Stochastic tendency perturbations for NWP ensembles

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European Centre for Medium-Range Weather Forecasts

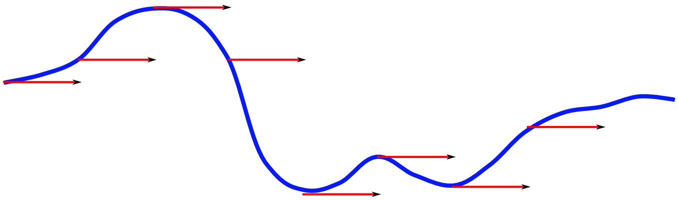
Workshop on Representing Model Uncertainty in numerical weather prediction (NWP) models and in climate models

Acknowledgements: Glenn Shutts, Martin Steinheimer & Peter Bechtold
Lars Isaksen, Massimo Bonavita & Roberto Buizza
Frederic Vitart, Tim Stockdale, Thomas Jung and Tim Palmer

Outline

1. Introduction
2. Tendency perturbations used in ECMWF ensembles
   - Stochastically Perturbed Parameterization Tendencies (SPPT)
   - Stochastic Kinetic Energy Backscatter (SKEB)
3. Impact of tendency perturbations on the EPS
4. Model uncertainty and analysis uncertainty
   - Kalman filter
   - Ensemble of 4D-Vars (EDA)
5. Summary

Estimating model error statistics



truth versus (unperturbed) model mismatches over interval Δt

- mismatches $\mathbf{x}_f - \mathbf{x}_t$ are state vectors
- spatial, multi-variate and temporal correlations matter
- error will be a function of the initial state

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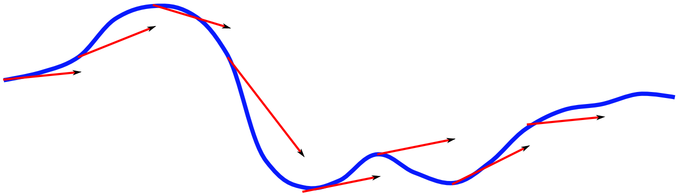

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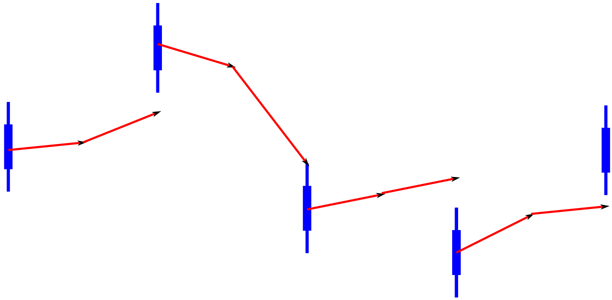

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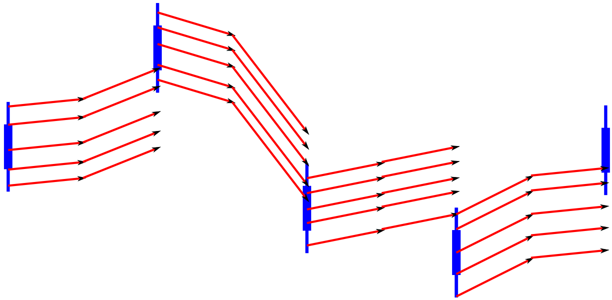

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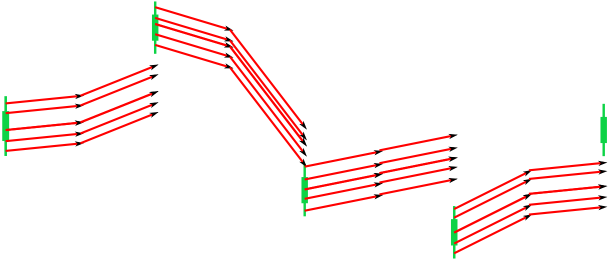

other estimate of truth versus model mismatches over interval Δt

Estimating model error covariances

observable are $\mathbf{G}_a = \langle(\mathbf{x}_f - \mathbf{x}_a)(\mathbf{x}_f - \mathbf{x}_a)^{\mathrm{T}}\rangle$ and
$\mathbf{G}_o = \langle(\mathbf{H}\mathbf{x}_f - \mathbf{y})(\mathbf{H}\mathbf{x}_f - \mathbf{y})^{\mathrm{T}}\rangle$
under some simplifying assumptions (linearity, temporally uncorrelated errors) we expect

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IF, initial uncertainty **A** (and **R**) precisely known, then

$$\mathbf{Q} = \mathbf{G}_a - \mathbf{MAM}^{\mathrm{T}} - \mathbf{A} \quad \text{and} \quad \mathbf{HQH}^{\mathrm{T}} = \ldots$$

yields the model error covariance **Q**. Vice versa, errors in **A** (and **R**) will alias into errors of our estimate of **Q** ($\mathbf{HQH}^{\mathrm{T}}$).

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yields the model error covariance $\mathbf{Q}$. Vice versa, errors in $\mathbf{A}$ (and $\mathbf{R}$) will alias into errors of our estimate of $\mathbf{Q}$ ($\mathbf{HQH}^{\mathrm{T}}$).
The analyis error covariance $\mathbf{A}$ depends on the assimilation technique α, $\mathbf{H}$, $\mathbf{R}$ *and* $\mathbf{Q}$. Thus, we have

$$\mathbf{G}_a = \mathbf{M}\,\mathbf{A}(\alpha, \mathbf{H}, \mathbf{R}, \mathbf{Q})\,\mathbf{M}^{\mathrm{T}} + \mathbf{Q} + \mathbf{A}(\alpha, \mathbf{H}, \mathbf{R}, \mathbf{Q}) \tag{1}$$

$\longrightarrow$ a nontrivial inverse problem!

Ambiguity between initial uncertainty and model uncertainty

- Without constraining the estimate of **A** (completely) by data assimilation, both the representation of initial uncertainties (**A**) and tendency perturbations (**Q**) need to be set for an ensemble forecasting system.

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- However, setting $\mathbf{A}$ *and* $\mathbf{Q}$ for ensemble forecasts may be an under-determined problem!
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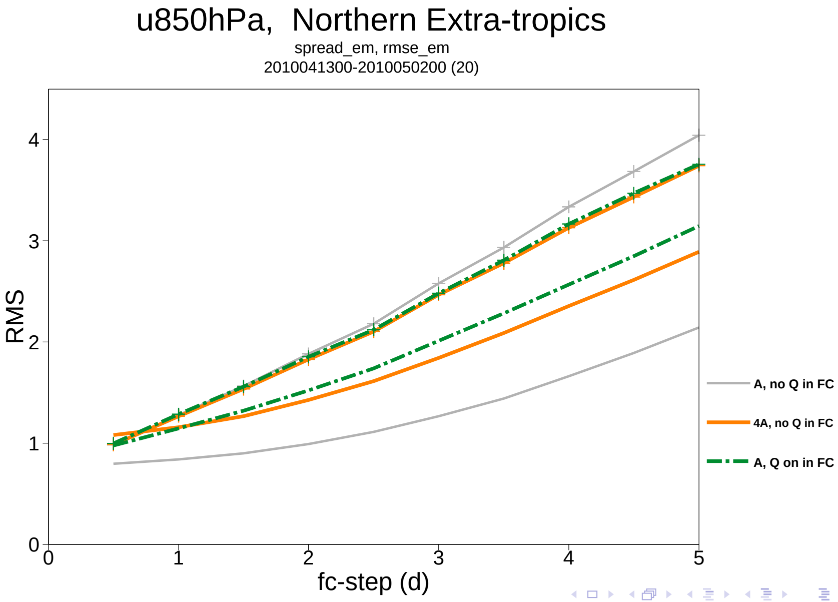
- Consider for instance $\alpha\tilde{\mathbf{A}}$ and $\beta\tilde{\mathbf{Q}}$ for two estimates of analysis error covariance and model error covariance.
  Can we determine *unambiguously* (α, β) for a NWP ensemble?

A-Q-Ambiguity (an example with the EPS)



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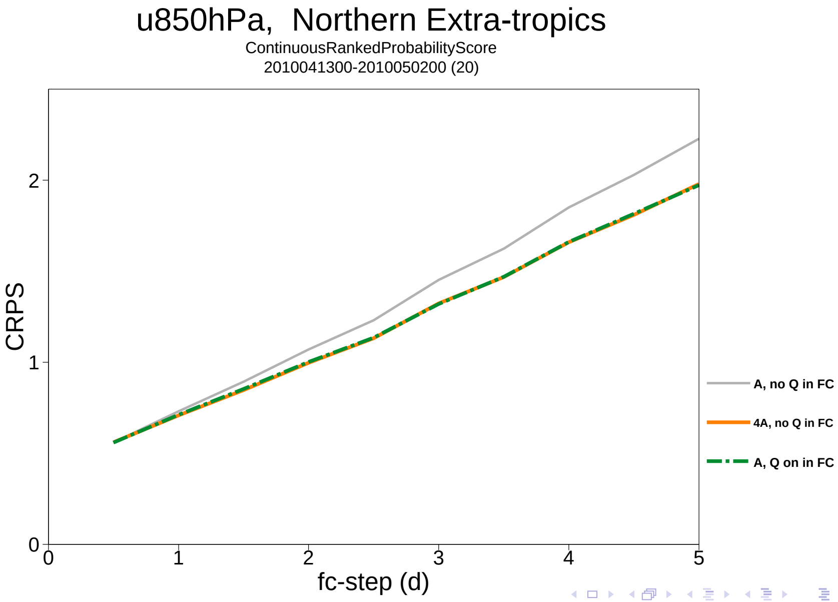

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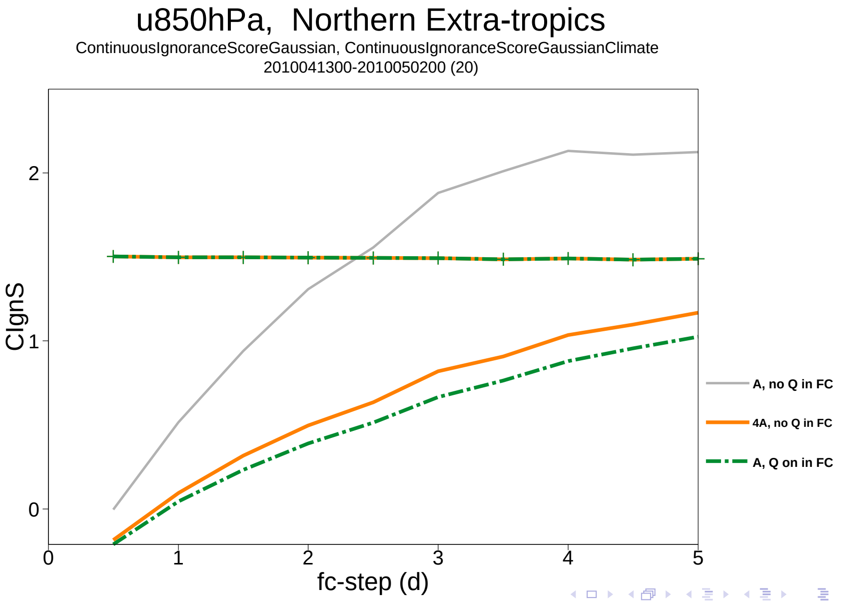

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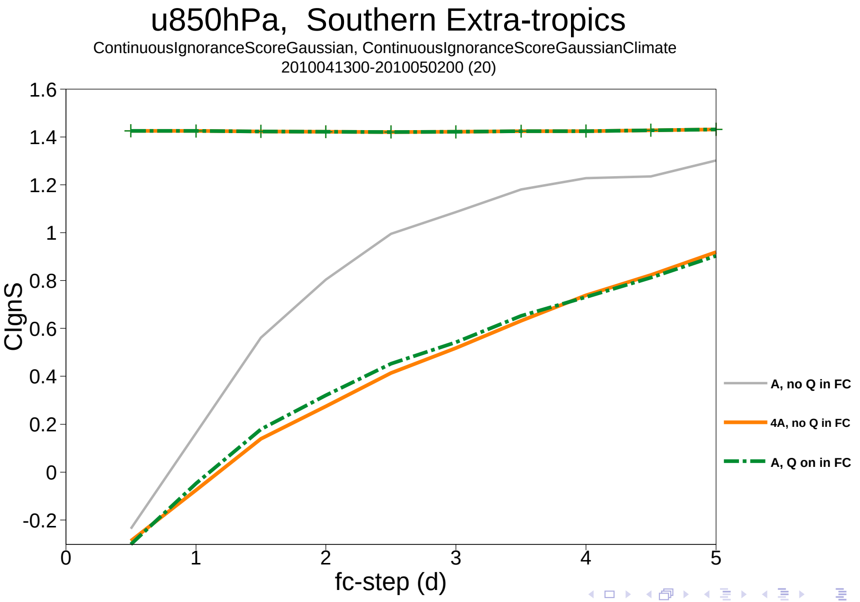

Model uncertainty representation at ECMWF

Status quo

The EPS uses

- Stochastically Perturbed Parameterization Tendencies (SPPT)
  a.k.a. stochastic physics
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The trajectory and the nonlinear forecast of the perturbed members of the EDA (Ensemble of 4D-Vars) use SPPT only.

- Work is in progress to make the representation of model uncertainties in the nonlinear forecasts in EPS and EDA consistent
- Full consistency requires more → weak-constraint 4D-Var

Stochastically Perturbed Parameterization Tendencies
SPPT

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- 2D Random pattern r uses AR-1 processes in spectral space and is smooth in space and time (instead of $10^\circ \times 10^\circ$ tiles changing every 6 time steps)
- Three components with different correlation scales:
  6 h, 3 d, 30 d and 500 km, 1000 km, 2000 km with standard deviations of 0.52, 0.18, 0.06, respectively

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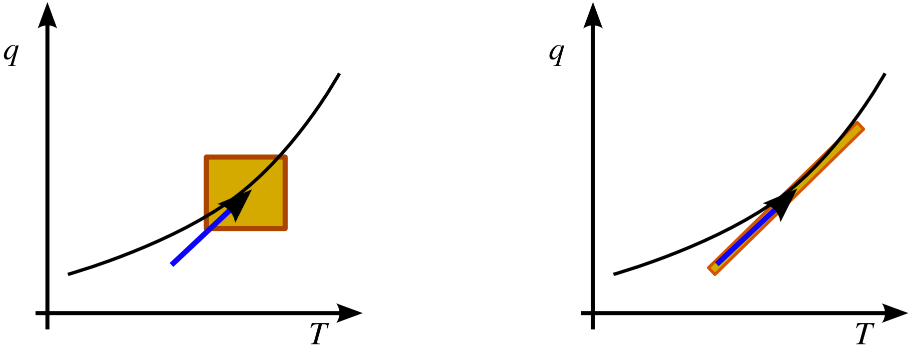
- Three components with different correlation scales:
  6 h, 3 d, 30 d and 500 km, 1000 km, 2000 km with standard deviations of 0.52, 0.18, 0.06, respectively
- Gaussian distribution, truncated at $\pm 2\sigma$ (instead of uniform distr.)
- Same pattern r for T, q, u, v (instead of an independent patterns for each variable)

see Tech Memo 598, Palmer et al. (2009) for more details

Multi-variate uniform versus univariate Gaussian



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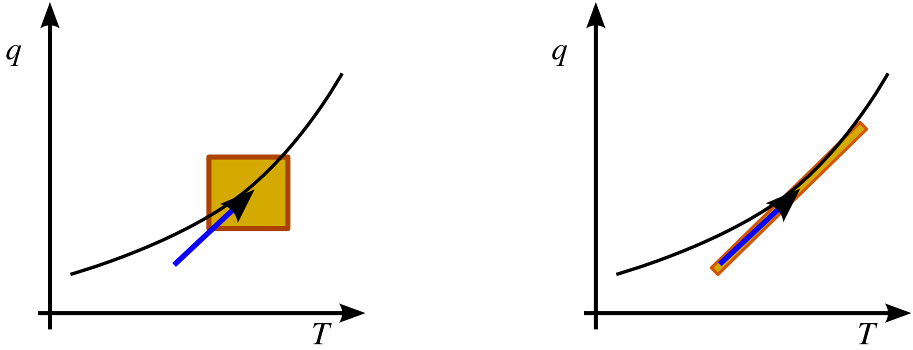
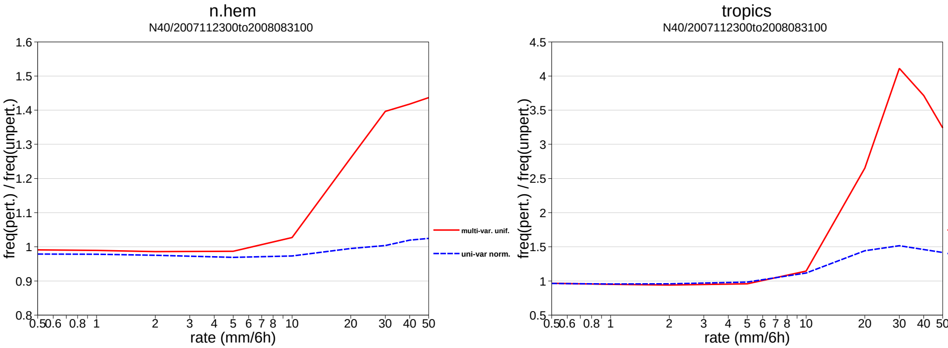


multi-variate uniform in 4 dimensions:

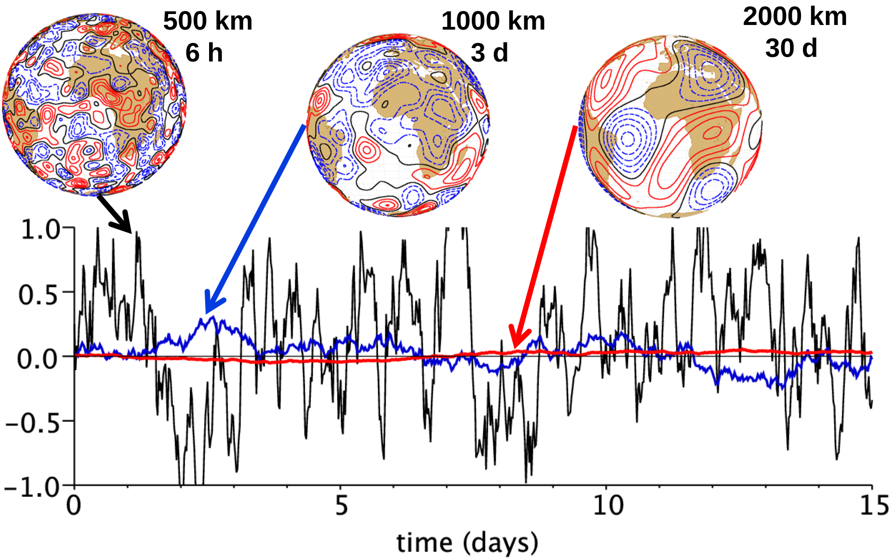
- probability to be within interquartile range for all four variables is $1/16$
- probability to perturb at least one of the four variables in excess of 0.92 of the maximum perturbation amplitude is $0.5 = (1 - 2 \times 0.08)^4$.

Tendency pert^{ns} and the frequency of heavy precipitation

multi-variate uniform distribution of (u, v, T, q) ten. perturbations
uni-variate Gaussian tendency perturbations



SPPT pattern



Stochastic Kinetic Energy Backscatter
SKEB

- Rationale: A fraction of the dissipated energy is backscattered upscale and acts as streamfunction forcing for the resolved-scale flow (Shutts and Palmer 2004, Shutts 2005, Berner et al. 2009)
- Streamfunction forcing $= [bD]^{1/2}\ F(\mathbf{x}, t)$,
  where b, D, F denote the backscatter ratio, the (smoothed) total dissipation rate and the 3-dim evolving pattern, respectively

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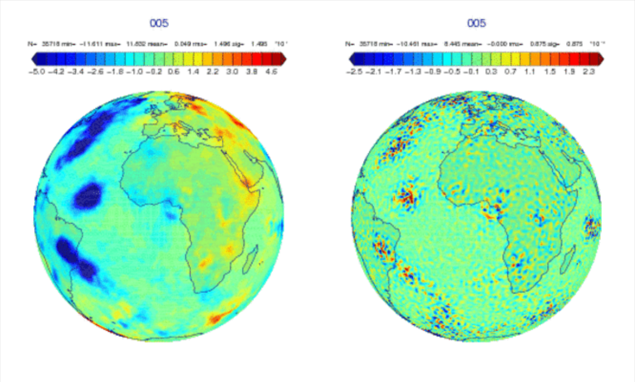
  where b, D, F denote the backscatter ratio, the (smoothed) total dissipation rate and the 3-dim evolving pattern, respectively
- Total dissipation rate: sum of
  - "numerical" KE dissipation by numerical diffusion + interpolation in semi-Lagrangian advection
  - dissipation from orographic gravity wave drag parameterization
  - an estimate of the deep convective KE production
- Boundary layer dissipation is omitted

see also Tech Memo 598, Palmer et al. (2009) for further details

SKEB forcing

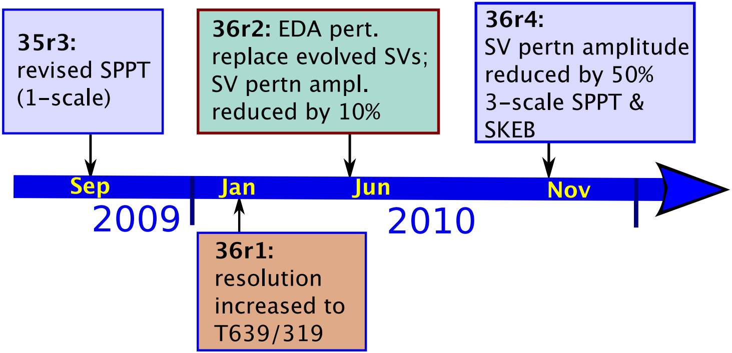
streamfunction | vorticity



- F uses AR-1 processes in spectral space with random vertical phase shifts
- decorrelation time of pattern F is set to 7 h
- structure of pattern constrained by results from coarse-graining studies with T1279 IFS and CRM

Recent operational implementations

affecting the EPS



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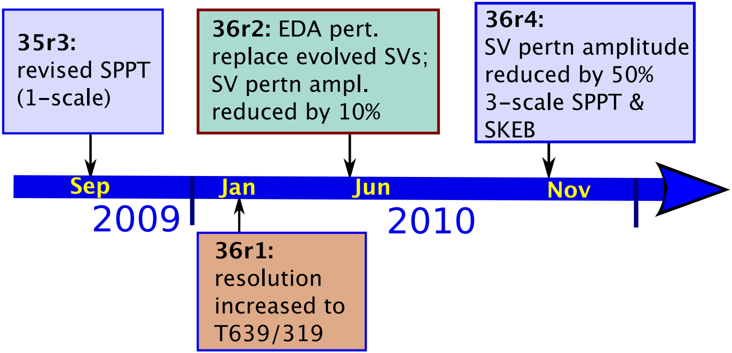


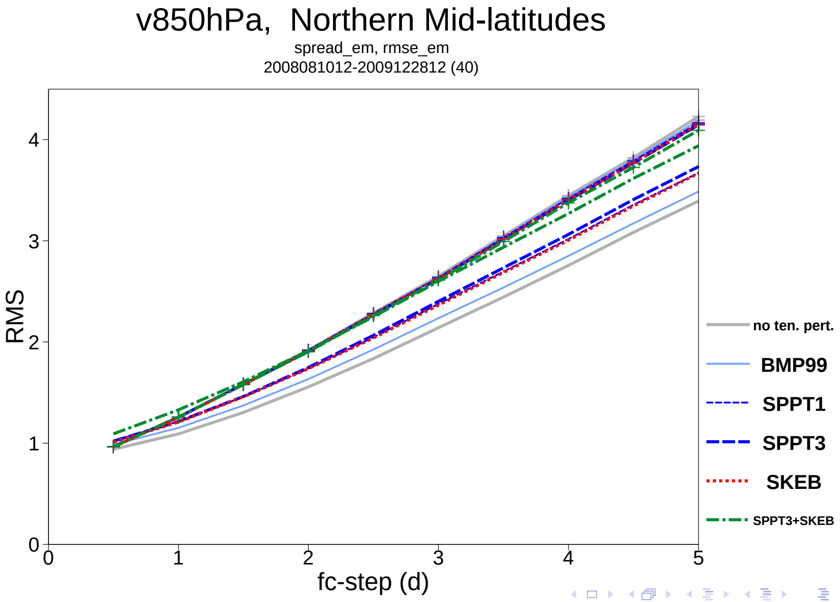
Note, EDA uses the 1-scale version of SPPT (as implemented in 35r3 in the EPS)

Impact of tendency perturbations on the EPS

for a fixed representation of initial uncertainties

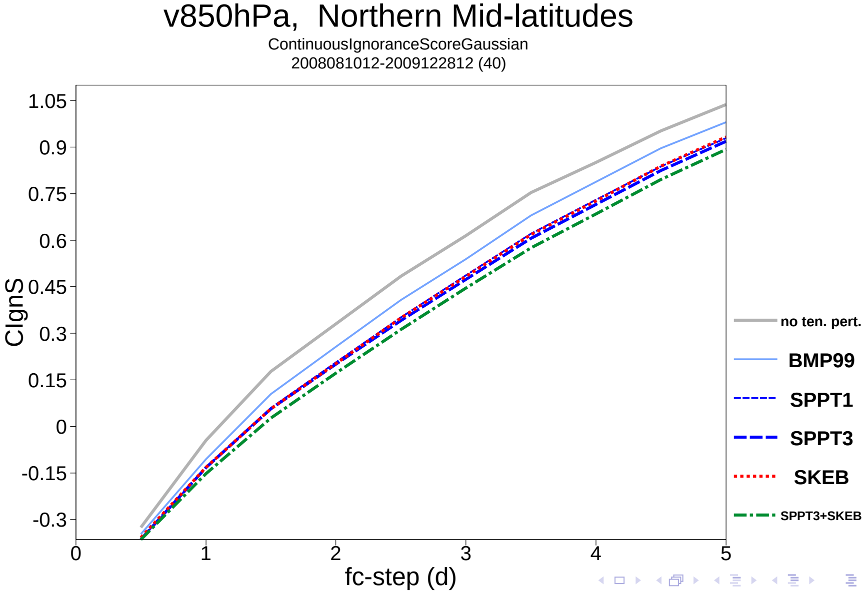
- initial perturbations as used since 36r4
  - EDA perturbations instead of evolved SV perturbations
  - 50 % reduced amplitude of initial SV perturbations
- 40 cases: Aug/Sep 2008 and Oct–Dec 2009
- T639, 50 member
- cycle 36r2
- 6 different tendency perturbations
  - no ten. perturbations
  - original SPPT, **BMP99** (Buizza, Miller & Palmer, 1999)
  - single-scale SPPT (**SPPT1** as implemented in 35r3)
  - three-scale SPPT (**SPPT3** as implemented in 36r4)
  - stochastic kinetic energy backscatter (**SKEB**)
  - **SPPT3+SKEB**

Ensemble standard deviation (no symbols), EM RMSE (+)



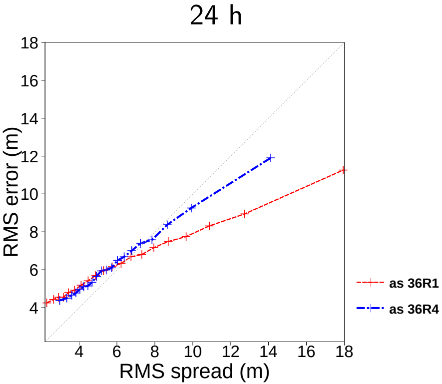
Ignorance score (=Logarithmic score)

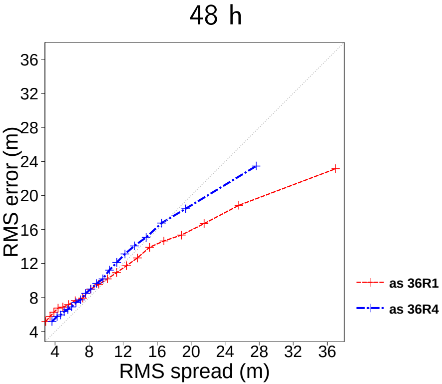
$\mathrm{CIgnS} = -\log(p_{\mathrm{fc}}(y))$; the smaller the better



Spread-reliability of 500 hPa height — 20°–90°N

Jan 2010 configuration versus Nov 2010 configuration



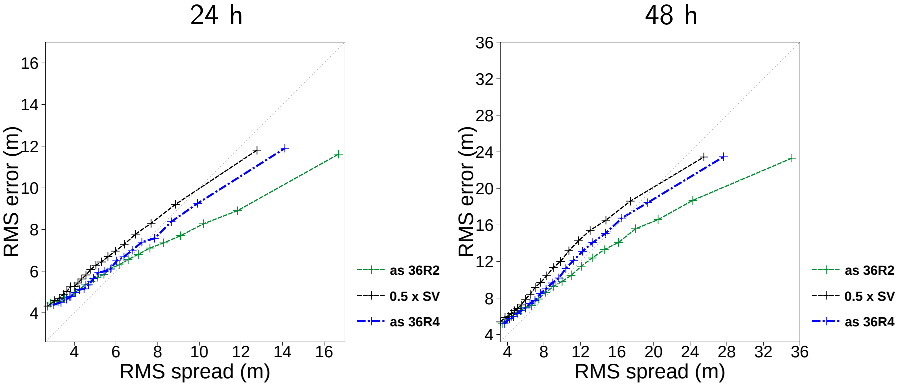


- 40 cases: Aug/Sep 2008 and Oct–Dec 2009
- T639, 50 member
- cycle 36r2

- Major improvement of a long-standing deficiency
- Which change of the EPS configuration is responsible?

Spread-reliability of 500 hPa height — 20°–90°N

Impact of halved SV perturbation amplitude



- Main improvement from reduced SV perturbation amplitude
- Probabilistic skill of 0.5 × SV is inferior to 36R2 configuration
- smaller contribution from 36R1 → 36R2
- upgraded tendency perturbations prevent underdispersion

Kalman filter and model uncertainty

see Daley & Menard (1993)

- variance evolution in the Kalman filter:

$$\text{forecast step} \qquad \mathbf{P}^f = \mathbf{M}\,\mathbf{P}^a\,\mathbf{M}^T + \mathbf{Q} \tag{2}$$

$$\text{analysis step} \qquad \mathbf{P}^a = (\mathbf{I} - \mathbf{K}\,\mathbf{H})\,\mathbf{P}^f, \qquad \text{where} \tag{3}$$

$$\text{gain matrix} \qquad \mathbf{K} = \mathbf{P}^f\,\mathbf{H}^T(\mathbf{H}\,\mathbf{P}^f\,\mathbf{H}^T + \mathbf{R})^{-1} \tag{4}$$

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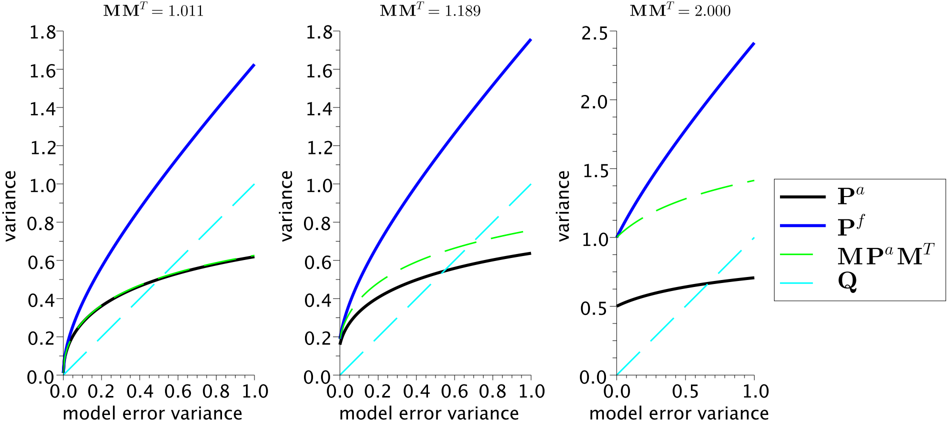
- Equivalence between 4D-Var and a Kalman smoother
- Many ensemble assimilation techniques aim at approximating the Extended Kalman filter
- What is the impact of model uncertainty in the simplest possible KF?
- DM93 studied properties of the KF with stationary **R M Q H** for the case where all matrices can be diagonalized simultaneously
  $\Rightarrow$ independent KF's, each provides the analysis for one scalar variable

Sensitivity to model error variance

Stationary Kalman filter for a scalar variable

slow pert^n growth
frequent obs^{ns}

fast pert^n growth
infrequent obs^{ns}



- all variances normalized by $\mathbf{R} = \sigma_o^2$

Impact of representing model uncertainties

in EDA and EPS on ensemble forecasts

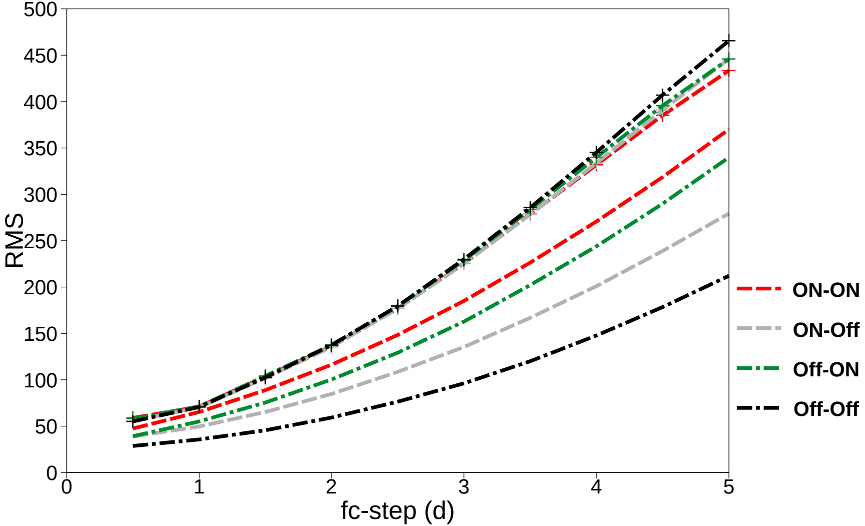
- 3 EDA experiments; (10 member, T399):
  - no tendency perturbations
  - SPPT
  - SPPT+SKEB
- 5 EPS experiments (20 member, T639):

pertn. in EDA	perturbation in EPS	
	None	SPPT+SKEB
None	Off-Off	Off-ON
SPPT+SKEB	ON-Off	ON-ON
SPPT		SPPT-ON

  - no SV perturbations
  - EDA perturbations defined with respect to EDA mean
  - analysis uncertainties accounted for in verification
- 20 cases in April/May 2010
- cycle 36r4
- see also earlier results in Sec. 3 of Tech Memo 598, Palmer et al. (2009)

Ensemble standard deviation (no symbols), EM RMSE (+)

500 hPa geopotential — 35°N–65°N



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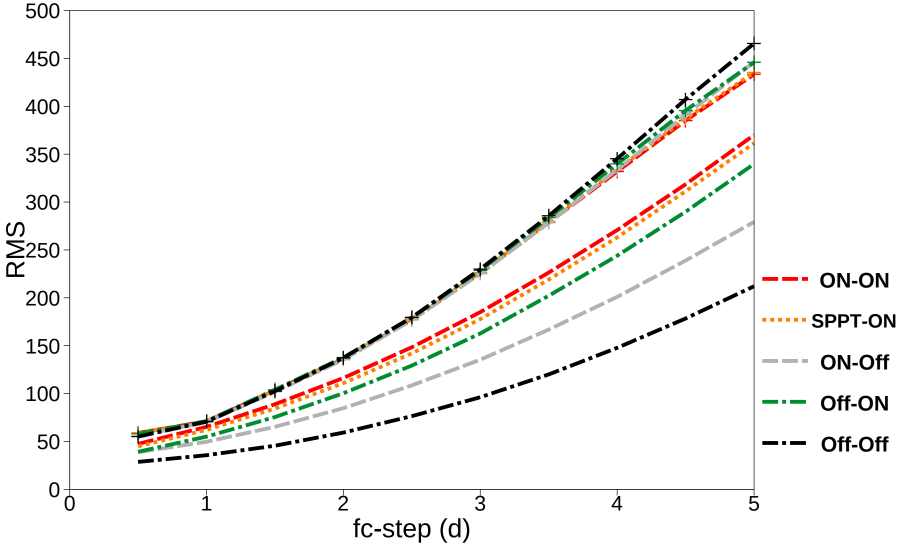

Continuous Ignorance Score

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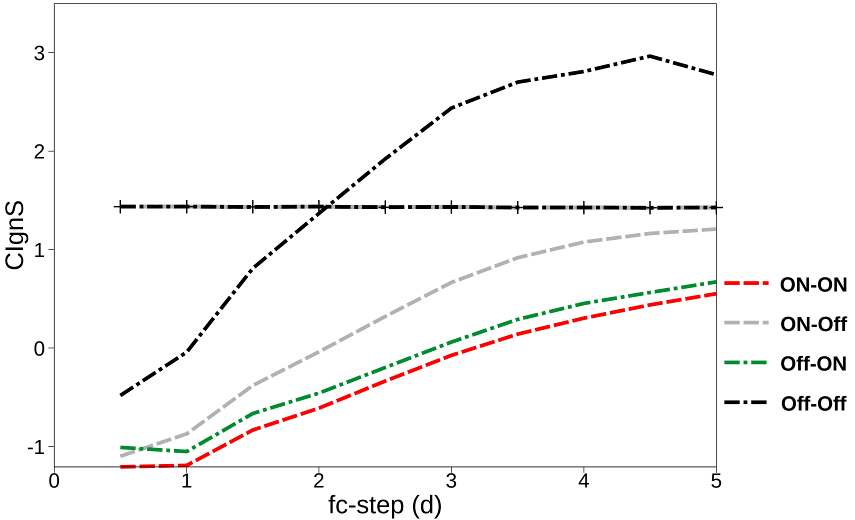

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the smaller the better

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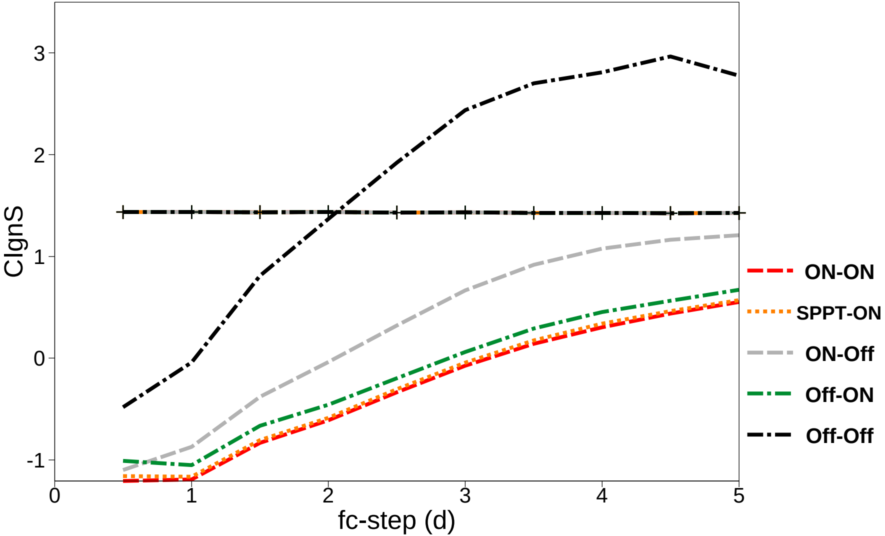

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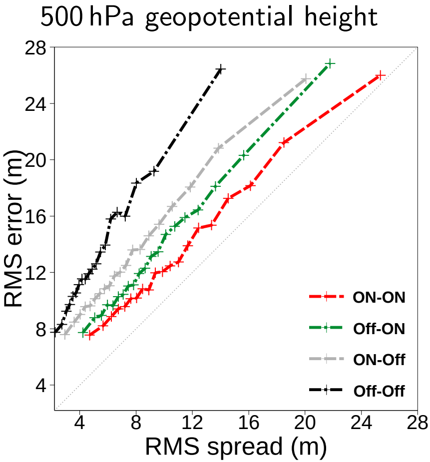
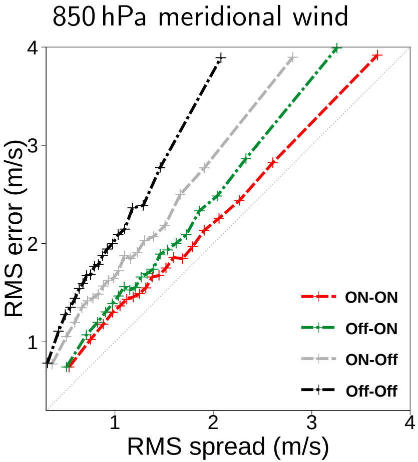
Spread-reliability: $t = 48$ h

Northern mid-latitudes 35°N–65°N





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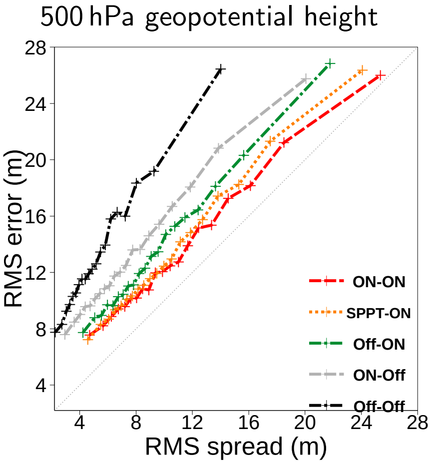
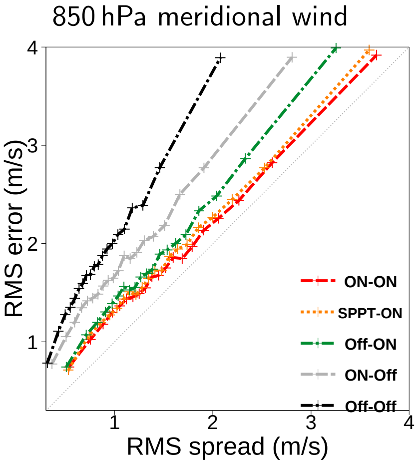
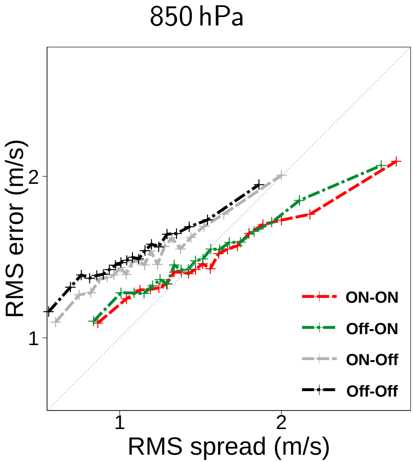
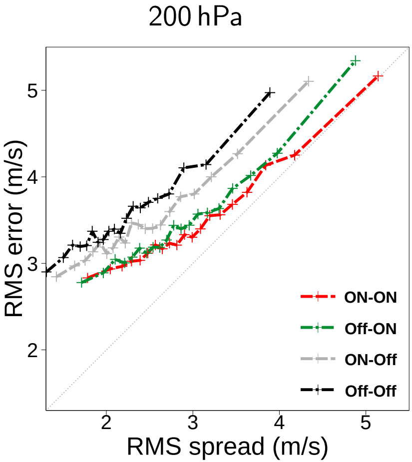




Spread-reliability: meridional wind $t = 48\,\text{h}$

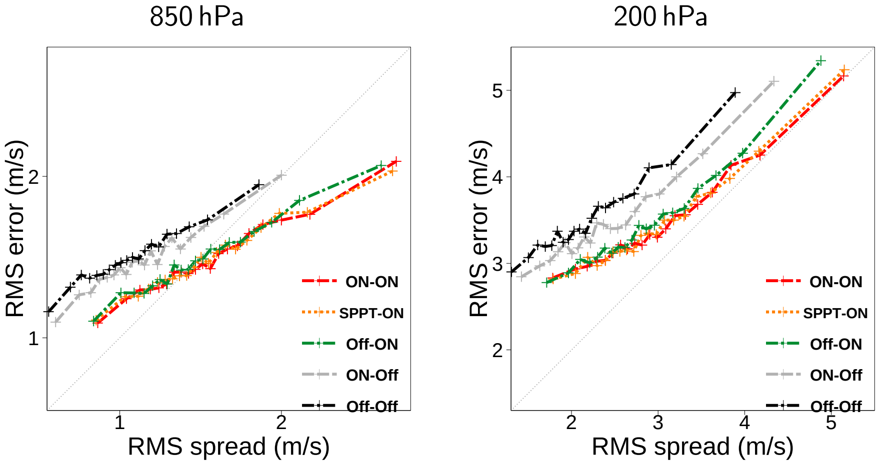
Tropics 20°S–20°N





Spread-reliability: meridional wind $t = 48$ h

Tropics 20°S–20°N



Conclusions

- Stochastic tendency perturbations used in the operational ECMWF ensembles contribute significantly to ensemble spread and improve probabilistic skill.
- Improved ensemble forecast variances and improved probabilistic skill through a combination of
  - introduction of EDA perturbations
  - reduced amplitude for SV perturbations
  - more active representation of model uncertainties

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- Stochastic tendency perturbations used in the operational ECMWF ensembles contribute significantly to ensemble spread and improve probabilistic skill.
- Improved ensemble forecast variances and improved probabilistic skill through a combination of
  - introduction of EDA perturbations
  - reduced amplitude for SV perturbations
  - more active representation of model uncertainties
- Not having precise estimates of initial error covariances hampers diagnostic of the characteristics of (random) model tendency errors
- Diagnostics may need to be improved to distinguish well different representations of model uncertainty.
- Model uncertainty contributes to initial uncertainty whereever a short-range forecast is used as prior information. A consistent representation of model uncertainty in data assimilation and forecast can help to better constrain the formulation of model uncertainty.

Plans

- Compare operational schemes with more basic tendency perturbations. For instance, additive noise, e.g. from scaled tendencies constructed from a tendency archive, (e.g. YOTC data)
- Diagnose tendency differences from different models started from the same initial conditions (resolution, different parameters, different parameterization schemes, . . .). What is the nature of the *random component* of the differences?
- Develop improved diagnostics that permit to evaluate better the realism of different tendency perturbations.