Planetary wave resonance, multiple equilibria and bifurcation

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June 8, 2017

Overview

Resonance basics

The multiple equilibria years

The return of resonance

Stationary Barotropic Rossby waves on the β-plane

$$U\partial_x(\nabla^2\psi + \eta) + \beta\partial_x\psi = 0$$

$$\nabla^2\psi + k_s^2\psi = -\eta$$

$$k_s^2 \equiv \frac{\beta}{U}$$

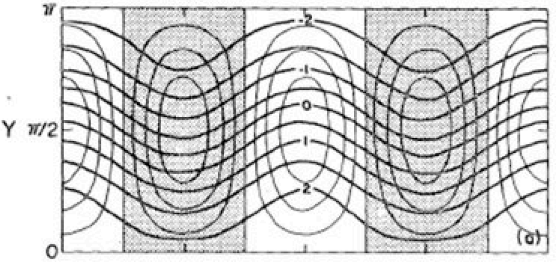
$$\eta = \tilde{\eta}e^{i(kx+\ell y)}, \psi = Ae^{i(kx+\ell y)}$$

$$A = \tilde{\eta}\frac{1}{K^2 - k_s^2}$$

With $U = 30\text{m/s}$, yields resonant wavelength of ≈ 8000 km.
(Zonal wavenumber 2-3, allowing for meridional confinement)

Standing waves in a β-channel

Superpose waves to satisfy rigid-wall (reflecting) boundary condition at N and S boundary



Charney and Eliassen *Tellus* 1949

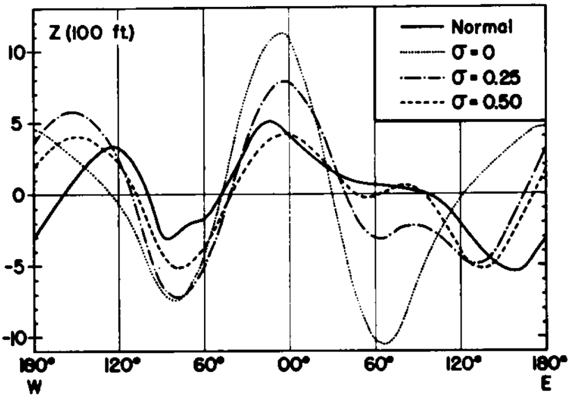
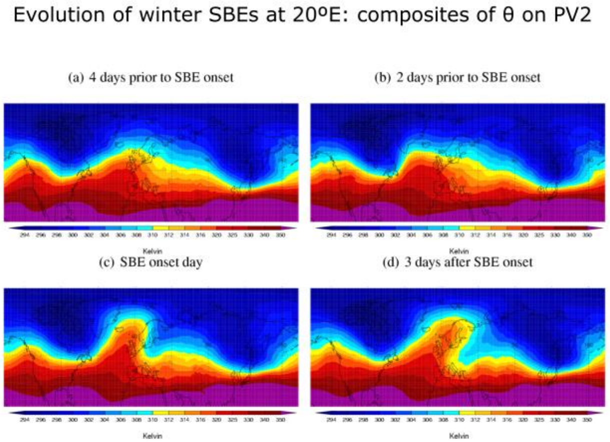


Fig. 4. Normal height profile of the 500 mb surface at 45° N for January together with computed stationary profiles for $\sigma = 0$ (no friction), for $\sigma = 0.25$ (moderate friction) and for $\sigma = 0.50$ (strong friction). For purposes

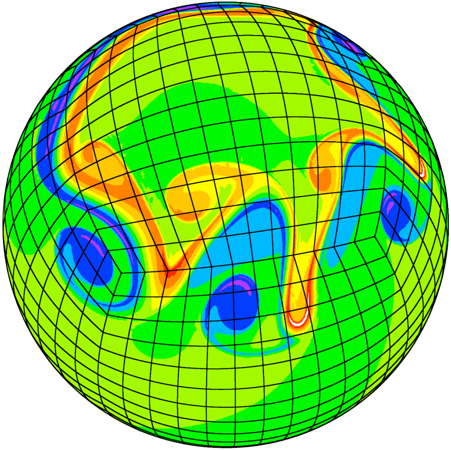
Horel & Wallace 1981: Wavetrains come down the track

Surf zone absorbs waves



Woolings *et al.*

Surf zone absorbs waves



(Idealized calculation, Jablonowski)

Charney & Devore 1979

Idea: *Couple wave to zonal mean flow and look for equilibria of coupled state.*

Form drag:

$$\partial_t U = -\overline{\psi \partial_x \eta}$$

In re-entrant channel, no form drag without friction!

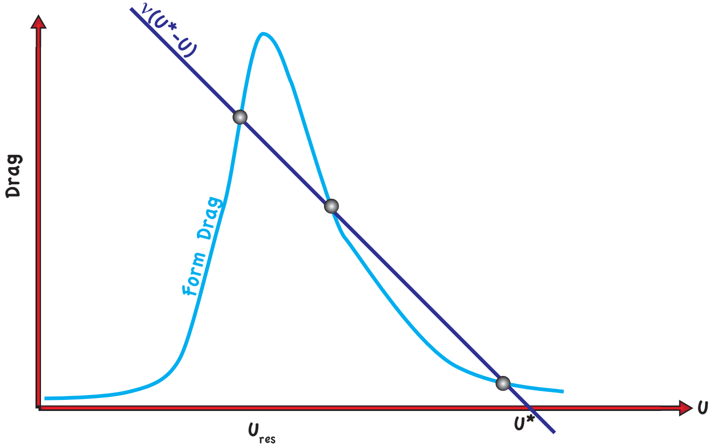
Form drag for forced wave in channel with Ekman friction:

$$\overline{\psi \partial_x \eta} = \tilde{\eta} \frac{\frac{\nu}{kU}}{((\frac{\beta}{U}) - K^2)^2 + (\frac{\nu}{kU})^2}$$

Add a forcing to maintain mean flow:

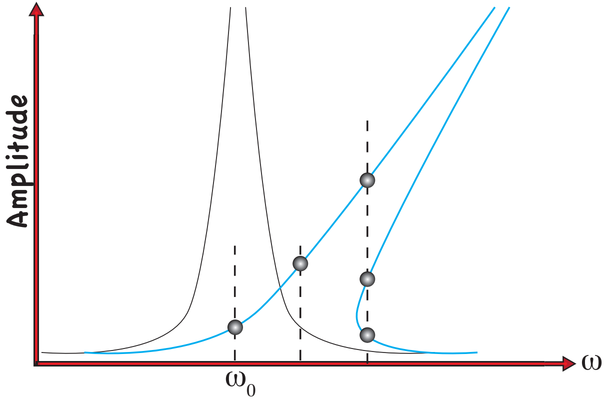
$$\overline{\psi \partial_x \eta} = \nu(U^* - U)$$

Self-tuning resonance



Anharmonic oscillator and "bent resonance"

$$\frac{d^2Y}{dt^2} + \omega_0^2(1+\epsilon Y^2)Y = \cos\omega t$$



All these equilibria are unstable

(... when you include baroclinicity)

- Need to consider interaction between planetary wave and synoptic eddies
- "Weather Regimes"
  (cf Reinhold and Pierrehumbert *MWR* 1981)

But modern dynamical systems theory also tells us that unstable fixed points can be very important in organizing the geometry of the attractor!

Planetary waves and resonance: Recurrent themes

714 MONTHLY WEATHER REVIEW VOLUME 107

A Theory of Stationary Long Waves. Part I: A Simple Theory of Blocking

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(Manuscript received 20 December 1977, in final form 24 February 1979)

ABSTRACT

A theory is presented that attempts to explain the tropospheric blocking phenomenon caused by the resonant amplification of large-scale planetary waves forced by topography and surface heating. It is shown that a wave becomes resonant with the stationary forcings when the wind condition in the lower atmosphere is such that the phase speed of the wave is reduced to zero. The resonant

Tellus (1988), *40A*, 177–187

On the resonance of an external planetary wave and its interaction with the mean zonal flow

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Planetary waves and resonance: Recurrent themes

Revisited with increasing sophistication, both for climatological mean and low frequency variability, with and without resonance theme

- Conditions for reflecting tropical boundary
  PV gradients and "index of refraction, PV homogenization (Haynes and McIntyre)
- Linearization of primitive equations on a sphere
  Nigam and Held; Jacqmin and Lindzen
- Linearization about zonally varying flows
- Forcing by rectified synoptic eddy fluxes

Resonance and weather extremes

PNAS

Quasiresonant amplification of planetary waves and recent Northern Hemisphere weather extremes

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Contributed by Hans Joachim Schellnhuber, January 16, 2013 (sent for review June 15, 2012)

In recent years, the Northern Hemisphere has suffered several devastating regional summer weather extremes, such as the European heat wave in 2003, the Russian heat wave and the Indus river flood in Pakistan in 2010, and the heat wave in the United States in 2011.

I. Quasiresonance Hypothesis

Generally the large-scale midlatitude atmospheric circulation is characterized by (*i*) traveling free synoptic-scale Rossby waves with zonal wave numbers $k > 6$ propagating predominantly in the

- Claim is that jet changes associated with global warming have made resonance more prevalent
- Trends in (barotropic) "resonance index" examined by Mann et al. 2017 *Nature Scientific Reports*

But it uses circa-1980 math!

II. Methods: Planetary Wave Quasiresonance Theory

In our study, we use a linearized nonstationary, nondivergent, barotropic vorticity equation on a sphere (23) at the equivalent barotropic level (EBL). Taking into account the assumed height [500–300 hPa, (20, 24)] of the EBL, we write this equation in a quasigeostrophic approximation as follows (see also Eq. **S1a**):

$$\left(\frac{\partial}{\partial t}+\alpha\frac{\partial}{\partial\lambda}\right)\Delta\Psi' + \left(2\Omega - \frac{\Delta\bar{u}}{a\cos\varphi}\right)\frac{\partial\Psi'}{\partial\lambda} = \frac{2\Omega\alpha a^2}{\tilde{T}}\sin\varphi\frac{\partial T'}{\partial\lambda} - \frac{2\Omega\sin\varphi\,\alpha_{or}a^2}{H}\frac{\partial h_{or}}{\partial\lambda} - \left(k_h\frac{\Delta\Psi'}{a^2}+k_z\frac{\Delta\Psi'}{H^2}\right). \quad [1]$$

Conclusions

I don't think Petoukhov has revived resonance

- Barotropic model insufficient to address resonance
- "Resonances" are shortwave – would they even exist in a baroclinic model?
- Shortwave stationary waves will interact very strongly with similar scale synoptic eddies

... but if resonance does revive

It brings back multiple equilibrium ideas with it. Note that we should pay attention to the importance of unstable fixed points in dynamical systems

Global warming and planetary waves

Without doubt, it is interesting and important to think about the effect of jet changes on the stationary waves.