Sensitivity of the ECMWF model to Semi-Lagrangian departure point iterations

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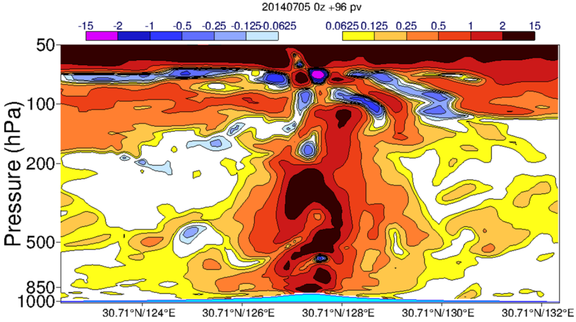
Thanks to: E. Holm, P. Lopez, S. Malardel, F. Vana

ECMWF

Workshop on numerical and computational methods for simulation of all-scale geophysical flows
3-7 October 2016

Motivation



- "Unusual" stratosphere above typhoon Neoguri at t+96hrs forecast

Advection equation and Departure Points

SL method solution to advection equation:

$$\frac{D\phi(\mathbf{r},t)}{Dt} = 0, \quad \frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla, \quad \mathbf{V} = (u,v,w)$$

$$\frac{D\mathbf{r}}{Dt} = \mathbf{V}(\mathbf{r},t)$$ SL trajectory eqn

is

$$\phi(\mathbf{r}_\mathbf{A}, t+\Delta t) = \phi(\mathbf{r}_\mathbf{D}, t)$$

$\mathbf{r}_\mathbf{A}$: "arrival points" (given gridpoints)
$\mathbf{r}_\mathbf{D}$: "departure points" d.p. (computed every timestep)

DP estimation crucial for accuracy of numerical solution

DP iterations in IFS

Computing DP with a mid-point scheme requires wind field at SL trajectory mid-points $(\frac{\mathbf{r_A}+\mathbf{r}_D}{2}, t^{n+1/2})$

IFS SETTLS extrapolation:

$$\frac{1}{2}\left[\mathbf{V}(\mathbf{r_A}, t) + 2\mathbf{V}(\mathbf{r_D}, t) - \mathbf{V}(\mathbf{r_D}, t - \Delta t)\right] \approx V(\frac{\mathbf{r_A}+\mathbf{r}_D}{2}, t^{n+1/2})$$

Iterate recurrence relation to compute DP:

$$\begin{aligned} \mathbf{r_D}^{[1]} &= \mathbf{r_A} - \Delta t \mathbf{V}(\mathbf{r_A}, t) \\ \mathbf{r_D}^{[\nu]} &= \mathbf{r_A} - \frac{\Delta t}{2}\left\{\mathbf{V}(\mathbf{r_A}, t) + \left[2\mathbf{V}(\mathbf{r}, t) - \mathbf{V}(\mathbf{r}, t - \Delta t)\right]\Big|_{\mathbf{r}=\mathbf{r_D}^{[\nu-1]}}\right\} \end{aligned}$$

for $\nu = 2, 3, \ldots, \nu_{max}$

DP iteration convergence

From Smolarkiewicz & Pudykiewicz MWR 1992 analysis:

$$\|\mathbf{r_D} - \mathbf{r_D}^{[\nu]}\| \leq \mathcal{L}^{\nu-1}\|\mathbf{r_D} - \mathbf{r_D}^{[1]}\|, \quad \nu = 2, 3, \ldots, \nu_{max}$$

or

$$\|\mathbf{r_D}^{[\nu]} - \mathbf{r_D}^{[\nu-1]}\| \leq \mathcal{L}\|\mathbf{r_D}^{[\nu-1]} - \mathbf{r_D}^{[\nu-2]}\|, \quad \nu = 2, 3 \ldots, \nu_{max}$$

$$\mathcal{L} \equiv \Delta t \|\frac{\partial \widetilde{\mathbf{V}}}{\partial \mathbf{r}}\| \qquad \text{Lipschitz number}$$

- $\mathcal{L} < 1$ is a sufficient condition for convergence
- $\mathcal{L}$ is an upper bound of the rate of convergence
- How large is $\mathcal{L}$ in a forecast and how fast DP converge?

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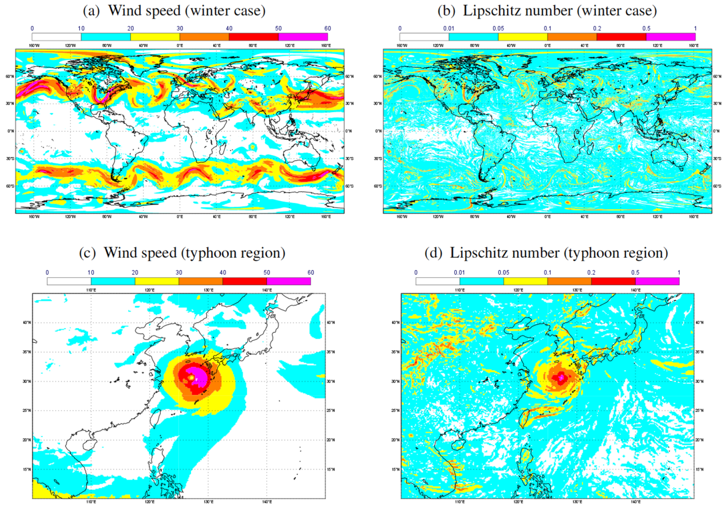
Lipschitz numbers in 16km res IFS forecasts



(a), (b): 00UTC 10 January 2014, t+48hrs fc at 500hPa. (c), (d): 00UTC 5 July 2014 t+96 hrs fc at 850hPa

Diagnostic for DP convergence

Define scaled DP displacement:

$$\delta x_{D,ik}^{[\nu]} = \frac{\left| x_{D,ik}^{[\nu]} - x_{D,ik}^{[\nu-1]} \right|}{d_{ik}}, \quad \nu = 2, 3, \ldots, \nu_{max}$$

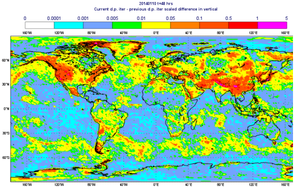
where

$$x_{D,ik}^{[\nu]} = \begin{cases} \eta_D^{[\nu]}, & \text{vertical} \\ \alpha\phi_{D,ik}^{[\nu]}, & \text{horizontal} \end{cases}, \quad d_{ik} = \text{resolution scaling factor}$$

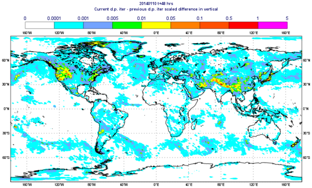
$\phi_{D,ik}^{[\nu]}$: angle between *ik* GP, its DP and the centre of the earth, α : Earth radius, $\Delta\eta_k$ thickness of k-layer, Δx: approximate gridlength.
Should be a converging sequence:

$$\delta x_{D,ik}^{[2]} > \delta x_{D,ik}^{[3]} > \cdots > \delta x_{D,ik}^{[\nu_{max}]} \rightarrow 0$$

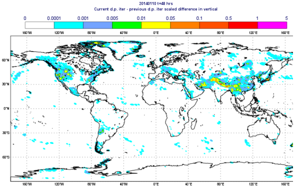
Winter case (16km res): t+48hrs 500hPa level $\delta x_{D,ik}^{(\nu)}$



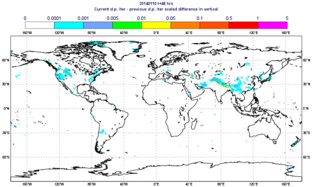
(a) iter 2 - iter 1



(b) iter 3 - iter 2

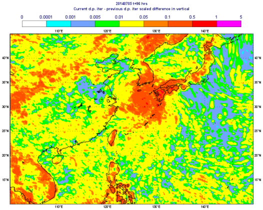


(c) iter 4 - iter 3



(d) iter 5 - iter 4

Neoguri: t+48hrs 850hPa level



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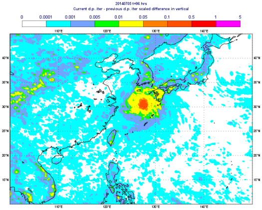

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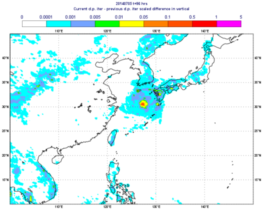

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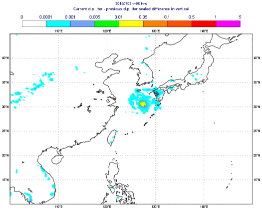
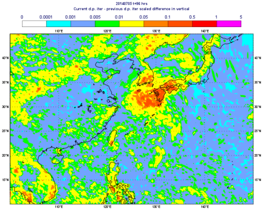


(d) iter 5 - iter 4

Neoguri: t+48hrs 850hPa $\Delta t/2 = 300s$



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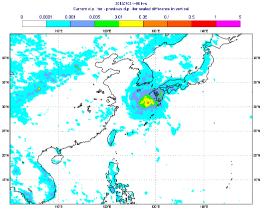
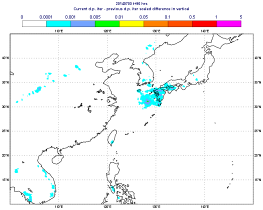


(b) iter 3 - iter 2



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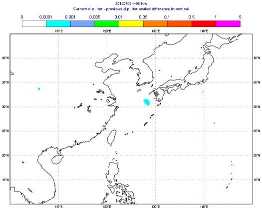

(d) iter 5 - iter 4

IFS upgrades and impact on Lipschitz number

$$\boxed{\mathcal{L} \equiv \Delta t \| \frac{\partial \widetilde{\mathbf{V}}}{\partial \mathbf{r}} \|}$$

- 2 DP iterations are sufficient for 2nd order accuracy and typically the recommendation in the literature has been 2 iterations. However,
- Higher resolution results in steeper velocity gradients
- Successive resolution updates and stretching of timestep for efficiency have shifted upwards mean Lipschitz numbers in IFS

Horizontal Res	Vertical levs	tstep (s)	$\Delta t/\Delta x$
TL399 (50km)	91	1200	0.02
TL511 (40km)	91	900	0.02
TL1279 (16km)	91	600	0.04
Tco1279 (9km)	137	450	0.05

ECMWF

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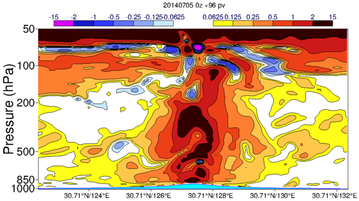
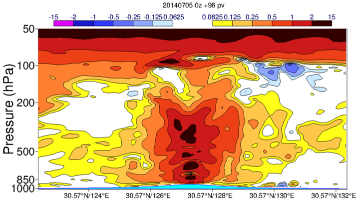
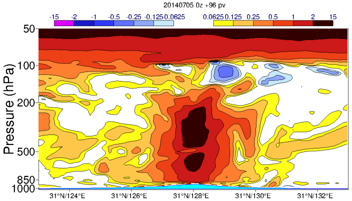
Neoguri: impact of DP iterations



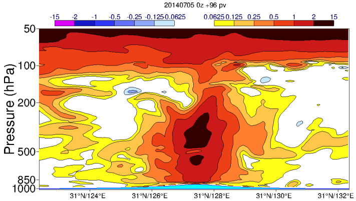
(a) forecast with iter=3



(b) forecast iter=5

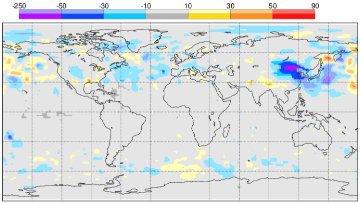


(c) forecast with iter=5, $\Delta t/2$

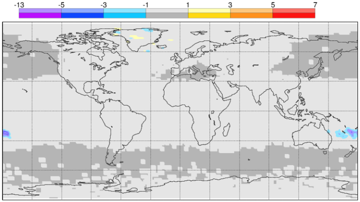


(d) forecast with iter=5, $\Delta t/3$

Verification: 5 it - 3 it difference at t+96hrs Tco1279 res



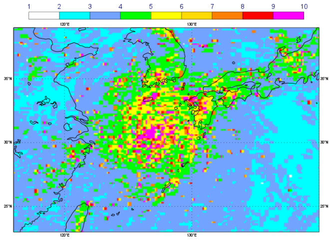
(a) z 500hPa RMSE difference (45 days)



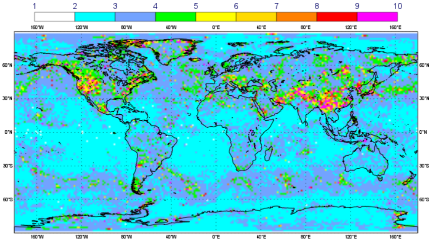
(b) RH 50 hPa RMSE difference

- Geopotential, winds and temperature errors reduce in extratropics, improved precipitation over China
- Improved TC prediction
- Allowed 12.5% increase of timestep from 400s to 450s without degradation (not possible at original 3 iteration setup)

"Dynamic selection" of number of DP iter



(a) # iterations (850hPa)



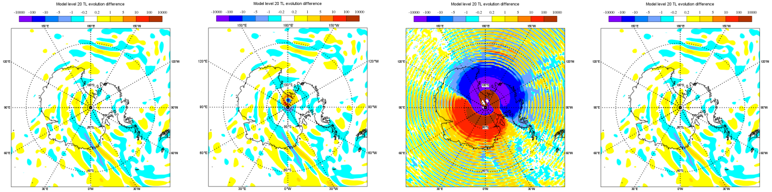
(b) # iterations (500hPa)

Stop iterating if iterations converged within a tolerance or the estimated convergence rate cr_{ik} is above a threshold:

$$cr_{ik} = \frac{\left\| x_{D_{ik}^{[\nu]}} - x_{D_{ik}^{[\nu-1]}} \right\|}{\left\| x_{D_{ik}^{[\nu-1]}} - x_{D_{ik}^{[\nu-2]}} \right\|}$$

- large #iterations selected at neighborhood of TCs, storm track, above orography

Increasing DP iterations in 4DVAR TL perturbation model



(c) $\nu_{max}=2$ (d) $\nu_{max}=5$ (e) $\nu_{max}=10$ (f) $\nu_{max}=10$ + dynamic iteration

(Fig: strong cross polar flow case)

Dynamic iteration

1. # of iter at each gridpoint are determined by the nonlinear forecast model and stored
2. Nonlinear model, TL and adjoint models do same # of iterations at each gidpoint (determined by nonlinear model)
3. Dynamic iteration helps control instability by stopping diverging iter

TL perturbation DP and multiple iterations

1D nonlinear advection equation: $\frac{Du}{Dt} \equiv \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0.$

TL perturbation DP (u^*: extrapolated velocity, α: lin interp weight):

$$\delta x_{d(j)}^{[\nu]} = -\frac{\Delta t}{2}\left[\delta u_j^n + (1-\alpha_j^{[\nu-1]})\delta u_{j-p}^* + \alpha_j^{[\nu-1]}\delta u_{j-p-1}^* + \delta\alpha_j^{[\nu-1]}(u_{j-p-1}^* - u_{j-p}^*)\right]$$

$$\alpha_j^{[\nu]} = \frac{x_{j-p} - x_{d(j)}^{[\nu]}}{\Delta x}, \qquad \delta\alpha_j^{[\nu]} = -\frac{\delta x_{d(j)}^{[\nu]}}{\Delta x}, \quad x_d^{[\nu]} \in [x_{j-p-1}, x_{j-p}]$$

or,

$$\delta x_{d(j)}^{[\nu]} = -\frac{\Delta t}{2}\left[\delta u_j^n + (1-\alpha_j^{[\nu-1]})\delta u_{j-p}^* + \alpha_j^{[\nu-1]}\delta u_{j-p-1}^*\right] - \frac{1}{2}\delta x_{d(j)}^{[\nu-1]} L_{j-p-1/2}^*$$

Large Δt, L, $\nu \Rightarrow$ large $\delta x_d \Rightarrow x_d + \delta x_d$ may be at different interval than $x_d \Rightarrow$ TL approx may become invalid (Li et al 1993)

Summary

- Inadequate convergence of DP in high res IFS in high CFL regions with strong shear with small number of iterations
- Increasing number of iterations improves convergence and forecast accuracy and improves structure of hurricanes allowing even larger timesteps which offsets extra cost
- Need for increasing iterations is confirmed by an experimental "dynamic DP iteration" code
- Increasing iterations in IFS tangent-linear model led to instabilities in TL model but these can be controlled by "dynamic selection of iteration number" which stops diverging iterations

More details in MWR Sep 2016 "Sensitivity of the ECMWF model to Semi-Lagrangian departure point iterations"