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From 1D to 4D: Towards a gravity-wave parameterization for NWP and climate models beyond the wave-dissipation paradigm

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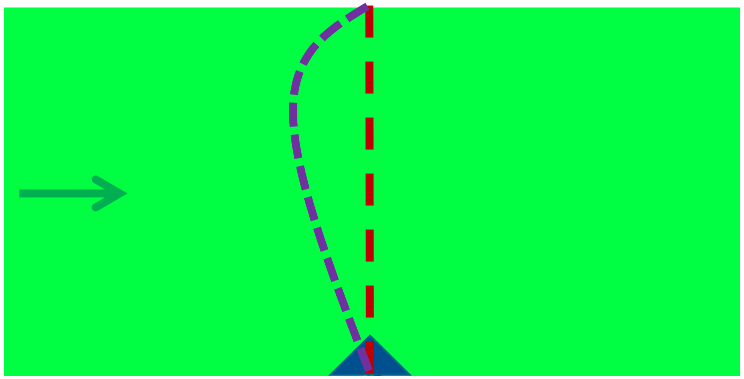
F. Senf (TROPOS Leipzig) & …

Motivation:
Gravity-wave (GW) parameterizations

GW parameterizations based on **WKB theory** (e.g. Grimshaw 1975)



- **Simplifications** for efficiency:
  - **Single column**
  - **Steady state (turbulence needed)**
- **Limitations**:
  - Without these simplifications non-interaction theorem does not need turbulence for wave/mean-flow interaction (Bühler & McIntyre 1998, 2003, 2005)
  - Transience and horizontal propagation have effects (Dunkerton1984, …, Alexander et al 2010, Kawatani et al 2010, Senf & Achatz 2011, Ribstein et al 2015)
- More **general approach** yet to be fully implemented
- Some issues:
  - **Dependence on stratification**
  - **Vortical/geostrophic mode**
  - **Numerical implementation**
  - **Direct wave-mean-flow interaction/impact by wave breaking**
  - **Mesoscale/submesoscale interaction**

Ray tracing with caustics:
Numerics for fully coupled WKB

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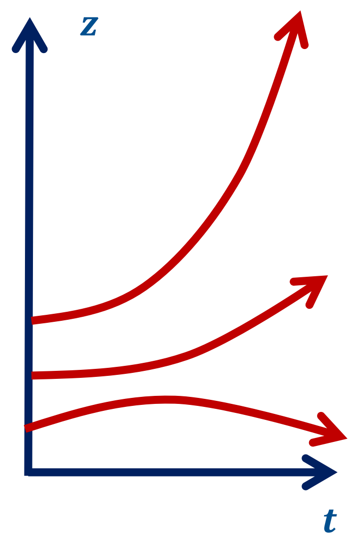
Classic WKB (Grimshaw 1975, …) for illustration 1D:

Locally monochromatic fields of the form $b'(\boldsymbol{x}, t) = \Re\, B(z,t) e^{i\phi(\boldsymbol{x},t)}$

local wavenumber and frequency: $\boldsymbol{k}(z,t) = k\boldsymbol{e_x} + m\boldsymbol{e_z} = \nabla\phi, \quad \omega(z,t) = -\partial\phi/\partial t$

wave-action density $\boldsymbol{A(z,t)}$ so that (e.g.)

$$\boldsymbol{E_{GW}(z,t) = A(z,t)\, \widehat{\omega}(m)}$$



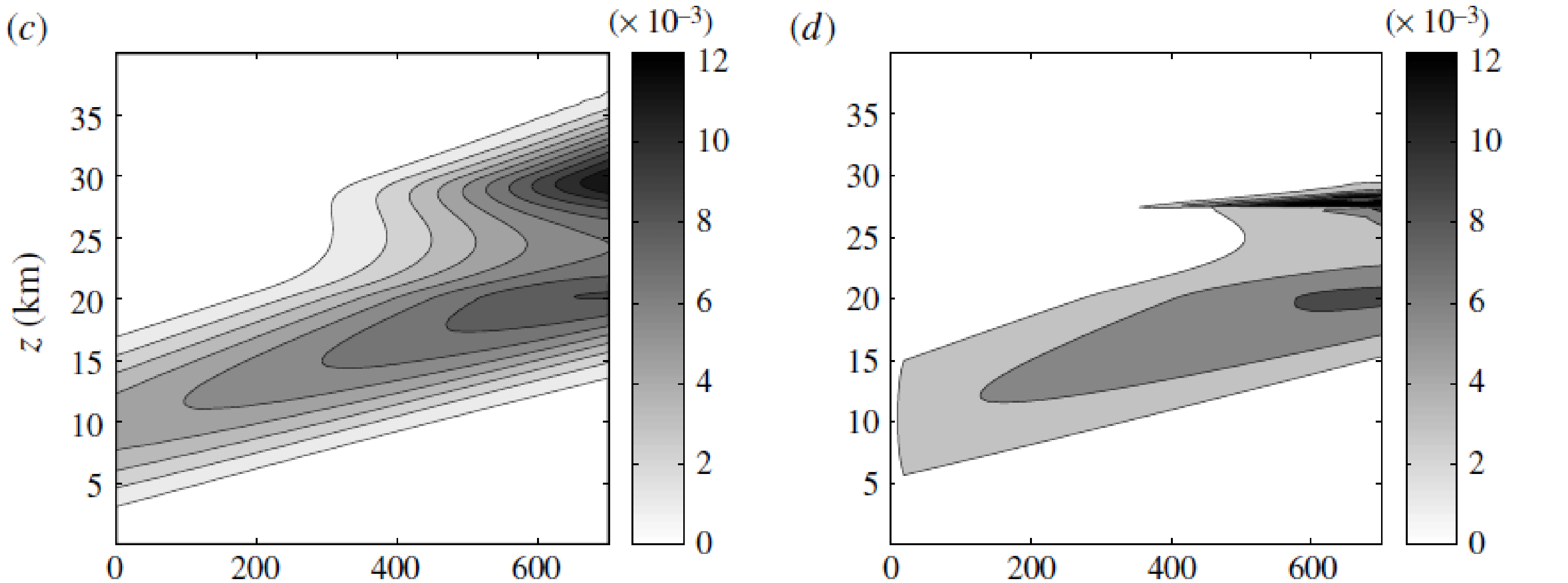
Along **rays**, defined by $\boldsymbol{dz/dt = c_g}$

$$\boldsymbol{\frac{dm}{dt} = -k\frac{\partial U}{\partial z}}, \qquad \boldsymbol{\frac{dA}{dt} = -A\frac{\partial c_g}{\partial z}}$$

Mean flow: $$\frac{\partial \boldsymbol{U}}{\partial t} = -\frac{1}{\bar\rho}\frac{\partial}{\partial z}\left(\bar\rho\, \overline{u'w'}\right) = -\frac{1}{\bar\rho}\frac{\partial}{\partial z}\left(\boldsymbol{c_g k\, A}\right)$$

Ray tracing with caustics:
Numerics for fully coupled WKB

GW packet refracted by a jet



time

LES **WKB**

Rieper et al (2013)

Ray tracing with caustics:
1D for illustration

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Locally monochromatic fields

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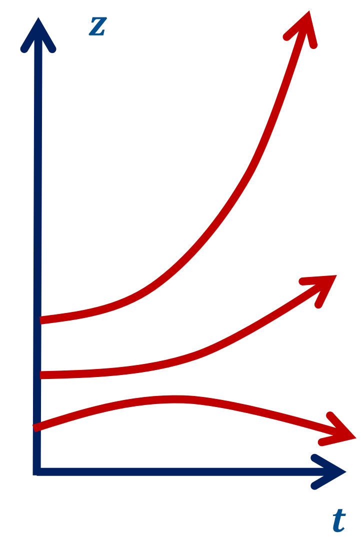

Ray tracing with caustics: 1D for illustration

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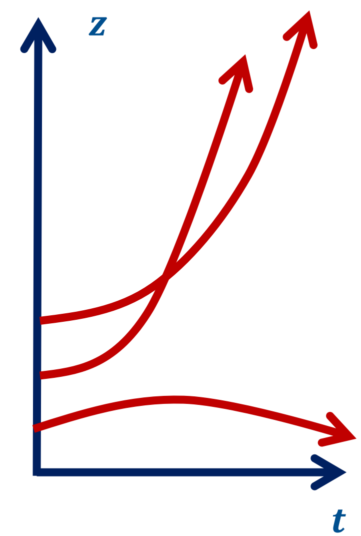

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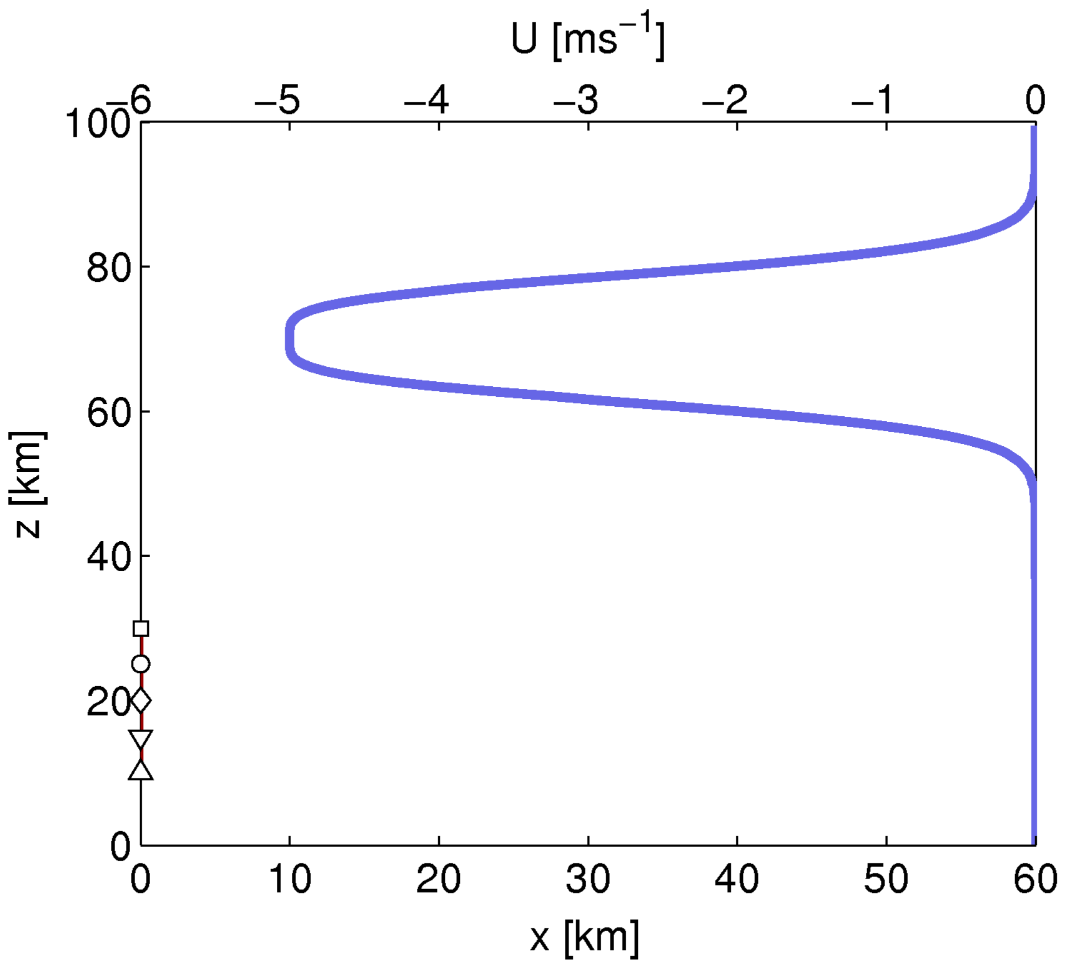
Crossing rays (caustics): uniqueness problem for A and m!

Ray tracing with caustics: examples for caustic situations

Nonuniqueness of wave number and wave-action density arises easily:

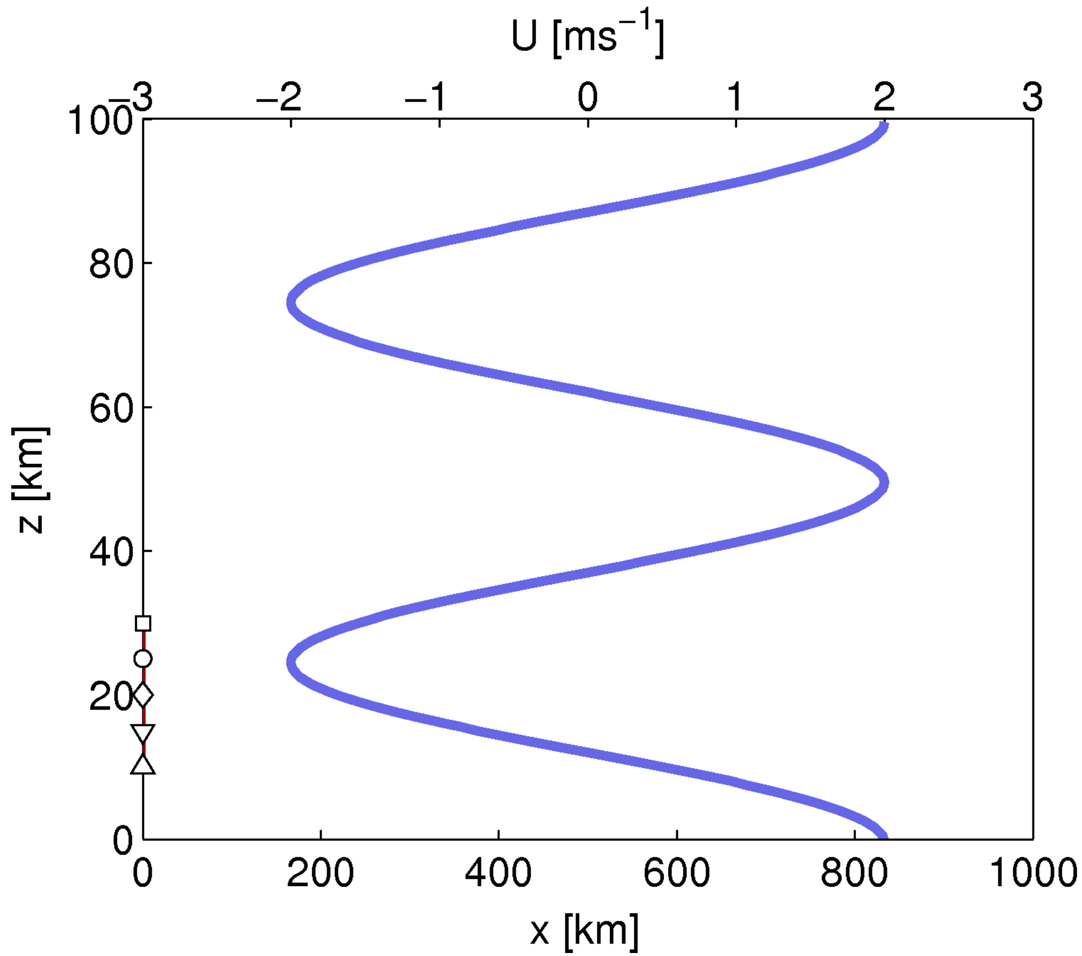
e.g. reflection at a jet



Ray tracing with caustics: examples for caustic situations

Nonuniqueness of wave number and wave-action density arises easily:

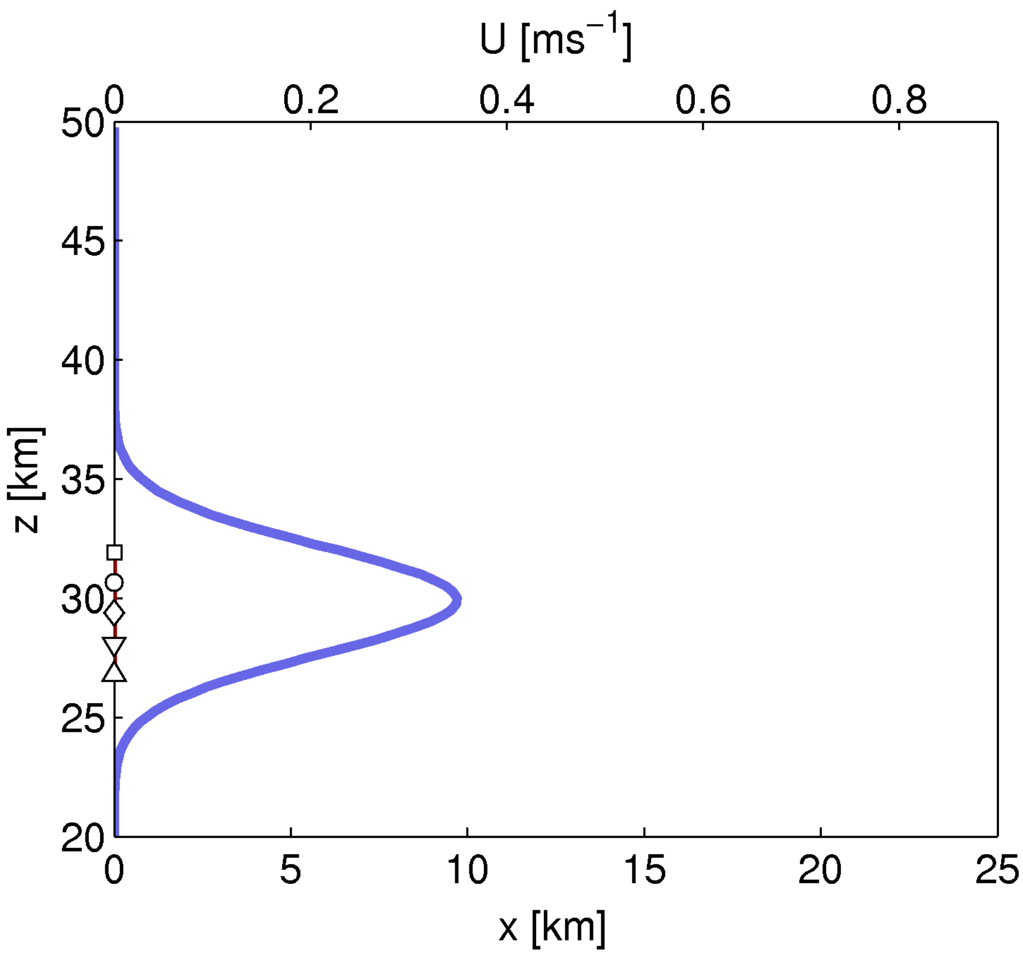
e.g. overtaking rays



Ray tracing with caustics: examples for caustic situations

Nonuniqueness of wave number and wave-action density arises easily:

e.g. by
wave-induced mean flow



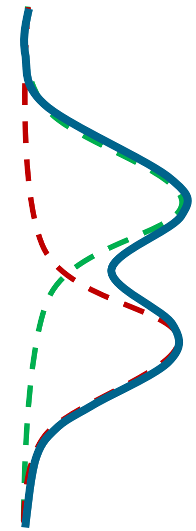
Ray tracing with caustics:
1D for illustration

linear limit: wave field can be **decomposed** into fields with singlevalued wavenumbers

$$\widehat{B} = \Re\left\{ b_1(\zeta,\tau)\, e^{i\,[kx + \phi_1(\zeta,\tau)/\epsilon]} + b_2(\zeta,\tau)\, e^{i\,[kx + \phi_2(\zeta,\tau)/\epsilon]} \right\}$$

$$\frac{\partial \phi_1}{\partial \zeta} = m_1 \qquad\qquad \frac{\partial \phi_2}{\partial \zeta} = m_2$$

$$\frac{D_{g\alpha} A_\alpha}{D\tau} = \frac{\partial A_\alpha}{\partial \tau} + c_{g\alpha} \frac{\partial A_\alpha}{\partial \zeta} = -\frac{\partial c_{g\alpha}}{\partial \zeta} A_\alpha + D_\alpha \qquad (\alpha = 1, 2)$$



case dependent surgery: **very complex**

Ray tracing with caustics: spectral approach

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linear limit: wave field can be **decomposed** into fields with singlevalued wavenumbers

spectral description in phase space (Dewar 1970, Dubrulle & Nazarenko 1997, Bühler & McIntyre 1999, Hertzog et al 2000, Muraschko et al 2015) does this automatically

wave-action density

$$\mathcal{N}(m, z, t) = \int d\alpha\, A_\alpha(z, t)\;\; \delta[m - m_\alpha(z, t)]$$

satisfies conservation equation

$$\frac{\partial \mathcal{N}}{\partial t} + \frac{\partial}{\partial z}(c_g\, \mathcal{N}) + \frac{\partial}{\partial m}(\dot{m}\, \mathcal{N}) = 0 \qquad\qquad \dot{m} = -k\frac{\partial U}{\partial z}$$

Mean flow: $\dfrac{\partial U}{\partial t} = -\dfrac{1}{\bar{\rho}}\dfrac{\partial}{\partial z}(\bar{\rho}\, \overline{u'w'}) = -\dfrac{1}{\bar{\rho}}\dfrac{\partial}{\partial z}\left(\int dm\, c_g k\, \mathcal{N}\right)$

Generalization to 3D straightforward

Ray tracing with caustics: efficient numerics (Muraschko et al 2015)

phase-space velocity is **non-divergent**

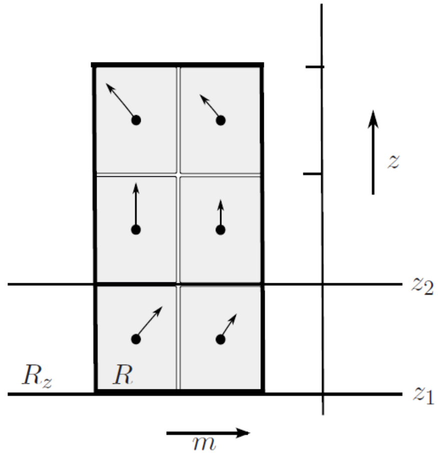
$$\frac{\partial c_g}{\partial z} + \frac{\partial \dot{m}}{\partial m} = \frac{\partial}{\partial z}\frac{\partial \Omega}{\partial m} + \frac{\partial}{\partial m}\left(-\frac{\partial \Omega}{\partial z}\right) = 0$$

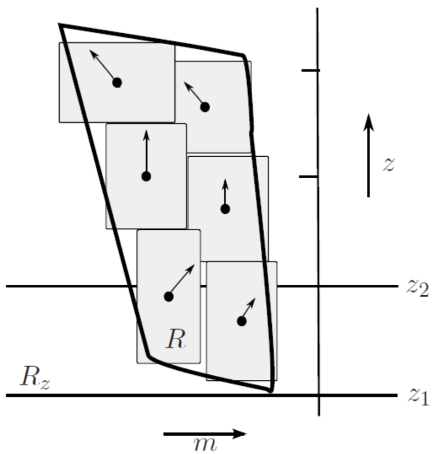
hence

- flow is **volume preserving**
- **rays cannot cross**
- Wave-action density **conserved on rays**

$$\frac{D\mathcal{N}}{Dt} = \frac{\partial \mathcal{N}}{\partial t} + c_g \frac{\partial \mathcal{N}}{\partial z} + \dot{m}\frac{\partial \mathcal{N}}{\partial m} = 0$$

- region of **nonzero $\mathcal{N}$** approximated by **rectangular ray volumes**
- ray volumes move with central ray
- ray volumes change height (Δz) and width (Δm) in area-preserving manner





Ray tracing with caustics: efficient numerics (Muraschko et al 2015)

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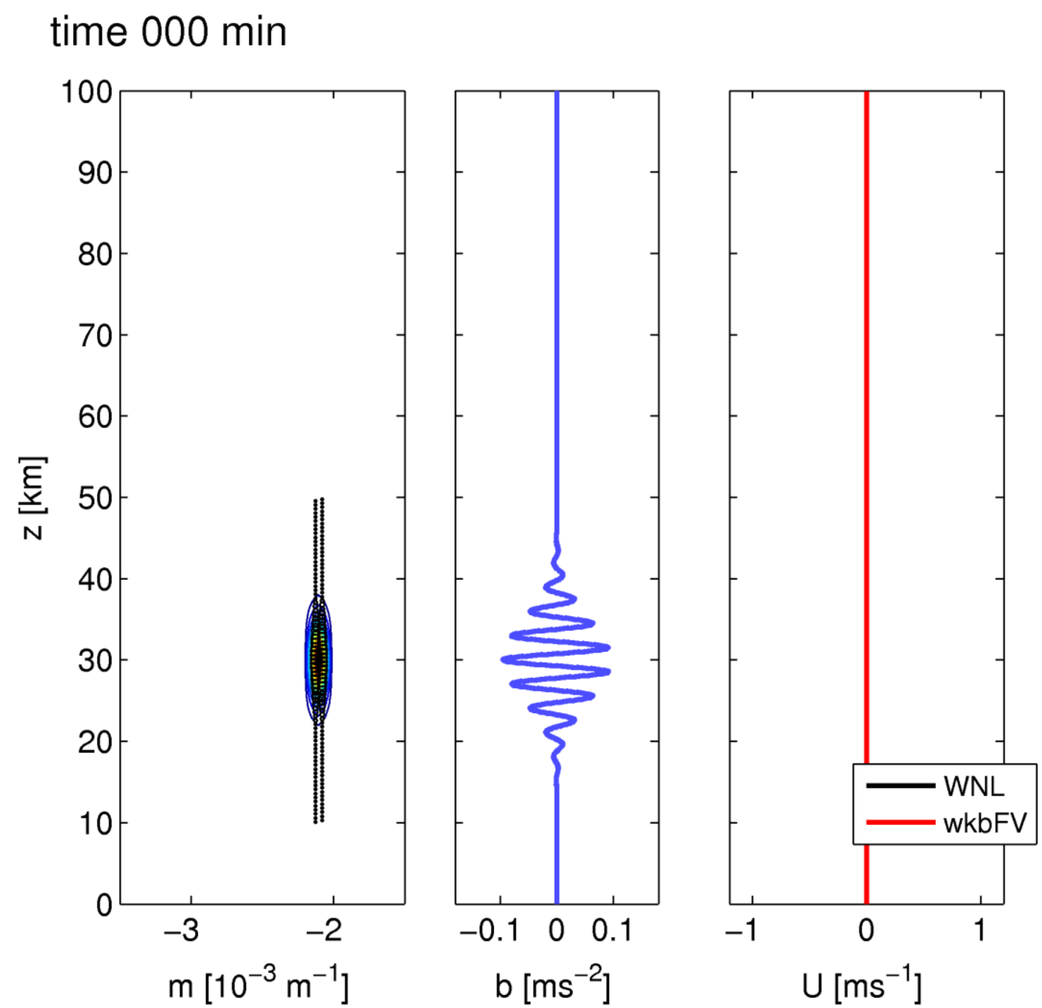
hydrostatic wave packet (Boussinesq)

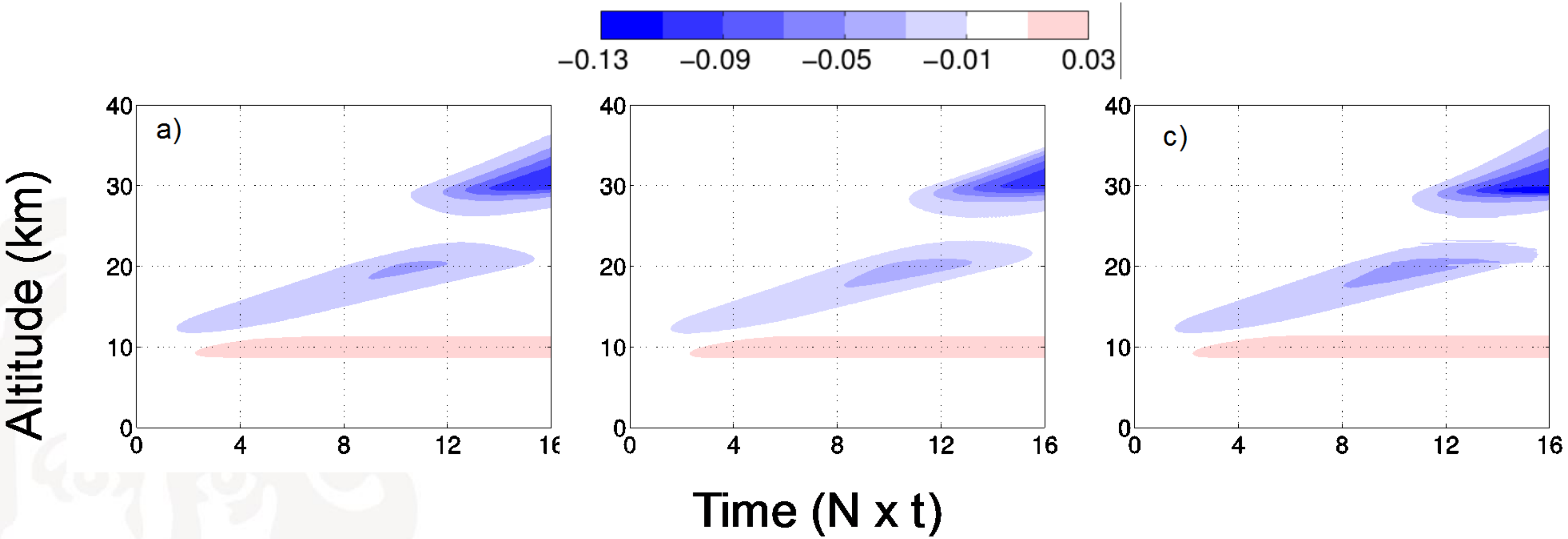


Rays are no wavepackets

No turbulence taken into account!

Ray tracing with caustics:
no numerical instabilities (Bölöni et al 2016, submitted)

GW packet refracted by a jet



LES

WKB ray tracer

WKB finite volume

Direct wave-mean-flow coupling: comparsion with role of wave breaking

- **transient GWs** can interact with the mean flow without the onset of turbulence (eg Dosser & Sutherland 2011)
- GW parameterizations (steady-state approximation) only rely on **wave breaking**

comparative role of wave transience (direct interaction) vs wave breaking?

direct wave-mean-flow interaction vs wave breaking (Bölöni et al 2016)

horizontally infinite GW packets in interaction with mean flow

- **1D**: $U(z,t), A(z,t), m(z,t)$
- direct GW-mean-flow interaction always active
- WKB: $\boldsymbol{E_{mean} + E_{wave} = const}$.

tools:

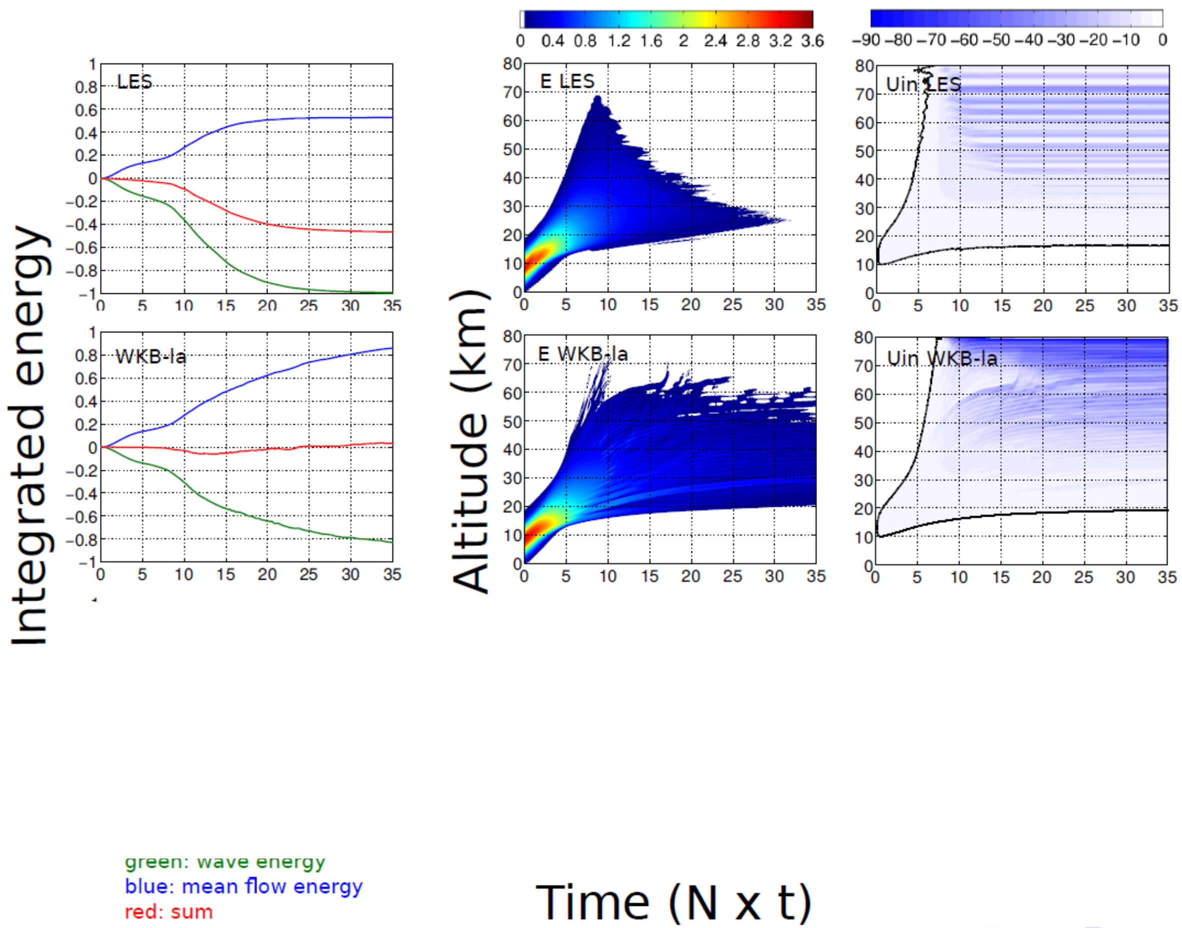
- wave resolving **LES** (reference data)
- fully coupled **WKB**
- **turbulence** onset
  - once static instability threshold can be surpassed

$$\int dm\ m^2 \frac{d|B|^2}{dm} = \int dm\ \mathcal{N}\ f(m) > \alpha N^2$$

  - parameter $\alpha \in [1,2]$ accounting for phase cancellations between spectral components
  - (scale selective) **eddy viscosity/diffusivity** reduces wave amplitude to inst. threshold

direct wave-mean-flow interaction vs wave breaking (Bölöni et al 2016)

static instability hydrostatic wave packet



LES (wave-resolving)

WKB

direct wave-mean-flow interaction vs wave breaking (Bölöni et al 2016)

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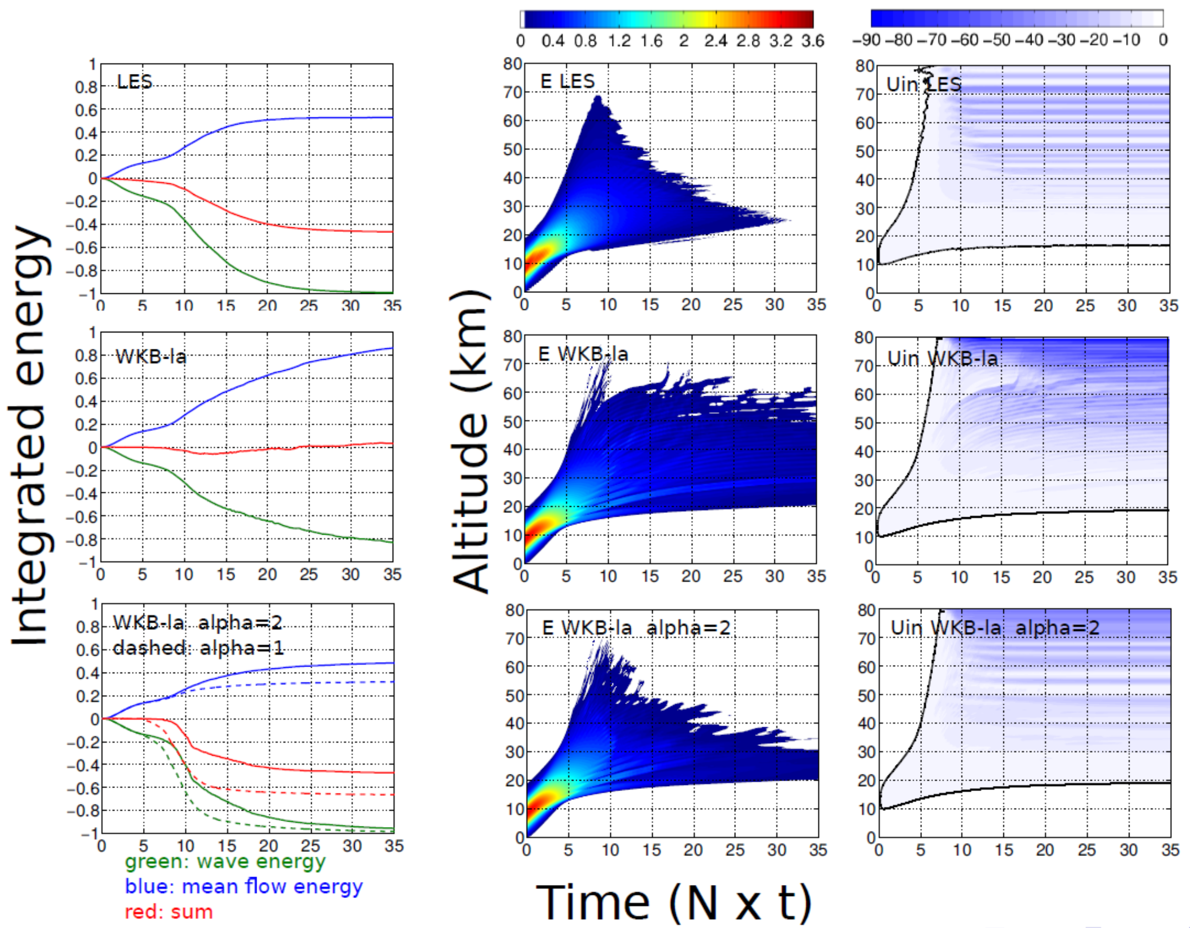
static instability hydrostatic wave packet



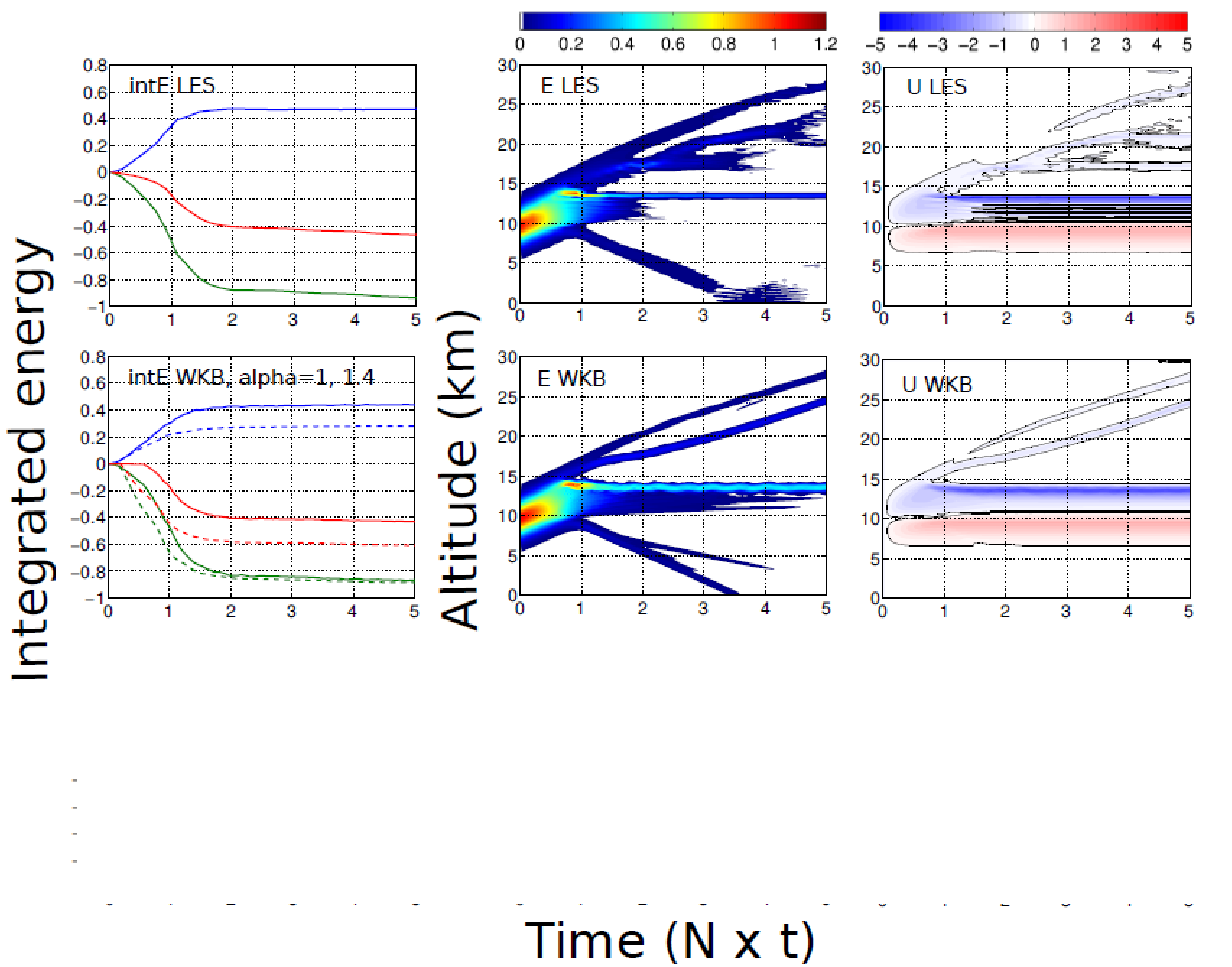
LES (wave-resolving)

WKB

WKB with saturation (turbulence param.)

direct wave-mean-flow interaction vs wave breaking (Bölöni et al 2016)

static instability non-hydrostatic wave packet



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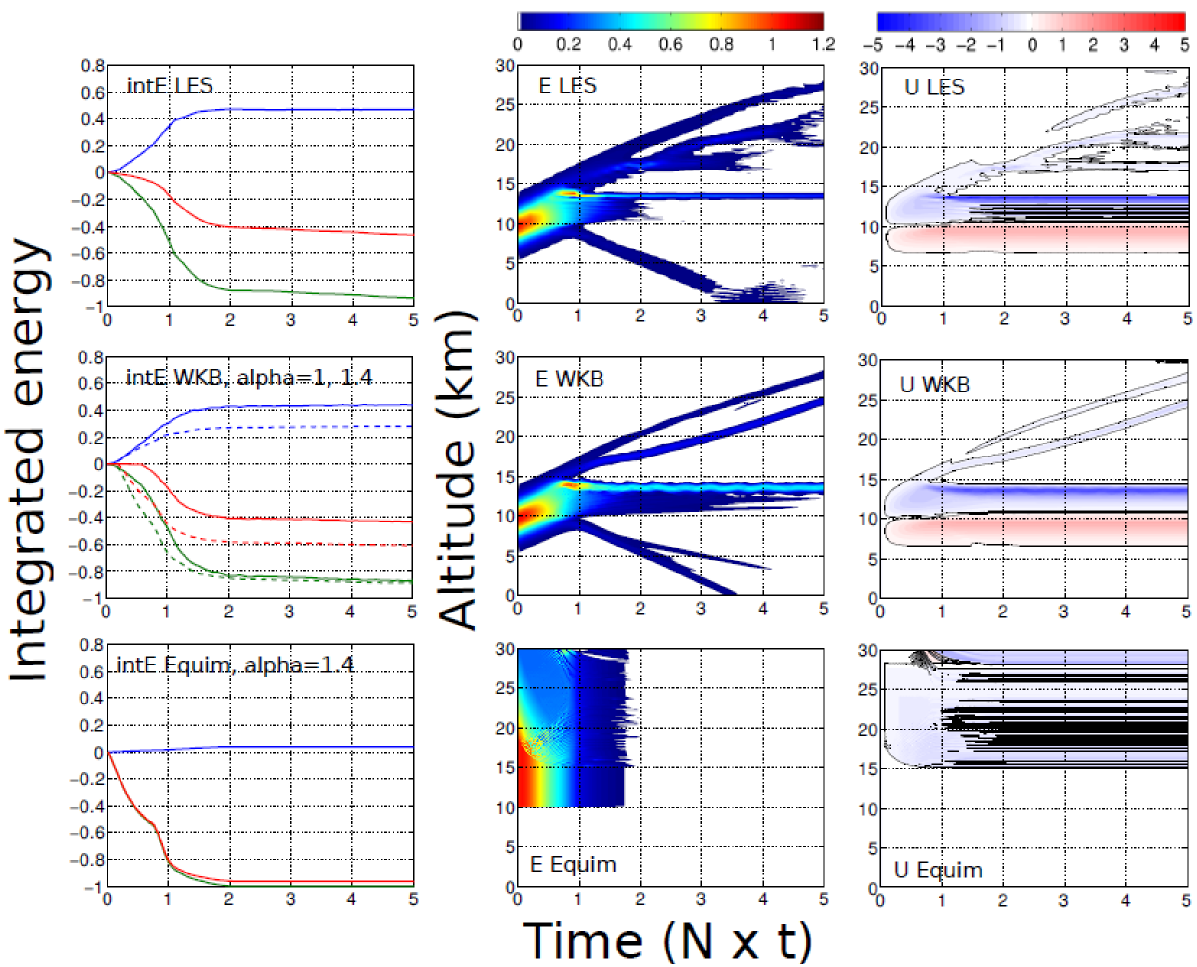

LES (wave-resolving)

WKB with saturation (turbulence param.)

steady-state (GW parameterization)

role of lateral propagation
linear large-scale dynamics in interaction with GWs

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From **GCM data** (HAMMONIA, Schmidt et al 2006):

- Seasonally dependent reference climatology $\overline{\boldsymbol{u}}(\lambda,\phi,z),\ \overline{T}(\lambda,\phi,z)$
- Diurnal heating cycle $\Re \sum_n Q_n(\lambda,\phi,z)\, e^{in\Omega t}$

Linear model (Achatz et al 2008, based on KMCM, Becker and Schmitz 2003)

$$\boldsymbol{u} = \overline{\boldsymbol{u}} + \boldsymbol{u}'(\lambda,\phi,z,t)$$
$$T = \overline{T} + T'(\lambda,\phi,z,t)$$

$$\left(\frac{\partial}{\partial t} + \overline{\boldsymbol{u}}\cdot\nabla_h\right)\boldsymbol{u}' + \cdots = -\frac{1}{\bar\rho}\nabla\cdot\left(\bar\rho\ \overline{\boldsymbol{v}_{GW}\boldsymbol{u}_{GW}}\right)$$

$$\left(\frac{\partial}{\partial t} + \overline{\boldsymbol{u}}\cdot\nabla_h\right)T' + \boldsymbol{v}'\cdot\nabla\overline{T} + \cdots = \Re\sum_n Q_n(\lambda,\phi,z)\, e^{in\Omega t} - \nabla_h\cdot\left(\overline{\boldsymbol{u}_{GW}\, T_{GW}}\right)$$

GW fluxes from **4D WKB model with rays propagating on** $(\overline{\boldsymbol{u}} + \boldsymbol{u}', \overline{\boldsymbol{T}} + \boldsymbol{T}')$

First implementation of a fully coupled transient ray tracer into a global model

linear large-scale dynamics in interaction with GWs (Ribstein et al 2015, Ribstein & Achatz 2016)

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3D effects (beyond single column)

- **Horizontal GW propagation**

$$\frac{d\boldsymbol{x}_h}{dt} = \boldsymbol{c}_{gh}, \qquad \frac{dz}{dt} = c_{gz}$$

- **Horizontal gradients in reference climatology and tides**

$$\frac{d\boldsymbol{k}_h}{dt} = -k\,\nabla_h(\bar{u}+u') - l\,\nabla_h(\bar{v}+v'), \qquad \frac{dm}{dt} = -k\frac{d}{dz}(\bar{u}+u') - l\frac{d}{dz}(\bar{v}+v')$$

- **Horizontal GW flux convergence**

$$\left(\frac{\partial}{\partial t} + \bar{\boldsymbol{u}}\cdot\nabla_h\right)\boldsymbol{u}' + \cdots = -\frac{1}{\bar{\rho}}\frac{\partial}{\partial z}\left(\bar{\rho}\ \overline{\boldsymbol{w}_{GW}\boldsymbol{u}_{GW}}\right) - \frac{1}{\bar{\rho}}\ \nabla_h\cdot\left(\bar{\rho}\ \overline{\boldsymbol{u}_{GW}\boldsymbol{u}_{GW}}\right)$$

$$\left(\frac{\partial}{\partial t} + \bar{\boldsymbol{u}}\cdot\nabla_h\right)T' + \boldsymbol{v}'\cdot\nabla\bar{T} + \cdots = \Re\sum_n Q_n(\lambda,\phi,z)\,e^{in\Omega t} - \nabla_h\cdot\left(\overline{\boldsymbol{u}_{GW}\,T_{GW}}\right)$$

Tidal model in interaction with GWs
(Ribstein et al 2015, Ribstein & Achatz 2016)

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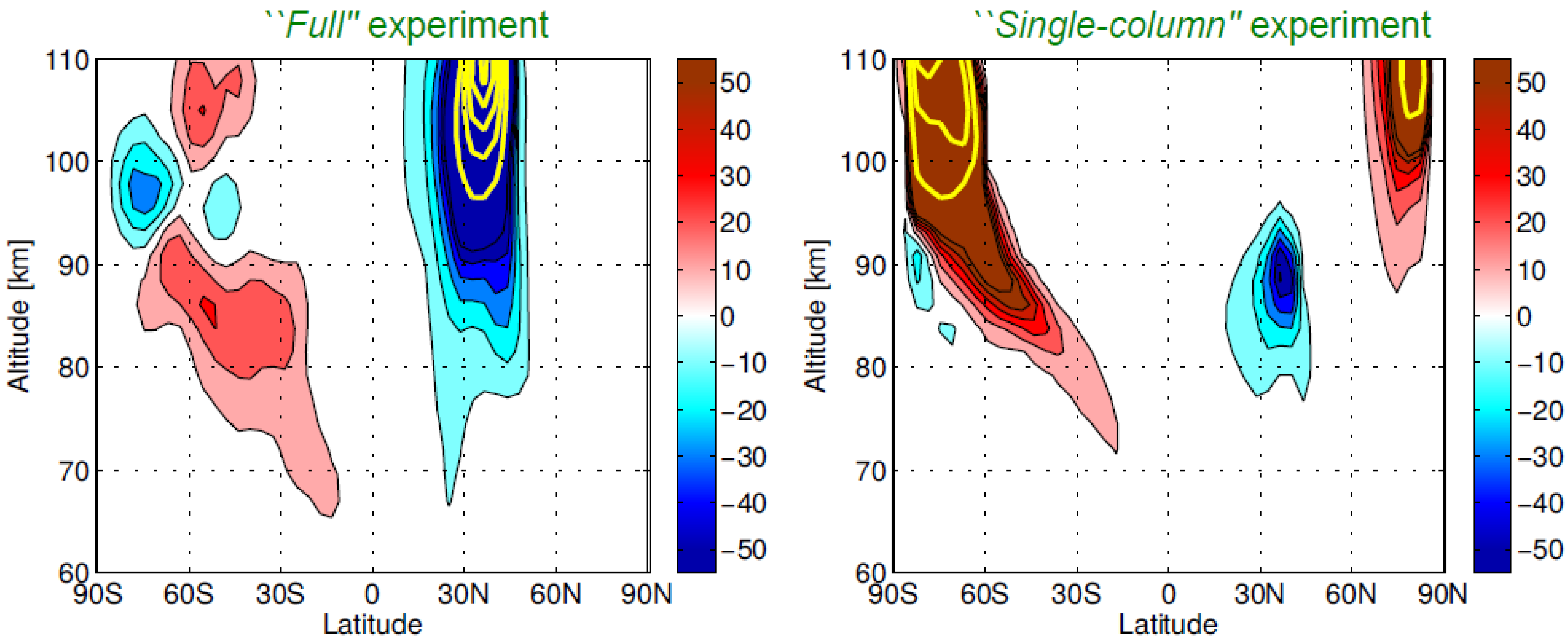
3D effects (beyond single column)



zonal-mean daily-mean GW forcing (December)

Summary

- **numerical implementation** fully interactive WKB
  - **no instabilities** due to caustics
  - very **efficient**
- **transient-GW dynamics**
  - direct GW-mean-flow interaction **dominates over GW breaking**
  - **GW parameterizations** not reliable
- **lateral-propagation effects matter in middle atmosphere**

Achatz, U., Ribstein, B., Senf, F., and R. Klein, 2016: The interaction between synoptic-scale balanced flow and a finite-amplitude mesoscale wave field throughout all atmospheric layers: Weak and moderately strong stratification. *Quart. J. R. Met. Soc.,* in revision

Bölöni, G., Ribstein, S., Muraschko, J., Sgoff, C., Wei, J., and U. Achatz, 2016: The interaction between atmospheric gravity waves and large-scale flows: an efficient description beyond the non-acceleration paradigm. *J. Atmos. Sci.*, doi: http://dx.doi.org/10.1175/JAS-D-16-0069.1

Muraschko J, Fruman M, Achatz U, Hickel S, Toledo Y. 2015: On the application of WKB theory for the simulation of the weakly nonlinear dynamics of gravity waves. *Quart. J. R. Met. Soc.* **141,** 676–697.

Ribstein, B., Achatz, U. und F. Senf, 2015: The interaction between gravity waves and solar tides: Results from 4D ray tracing coupled to a linear tidal model, *J. Geophys. Res.*, **120**, 6795–6817, doi:10.1002/2015JA021349

Ribstein, B. und U. Achatz, 2016: Gravity wave propagation and impacts on a diurnal middle atmosphere : results from 4D ray tracing directly coupled to a linear tidal model. *J. Geophys. Res.,* **121,** doi:10.1002/2016JA022478

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https://ms-gwaves.iau.uni-frankfurt.de/index.php

- Investigation **multi-scale dynamics of GWs** in 6 projects
- **prognostic WKB GW parameterization** to be developed for NWP and climate model
- To be addressed:
  - **Sources**
  - **Propagation**
  - **dissipation**
- Combined effort:
  - **Theory,**
  - **modelling,**
  - **measurements,**
  - **laboratory experiments**