Improving coupled model solution mathematical consistency through data assimilation.

Rémi Pellerej, Florian Lemarié, Arthur Vidard

LABORATOIRE JEAN KUNTZMANN
MATHÉMATIQUES APPLIQUÉES - INFORMATIQUE

Inria informatics mathematics

UNIVERSITÉ Grenoble Alpes

ERA CLIM2

Vienna, 17thof January 2017

ERA-CLIM2 General Assembly 2017

Coupled Data Asimilation: an opportunity

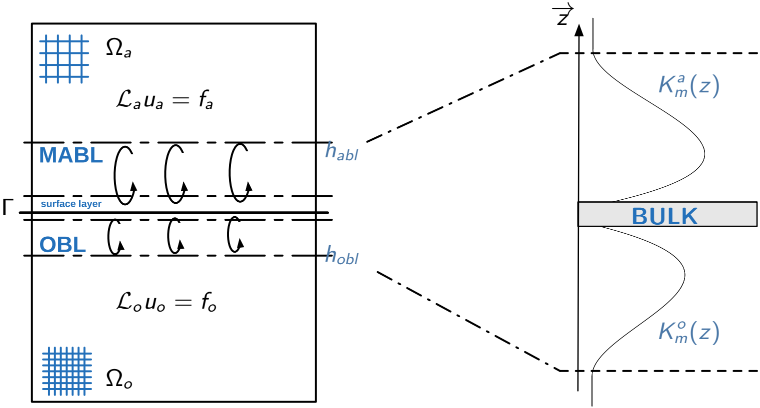
OA coupling is a complex matter with many sources of uncertainties

- time/space non-confomity
- interfaces may actually not be represented by any component
- multi physics with different characteristics.
- highly parameterised interface (Bulk formulae)
- coupling methods
- ...

Some of theses uncertainties are unavoidable, some others are linked to the way we implement things.

Coupled DA is an opportunity to account for or reduce them

Coupled modelling systems

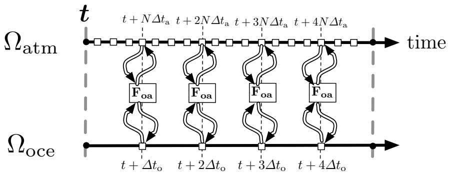


- K_m parameterization in the boundary layers
- $\boldsymbol{F}_{oa} = (\tau, Q_{net})$ is the interface flux

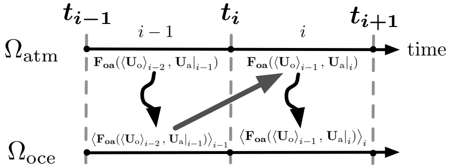
At the Interface Γ :

$$\mathcal{G}_o u_o = \mathcal{G}_a u_a$$

$$\mathcal{F}_a u_a = \mathcal{F}_o u_o$$



SYNCHRONOUS

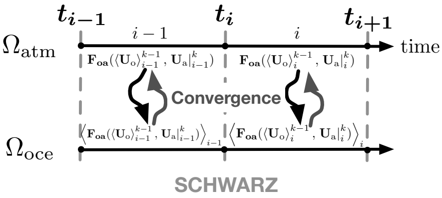


ASYNCHRONOUS

- Synchronous method.
  - Aliasing errors
  - Synchronicity issues
  - Physics-dynamics inconsistency error
- Asynchronous method:
  - Does not solve the original problem

Possible solutions

- Monolithic approach
- Iterative method to solve the coupling problem



Considering :

- $u_0 \in H^1(\Omega_a \cup \Omega_o)$ the initial condition
- k the iteration number
- $u_a^0(0,t)$ the *first-guess*

The SWR algorithm reads :

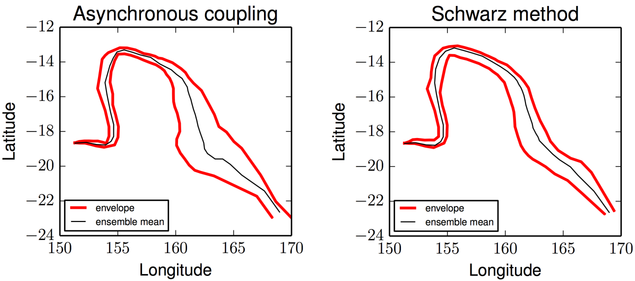
$$\begin{cases} \mathcal{L}_o u_o^k = f_o & \text{on } \Omega_o \times T_i \\ u_o^k(z,0) = u_0(z) & z \in \Omega_o \\ \mathcal{G}_o u_o^k = \mathcal{G}_a u_a^k & \text{on } \Gamma \times T_i \end{cases} \qquad \begin{cases} \mathcal{L}_a u_a^k = f_a & \text{on } \Omega_a \times T_i \\ u_a^k(z,0) = u_0(z) & z \in \Omega_a \\ \mathcal{F}_a u_a^k = \mathcal{F}_o u_o^{k-1} & \text{on } \Gamma \times T_i \end{cases}$$

where $T_i = [t_i; t_{i+1}]$

- At convergence, it provides a flux consistent solution : $\mathcal{F}_a u_a = \mathcal{F}_o u_o$ and $\mathcal{G}_o u_o = \mathcal{G}_a u_a$ on $\Gamma \times T_i$

Coupling methods

Why does it matter



Hurricane Erica's trajectory and ensemble spread
18 members of WRF/ROMS, generated through perturbations of initial conditions and coupling frequency (lemarié et al. 2014))

Main drawbacks :

- This is an iterative method
- Convergence speed greatly depends on $\mathcal{F}_d$, $\mathcal{G}_d$ and $u_a^0(0,t)$ $(d = a, o)$

Advantages :

- This is a non-intrusive coupling method
- At convergence, it provides a strongly coupled solution

Starting point of this ERA-CLIM2 task, in the framework of a variational system

- Can we improve the boundary conditions to accelerate the SWR convergence?
- Take benefit of the minimisation iterations for the SWR ones

Fully Iterative Method (FIM)

- $\mathbf{x}_0 = u_0(z),\ z \in \Omega = \Omega_a \cup \Omega_o$ is the controlled state vector
- $\mathbf{x}^{\mathrm{cvg}} = (u_a^{k_{\mathrm{cvg}}}, u_o^{k_{\mathrm{cvg}}})^T$ is the converge solution of the SWR algorithm : k_{cvg} iterations
- The first-guess u_a^0 in the SWR algorithm is updated after each minimisation iteration

$$J_{FIM}(\mathbf{x}_0) = J^b(\mathbf{x}_0) + \int_{t_i}^{t_{i+1}} \left\langle \mathbf{y} - H\left(\mathbf{x}^{\mathrm{cvg}}\right), \mathbf{R}^{-1}(\mathbf{y} - H\left(\mathbf{x}^{\mathrm{cvg}}\right)) \right\rangle_{\Omega} \mathrm{dt}$$

- The solution provided is ~~strongly~~ ~~fully~~ insanely coupled
- It requires the adjoint of the coupling
- It possibly requires a large number of Schwarz iterations

Truncated iterative method (TIM)

- $\mathbf{x}_0 = (u_0(z), \underline{u_o^0(0,t)})^T, \; z \in \Omega \setminus \Gamma$
- The Schwarz iterations are truncated at $k_{\max} < k_{\mathrm{cvg}}$ iterations
- $\mathbf{x}^{\mathrm{max}} = (u_a^{k_{\max}}, u_o^{k_{\max}})^T$
- Extended cost function :

$$J^s = \alpha_{\mathcal{F}} \| \mathcal{F}_a u_a^{k_{\max}}(0,t) - \mathcal{F}_o u_o^{k_{\max}}(0,t) \|_{T_i}^2 + \alpha_{\mathcal{G}} \| \mathcal{G}_a u_a^{k_{\max}}(0,t) - \mathcal{G}_o u_o^{k_{\max}}(0,t) \|_{T_i}^2$$

with $\|a\|_\Sigma^2 = \langle a, a \rangle_\Sigma$

$$J_{TIM}(\mathbf{x}_0) = J^b(\mathbf{x}_0) + \int_{t_i}^{t_{i+1}} \left\langle \mathbf{y} - H\left(\mathbf{x}^{\mathrm{max}}\right), \mathbf{R}^{-1}(\mathbf{y} - H\left(\mathbf{x}^{\mathrm{max}}\right)) \right\rangle_\Omega \mathrm{dt} + \boxed{J^s}$$

- The solution provided is quasi-strongly coupled
- It requires the adjoint of the coupling
- It requires fewer number of Schwarz iterations than the FIM

Weakly Interfaced Models (WIM)

- $\mathbf{x}_0 = (\mathbf{x}_{0,a}, \mathbf{x}_{0,o})^T$ with $\mathbf{x}_{0,d} = \left(u_0|_{z\in\Omega_d}, u_d^0(0,t) \right)$
- The direct coupling between both models is suppressed
- Models are coupled during the assimilation process

$$J_{WIM}(\mathbf{x}_0) = \left\{ \sum_{d=a,o} \left(J^b(\mathbf{x}_{0,d}) + J^o(\mathbf{x}_{0,d}) \right) \right\} + J^s$$

- The solution provided is weakly coupled (as coupling is a weak constraint)
- It requires only the adjoints of the uncoupled models
- There is no coupling iterations

Considered schemes - Summary

Algo	Control vector	# of coupling iterations	extended cost function	Adjoint of the coupling	Coupling
FIM	$(u_0(z))$	k_{cvg}	no	yes	strong
TIM	$(u_0(z), u_o^0)^T$	k_{max}	possibly	yes	∼strong
WIM	$(u_0(z), u_a^0, u_o^0)^T$	1	yes	no	weak

Table: Overview of the properties of the coupled variational DA methods described

Previous algorithms (in incremental formulation) are applied on a **1D linear diffusion problem**, using the OOPS framework.
Let us consider ($d = a, o$):

- $\mathcal{L}_d = \partial_t + \nu_d \partial_z^2$
- $\nu_a \neq \nu_o$ the diffusion coefficients
- $\mathcal{G}_d = \nu_d \partial_z$ and $\mathcal{F}_d = \mathrm{Id}$ the interface operators on Γ
- f_d the second member such that the analytical solution is $u_d^\star(z,t) = \frac{U_0}{4} e^{-\frac{|z|}{\alpha_d}} \left\{3 + \cos^2\left(\frac{3\pi t}{\tau}\right)\right\}$ on $\Omega_d \times T_i$

Algo	$\alpha_{\mathcal{F}}$	$\alpha_{\mathcal{G}}$	k_{max}	# of minimisation iterations	# of models runs	Interface imbalance indicator	RMSE in °C
FIM	-	-	k_{cvg}	14	1501	$5.\ 10^{-12}$	0.216
TIM	0	-	k_{cvg}	14	1509	$7.\ 10^{-11}$	0.216
TIM	0	-	5	15	165	15.07	0.282
TIM	0	-	2	16	68	123.	0.641
TIM	0.01	-	2	166	670	$4.\ 10^{-7}$	0.216
TIM	**0.01**	-	**1**	**97**	**197**	$\mathbf{1.\ 10^{-6}}$	**0.217**
WIM	0.01	40	1	416	835	$1.4\ 10^{-7}$	0.228
Uncoupled	0	0	1	24	51	2633.	8.495

Algo	$\alpha_{\mathcal{F}}$	$\alpha_{\mathcal{G}}$	$k_{\max}$	# of minimisation iterations	# of models runs	Interface imbalance indicator	RMSE in °C
TIM	**0.01**	-	**1**	**24**	**51**	**$5.\ 10^{-5}$**	**0.330**
WIM	0.01	40	1	24	51	$7.\ 10^{-1}$	3.092
Uncoupled	0	0	1	24	51	2633.	8.495

Algo	$\alpha_{\mathcal{F}}$	$\alpha_{\mathcal{G}}$	$k_{\max}$	# of minimisation iterations	# of models runs	Interface imbalance indicator	RMSE in °C
TIM	**0.01**	-	**1**	**24**	**51**	**$5.\ 10^{-5}$**	**0.330**
WIM	0.01	40	1	24	51	$7.\ 10^{-1}$	3.092
Uncoupled	0	0	1	24	51	2633.	8.495

Algo	$\alpha_{\mathcal{F}}$	$\alpha_{\mathcal{G}}$	$k_{\max}$	# of minimisation iterations	# of models runs	Interface imbalance indicator	RMSE in °C
CERA-like-2	—	—	2	$2 * 10$	48	10	0.3
CERA-like-4	—	—	2	$4 * 5$	54	$7.\ 10^{-1}$	0.3

Final comments

It is difficult to draw a clear conclusion from such a simplistic testcase but

- The way models are coupled should not be overlooked
- but controlling and/or penalising the interface mismatch could be a step toward stronger coupling

Before the end of the project

- more in depth theoretical study on convergence
- Apply these algorithms to a more realistic coupled SCM (Ocean/ABL, currently being implemented within OOPS)
- look into optimized interface conditions for SWR

In parallel:

- extend this work to ensemble smoother