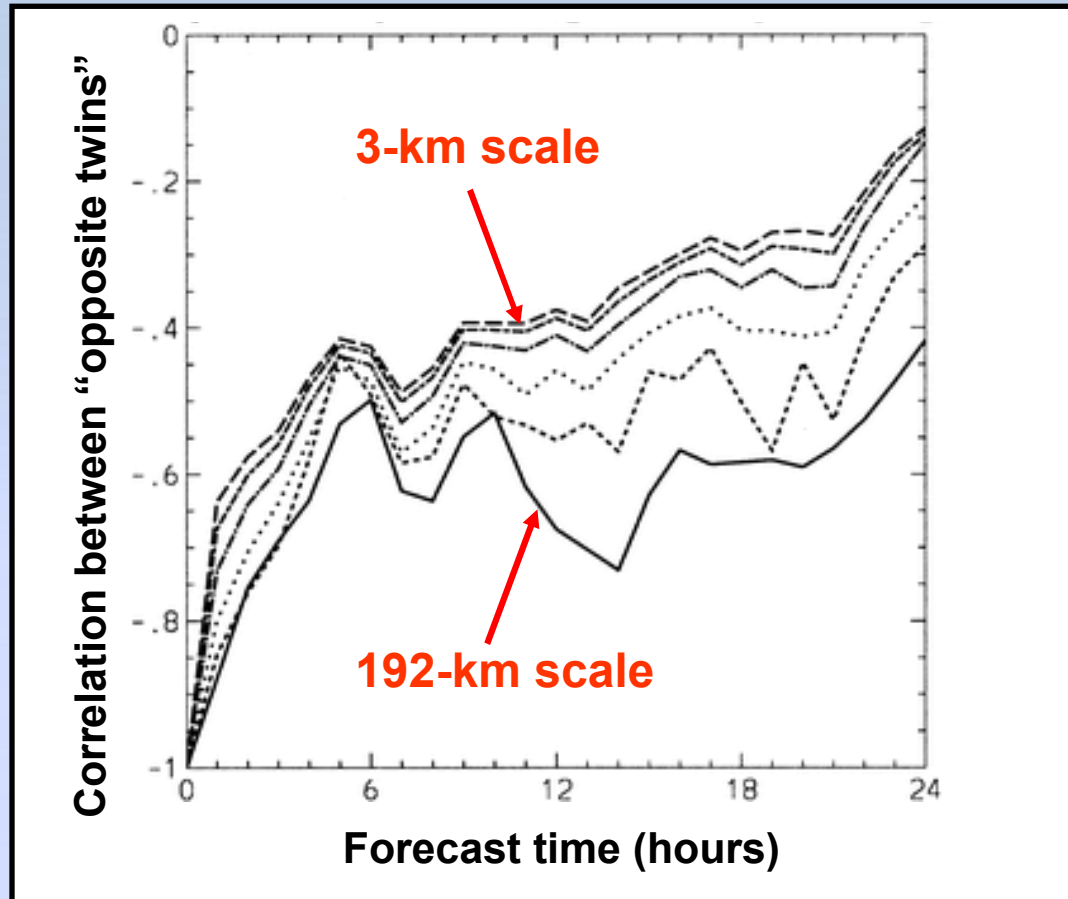
- **Better collaboration** needed between **NL modellers** and **LP people** (too late?)
- Investigate further potential benefits of **smoothing** moist processes in NL trajectory and/or cloud/precipitation observations to be assimilated.
- Study **applicability of LP at high horizontal & vertical resolutions** ( $\Delta x < 5$  km).
- Improve representation of microphysical processes in LP, but making sure that TL approximation is not degraded.
- Continue work towards inclusion of **cloud** (precipitation?) **variables in DA control vector and LP**.
- Study the possibility of using first-guess  $\neq$  background in 4D-Var to allow better assimilation of **“0-rain”** situations.
- Try to reduce the **asymmetry of analysis increments** (new control variables might help).

Thank you!



## Linearity assumption for various time and horizontal scales

Results from ensemble runs (opposite twins) with the MC2 model over the Alps, from Walser et al. (2004).

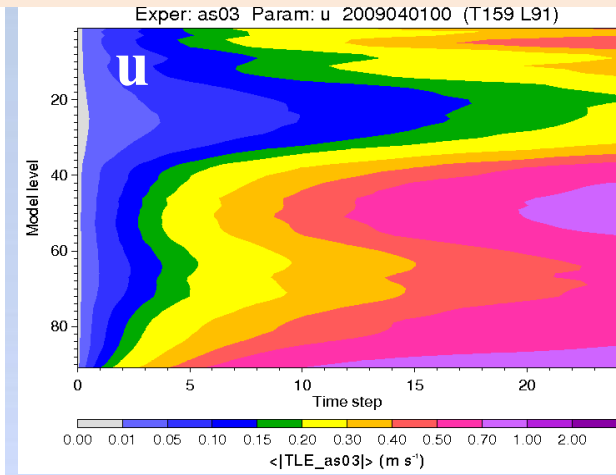
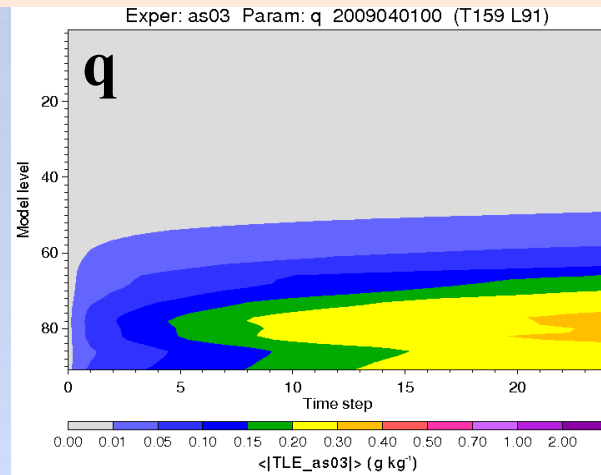
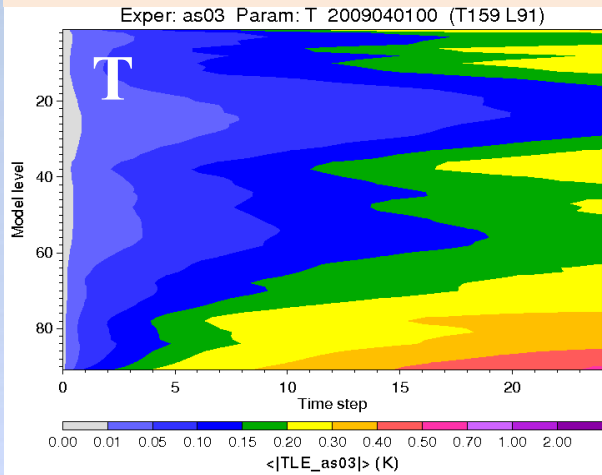


→ Validity of the linear assumption for precipitation quickly drops during first hours of forecast, especially for smaller scales.

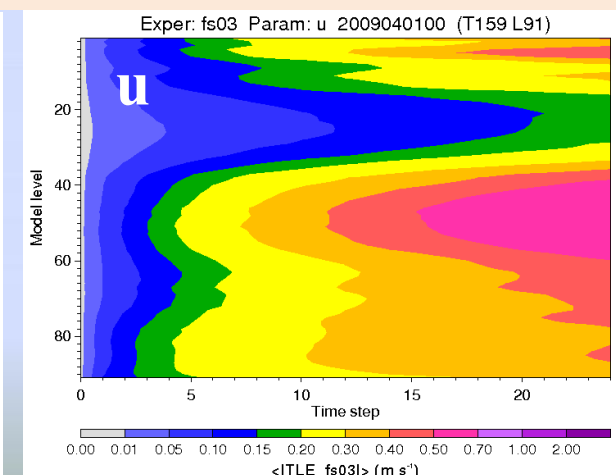
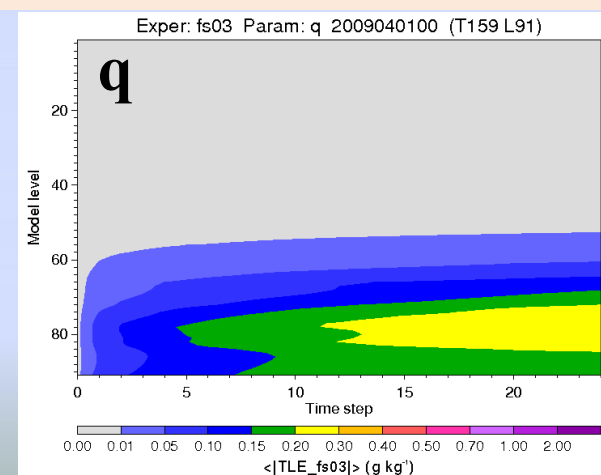
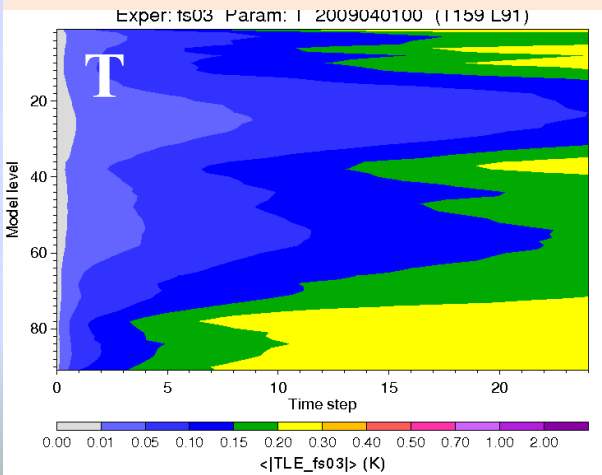
# Time evolution of TL approximation error

- Time evolution of TL approximation error (T159 L91 experiment with ECMWF LP).

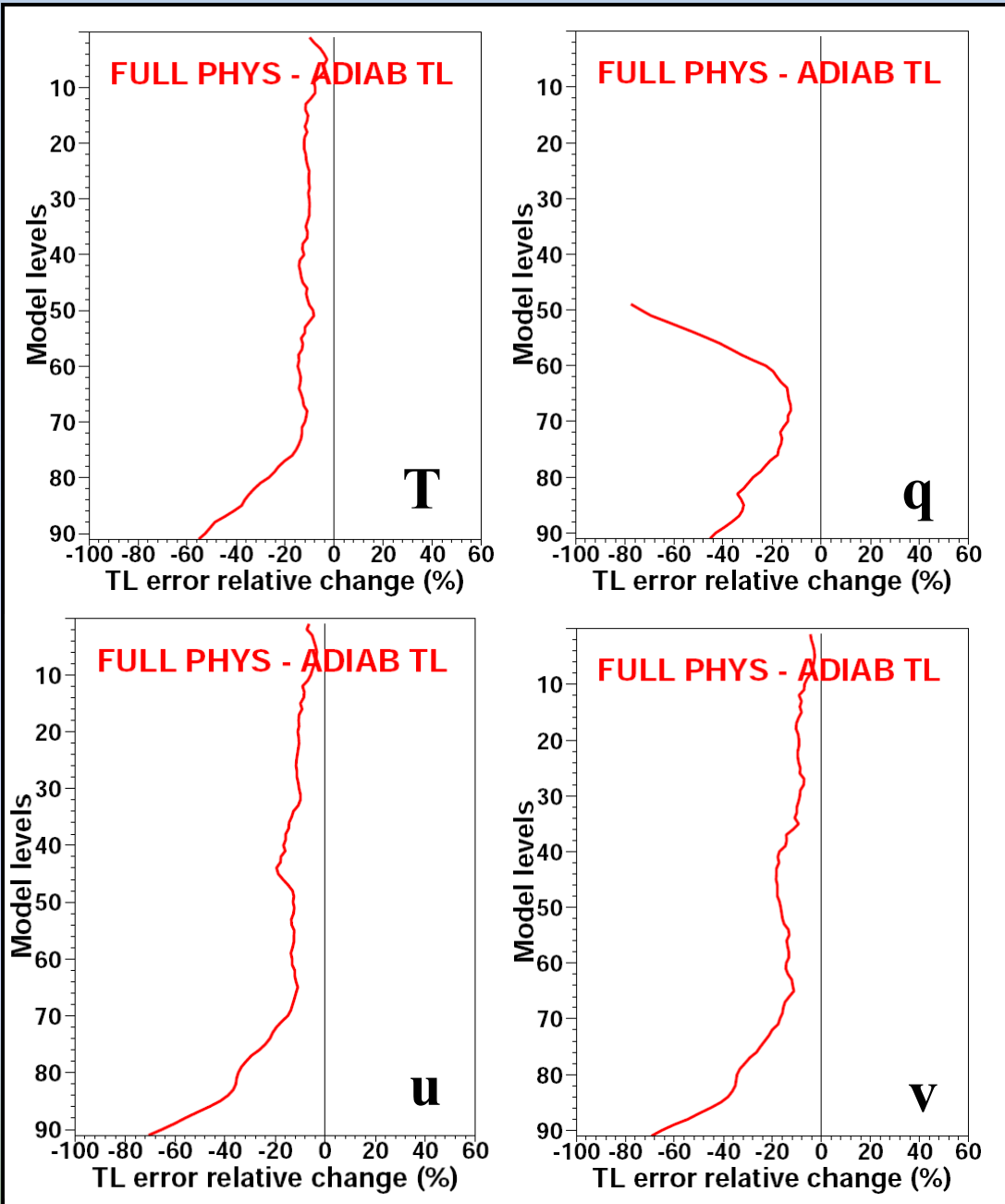
## Adiabatic TL



## Full physics TL



# Reduction of TL approximation error brought by linearized physics



Profiles of relative change in TL approx. error when full simplified physics is used in TL (w.r.t. adiabatic TL run)

12h integration

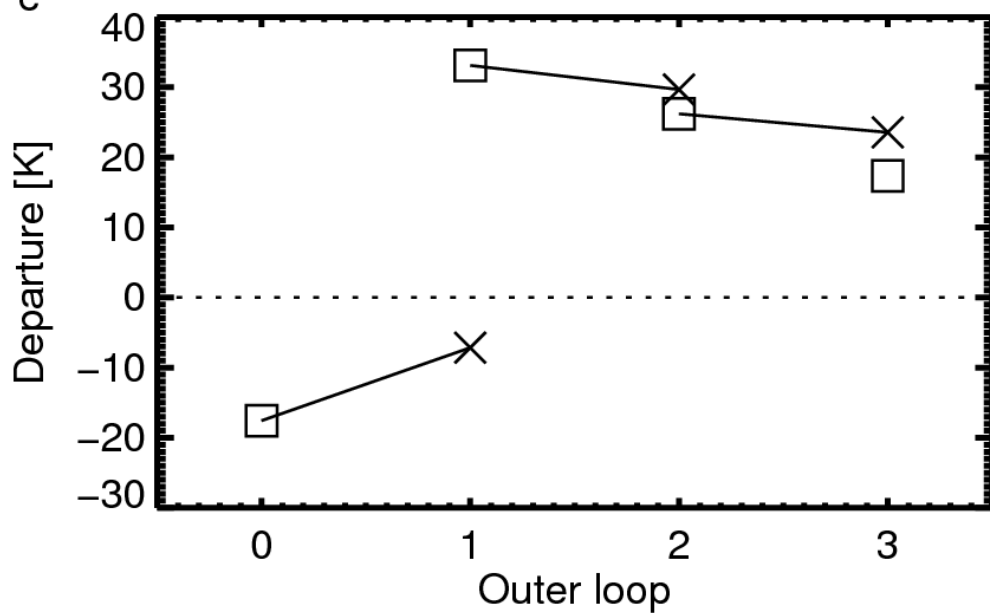
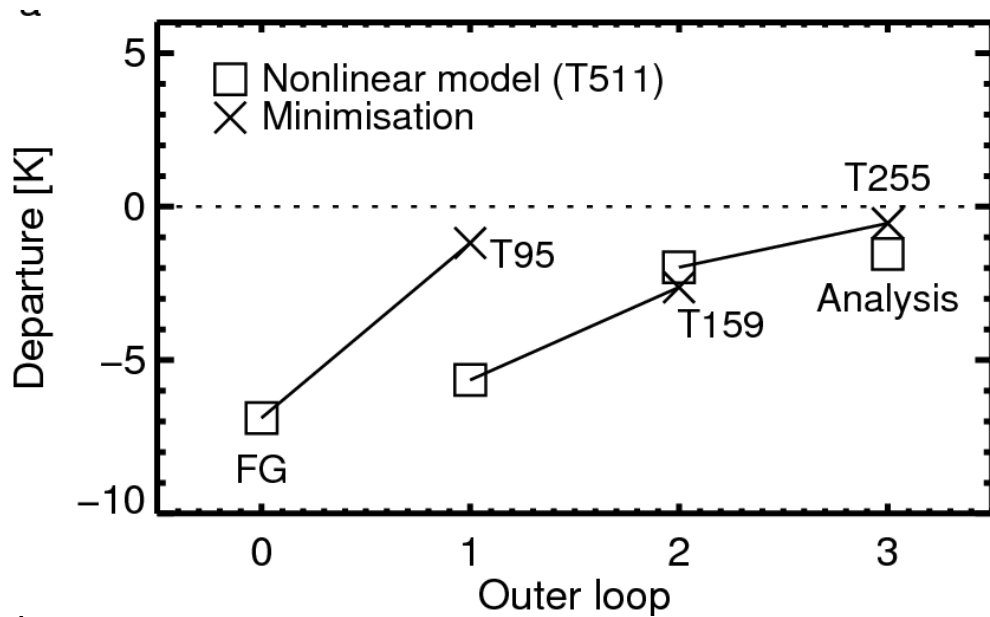
T511 L91

ECMWF model (cy35r2)

TL error ( $TLE$ ) defined as  $|M(\mathbf{x}+\delta\mathbf{x})-M(\mathbf{x})-\mathbf{M}'\delta\mathbf{x}|$

$$100 \times \left( \frac{|TLE_{full\_phys}|}{|TLE_{adiab}|} - 1 \right)$$

# Trajectory / Minimization (mis)match in 4D-Var (single obs exper.)



Single SSM/I observation used in single 4D-Var cycle (T95/T159/T255 minimizations, T511 trajectories)

## Good case:

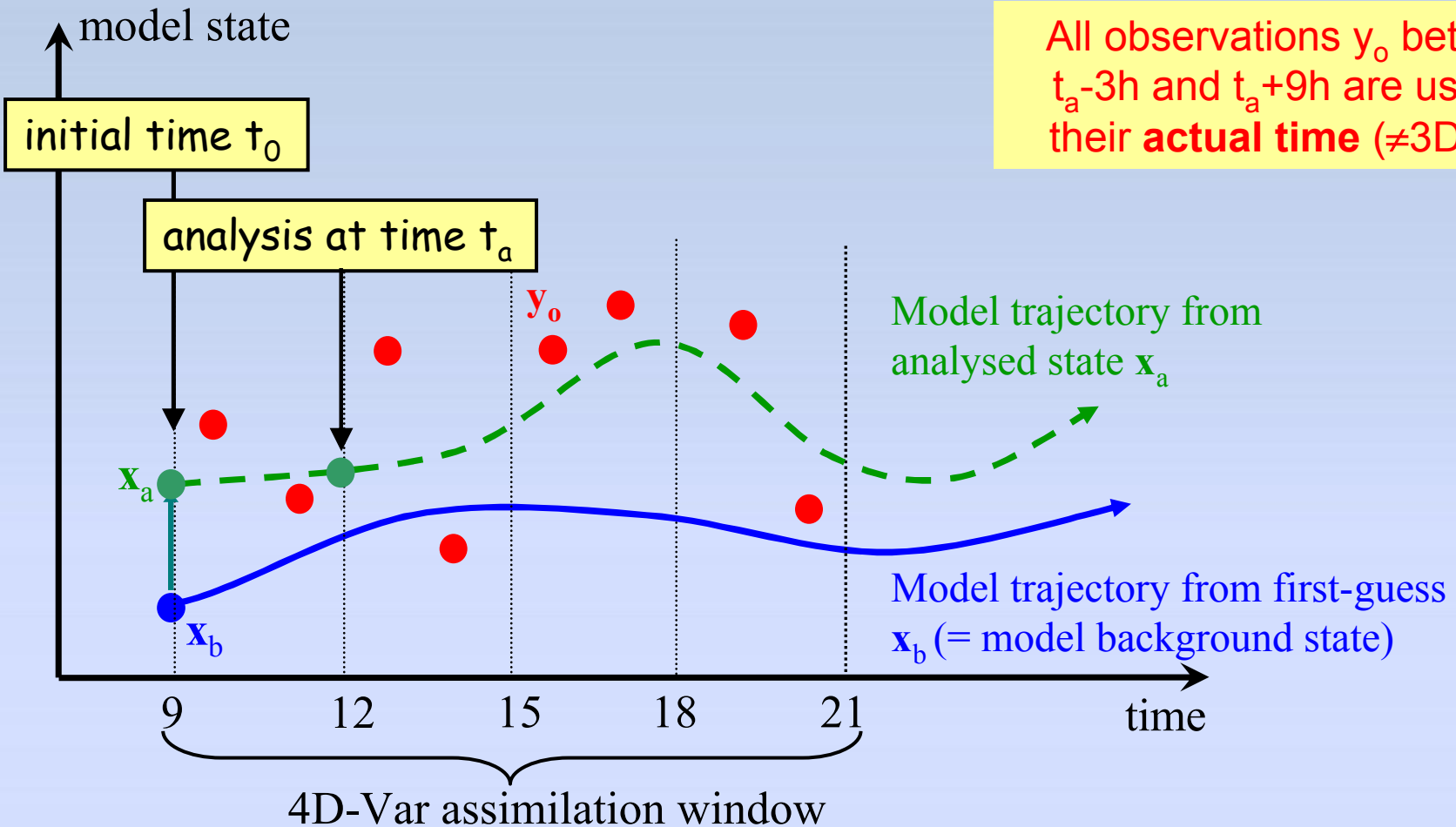
Successful reduction of model-obs departure after 3<sup>rd</sup> minimization

## Bad case:

No reduction of model-obs departure after 3<sup>rd</sup> minimization

*from Alan Geer et al. (2010)*

# 4D-Var (ECMWF's example)



## Smoothing is good...

It is better to assimilate precipitation observations accumulated over several hours than instantaneous ones, even though this implies some loss of information.

This is consistent with e.g. Mahfouf and Bilodeau (2007), Errico et al. (2003) and Errico and Raeder (1999).

### Questions:

Should some similar smoothing be applied to other measurement types (Z, TBs)?

Should moist processes in non-linear trajectory be smoothed (e.g. Stiller 2009)?



## The non-linear model issue (2)

This excessive non-linear amplification:

- occurs when activating either vertical diffusion or moist physics.
- does not occur with dynamics, radiation and gravity wave drag on.
- does not seem to affect 4D-Var (yet) because increments are still “large”.
- is bound to become worse as non-linear model becomes more complex and as 4D-Var increments get smaller.

### Solution?

Some smoothing (in time or space or through an ensemble approach) ought to be applied to the non-linear model (in 4D-Var trajectory & singular vector computations).

Consistently with these findings, the Met Office implemented a beneficial smoothing of convective tendencies over several time steps in the trajectory (Stiller 2009).

## Cloud/precipitation prognostic variables in DA control vector? (1)

In operational global models: only temperature, moisture, wind and surface pressure.

### Without cloud/precipitation variables:

- The link between cloud/precipitation observations and the model is rather indirect.
- Spin-up of cloud fields could mean that cloud observations cannot be efficiently assimilated during the first hours of the 4D-Var assimilation window.
- Temperature and moisture increments might be too large in cloudy/rainy regions.
- Moisture increments are limited by saturation threshold.

### Which new variables?

**Total water (vapour + cloud):** (in ops at Met Office)

- Distributions remain more or less Gaussian.
- Requires a diagnostic splitting between vapour and cloud in physics.

**Cloud condensate:** (e.g. under development at ECMWF and Météo-France)

- Completely separate from water vapour.
- Background error statistics need to be computed, distributions not necessarily Gaussian.

## Cloud/precipitation prognostic variables in DA control vector? (2)

**Precipitation:** (only in mesoscale assimilation, so far)

- Closer link to observed precipitation retrievals (e.g. radar).
- Same as for cloud condensate, short-lived increments (fall speed).

**Cloud fraction:**

- Frequent source of noise in LP,  
Not well observed.

In mesoscale models, few attempts were made to include microphysical variables in the DA control vector, e.g.:

Vukicevic et al. (2004): Cloud mixing ratio in RAMDAS (3h-window 4D-Var) using GOES VIS and IR obs.

Sun and Zhang (2008): Rain water mixing ratio in 12mn-window 4D-Var using radar Doppler wind and reflectivity observations.

Zupanski et al. (2009): Positive impact in Ensemble DA experiments using synthetic GOES-R IR radiances.

## The frustrating world of LP...

- Work on LP is:
  - **time consuming:** TL regularization, adjoint debugging.
  - **tedious:** constant maintenance and multiple verification for each release of a new model version (TL approx, adjoint test, 4D-Var, singular vectors, forecast sensitivity, making sure that LP does not depart too much from NL physics).
  - **frustrating:** lack of appreciation, LP=easy culprit, not a fashionable topic for publication, limited readership/audience.
- **The LP community is shrinking**, while more people would be needed to address future challenges posed by new observation types, new control variables, higher resolutions, extended 4D-Var window,
- Developments in NL parameterizations are usually conducted with no consideration for their impact on the validity of the TL and AD codes.

However, the extent to which a linearized physical package can actually match a full NL physics package is limited (if TL and NL diverge too much, 4D-Var scores degrade).