

On the use of a TKE equation to compute subgrid fluxes in the 'Grey Zone'

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&

CONSTRAIN case collaborators

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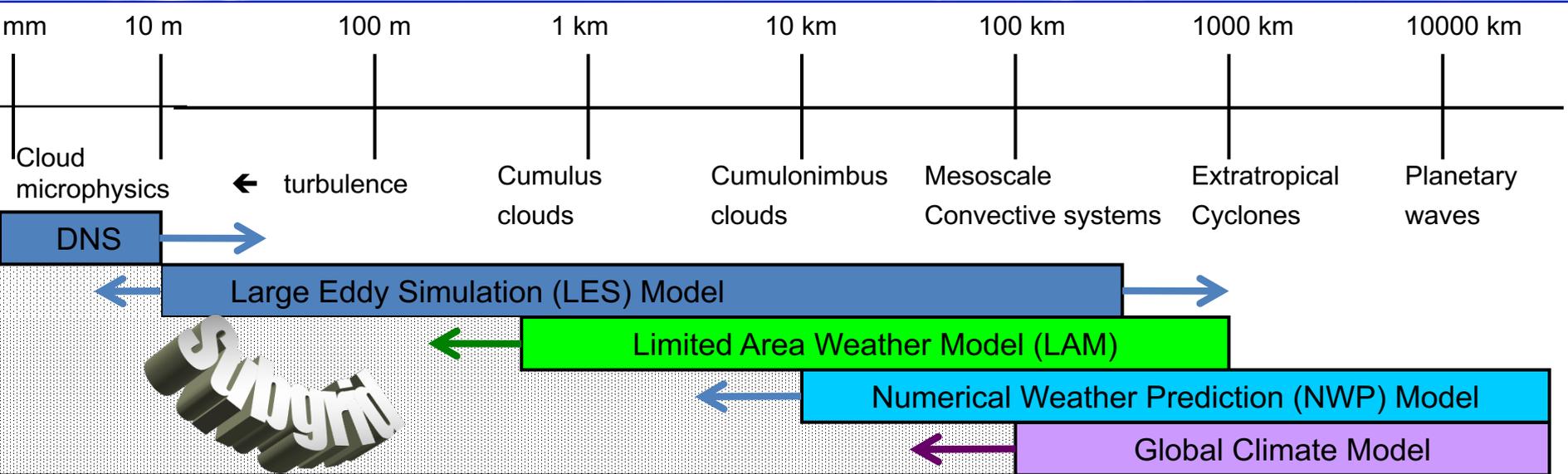
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Rachel Honnert, Christine Lac, *Meteo France*, Toulouse, France

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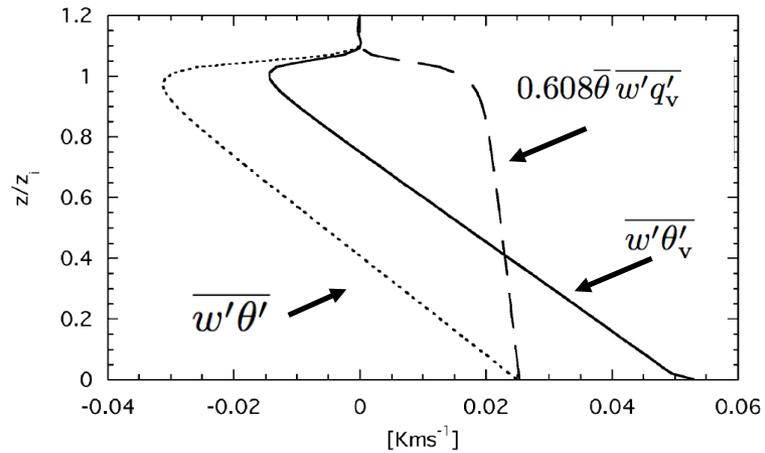
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Current developments



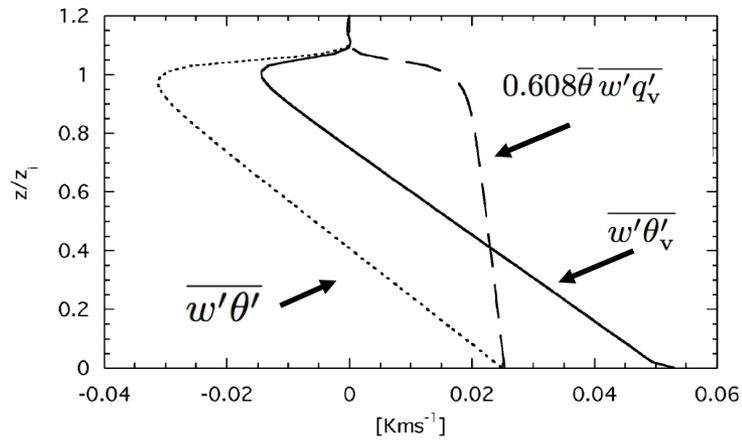
- when does turbulence cause the growth of mesoscale fluctuations?
- how does LES behave if Δx becomes very large ("LAM" or "NWP" limit)?
- is the Grey Zone case dependent?
examples for the stable, convective and cloud-topped boundary layer

A Convective Boundary Layer driven by a surface temperature and humidity flux

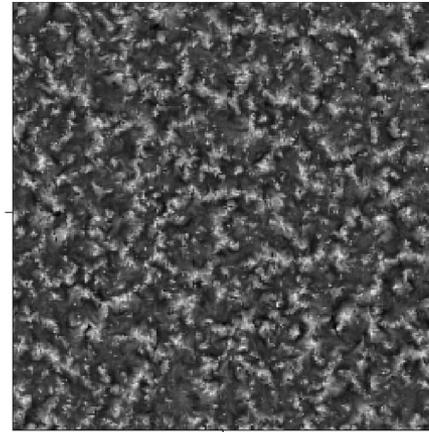


$$\overline{w'\theta'_v} \approx \overline{w'\theta'} + 0.608\overline{\theta} \overline{w'q'_v}$$

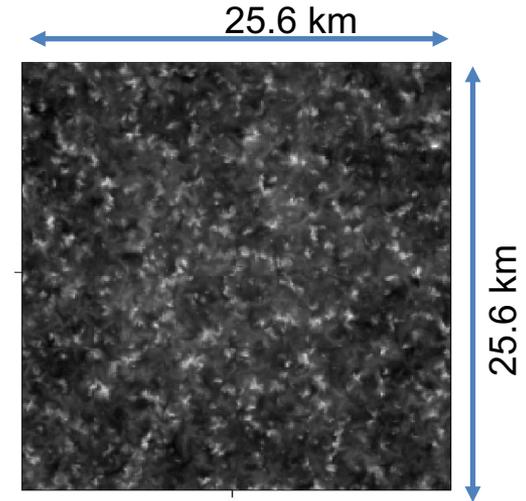
A Convective Boundary Layer driven by a surface temperature and humidity flux



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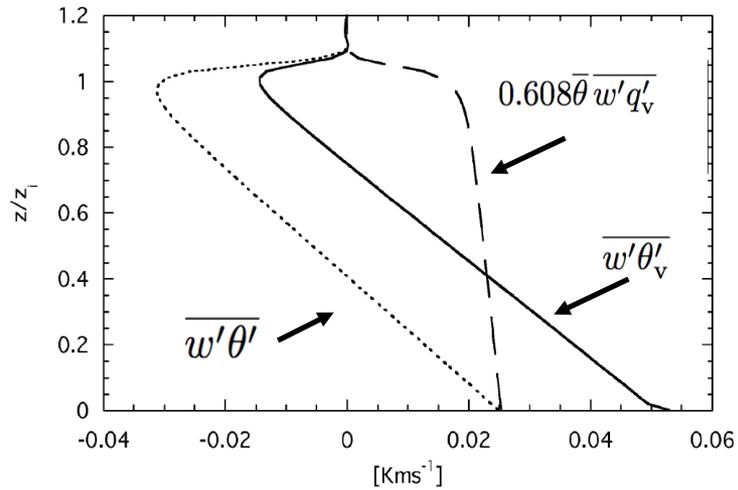


-1.0 0.0 1.0 2.0 3.0
vertical velocity (m/s)



0.0 0.1 0.2 0.3
virtual potential temperature (K)

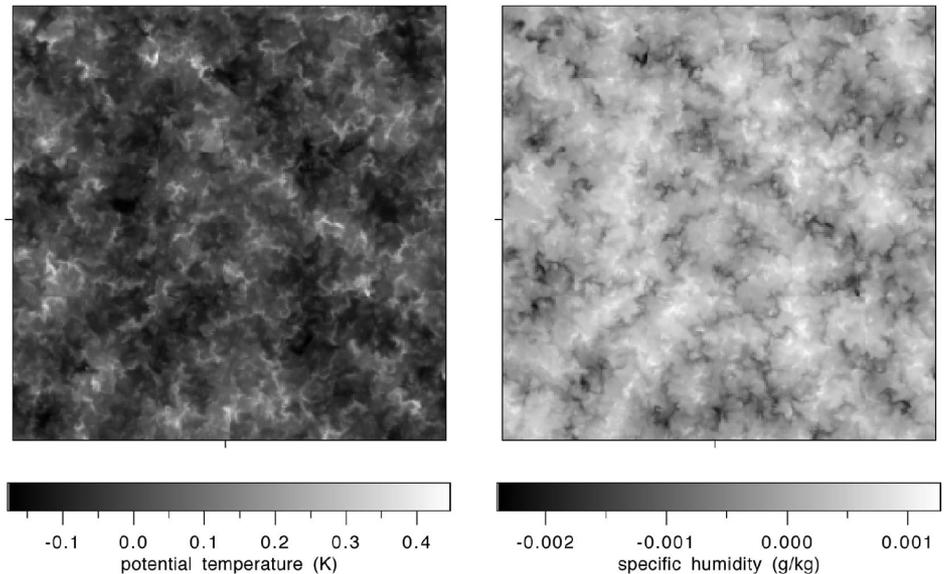
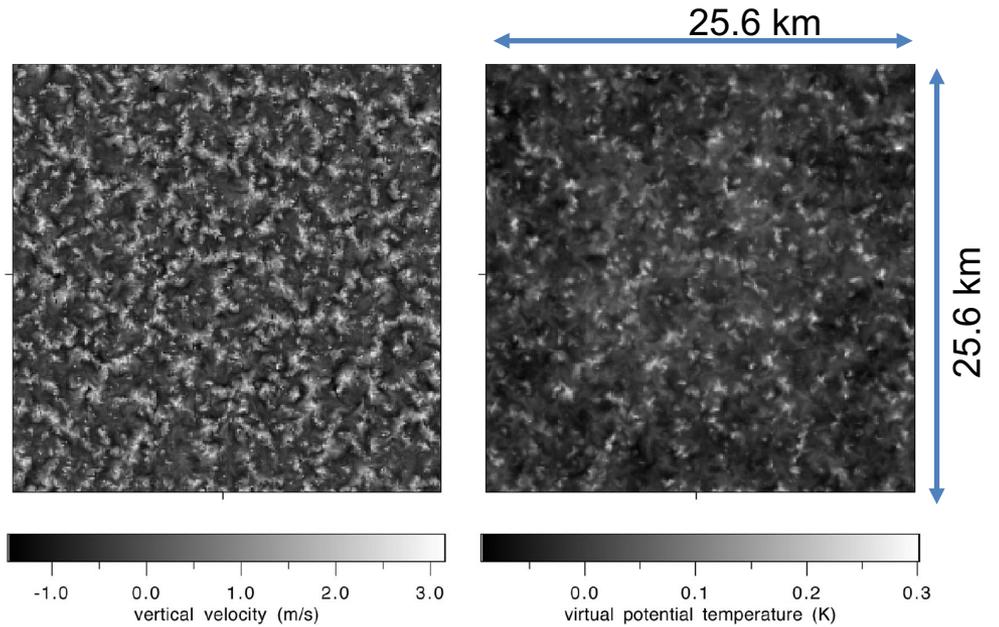
A Convective Boundary Layer driven by a surface temperature and humidity flux



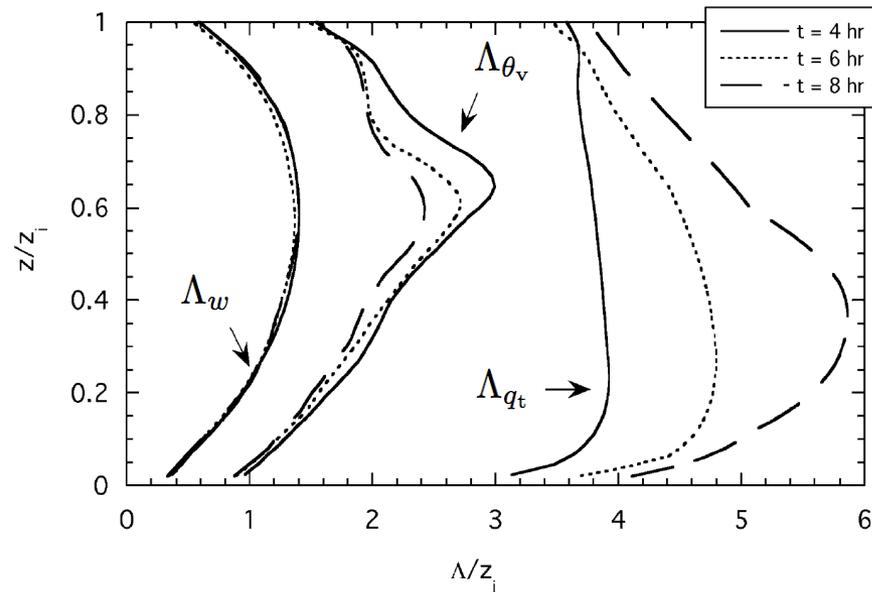
$$\overline{w'\theta'_v} \approx \overline{w'\theta'} + 0.608\theta'\overline{w'q'_v}$$

typical length scales

boundary layer size z_i : w, θ_v
 mesoscales $> z_i$: θ, q_v



Length scale (Λ , from spectra) evolution in time



Variance production terms

dynamics

$$\frac{\partial \overline{w'w'}}{\partial t} = \beta \overline{w'\theta'_v}$$

$$\frac{\partial \overline{\theta'_v\theta'_v}}{\partial t} = -2\overline{w'\theta'_v} \frac{\partial \overline{\theta}_v}{\partial z}$$

arbitrary scalar

$$\frac{\partial \overline{\chi'\chi'}}{\partial t} = -2\overline{w'\chi'} \frac{\partial \overline{\chi}}{\partial z}$$

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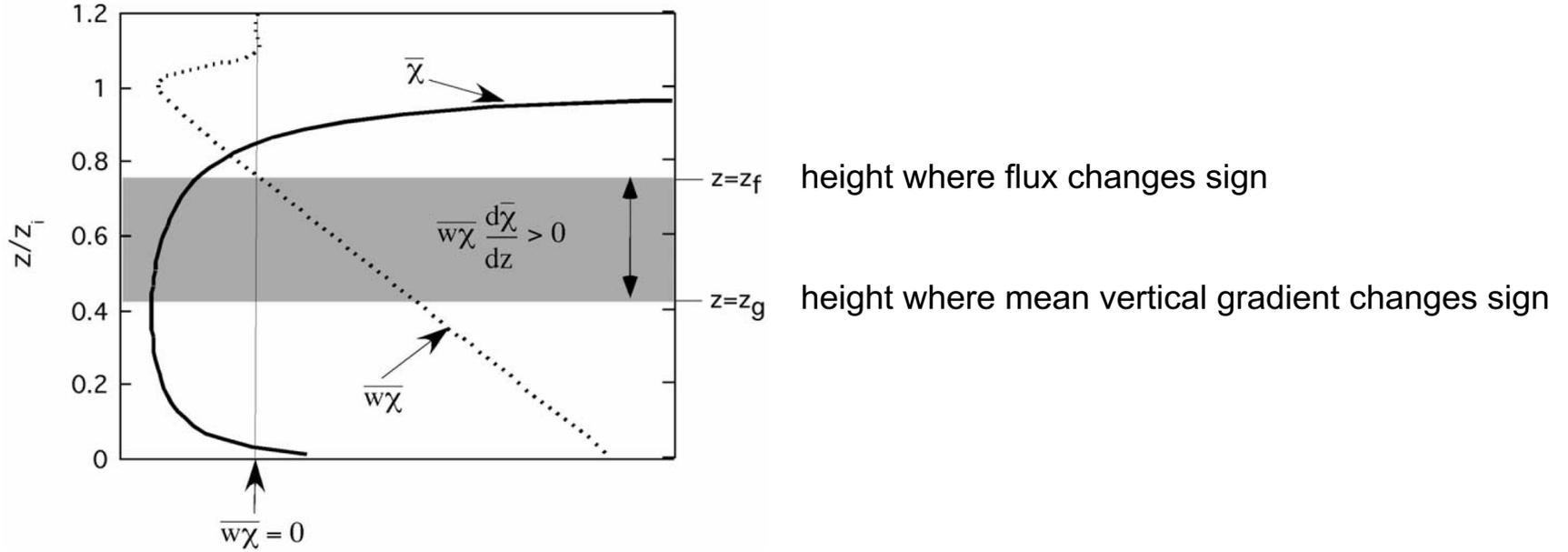
if flux is down the mean gradient

$$\overline{w'\chi'} = -K_h \frac{\partial \overline{\chi}}{\partial z}$$

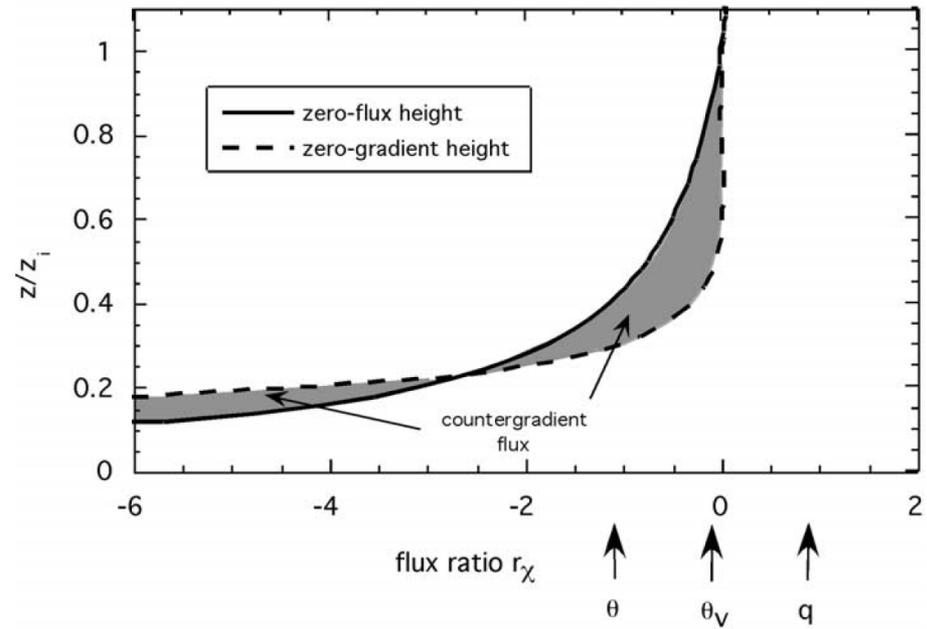
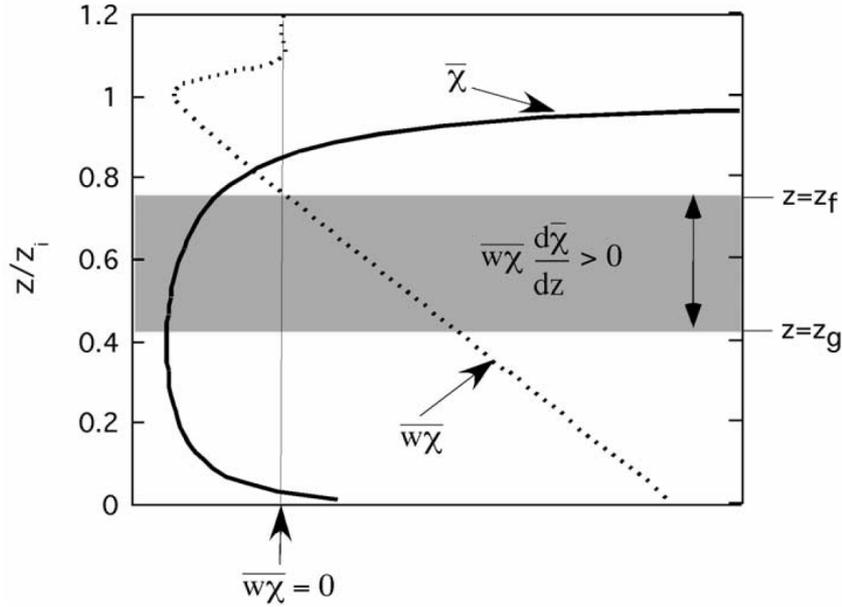
then variance will be produced

$$\frac{\partial \overline{\chi'\chi'}}{\partial t} = +2K_h \left(\frac{\partial \overline{\chi}}{\partial z} \right)^2 > 0$$

Countergradient regime

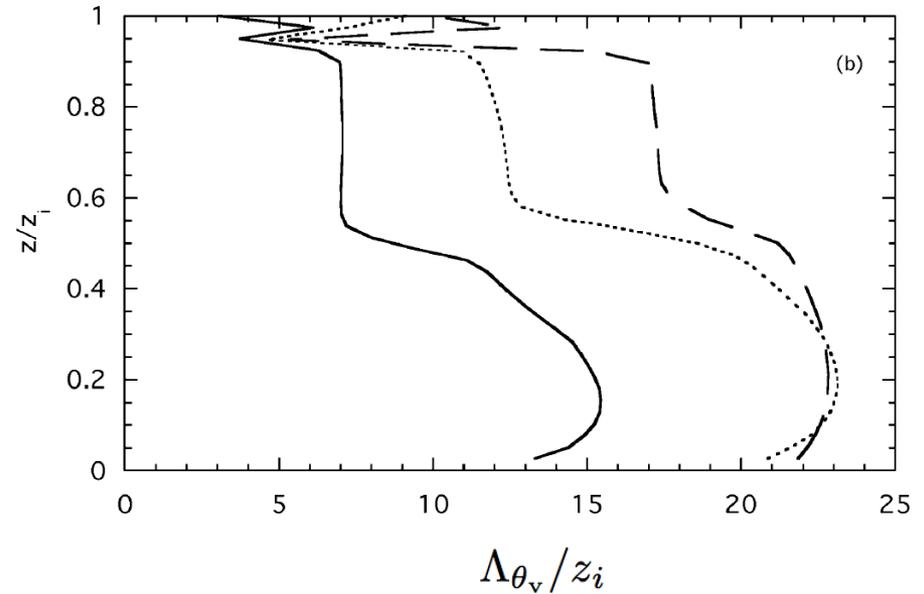
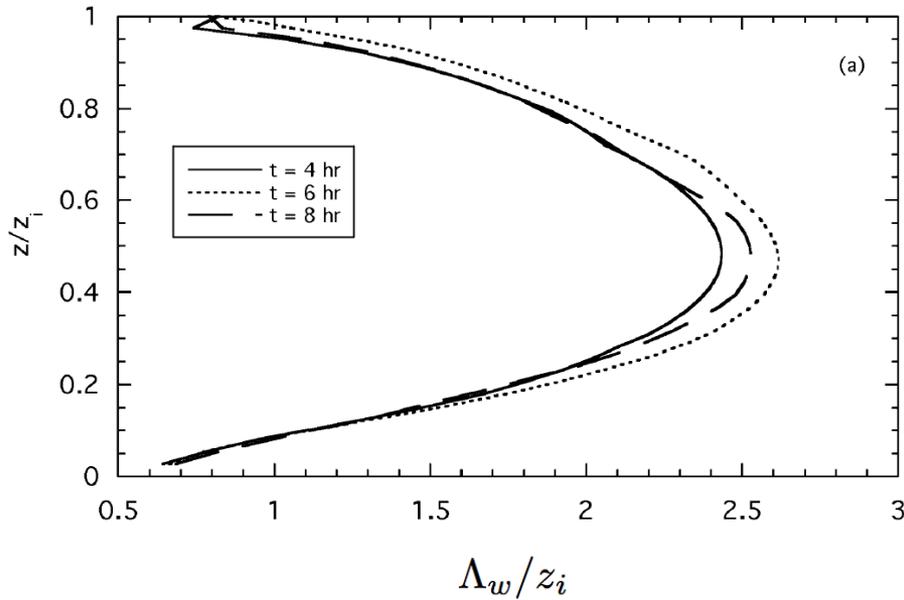


Countergradient regime as a function of the flux ratio flux ratio = top/bottom flux)



in the interior of the convective boundary layer the countergradient flux destroys θ_v variance

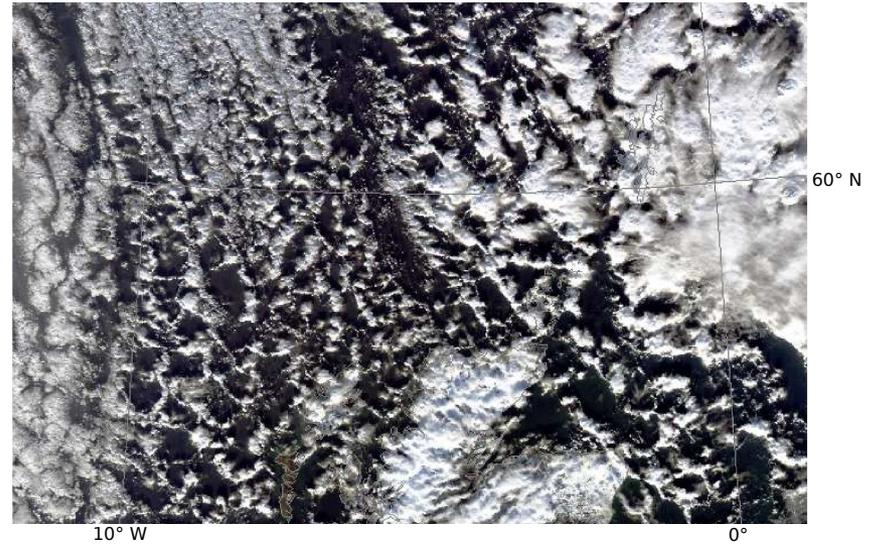
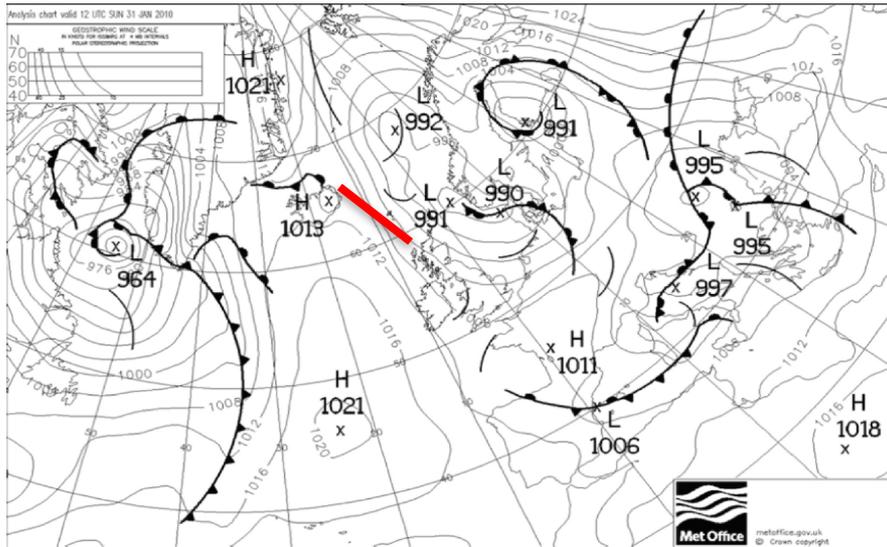
Stratocumulus



$$\theta'_v = A_w \theta'_1 + B_w q'_t \approx B_w q'_t$$

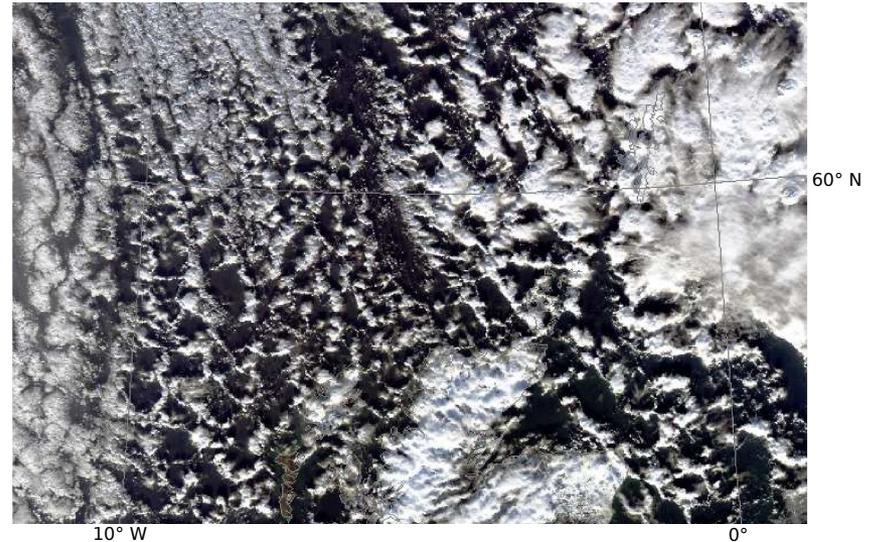
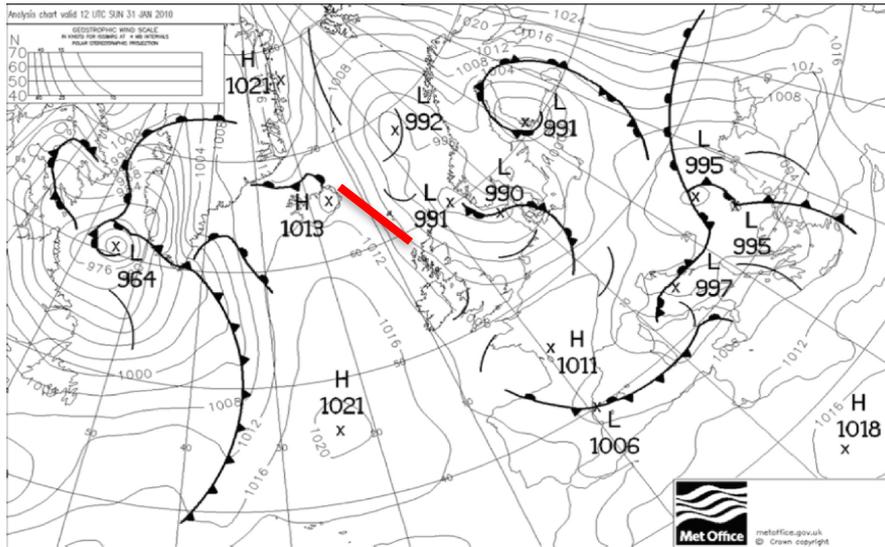
$$\frac{\partial \overline{q'_t q'_t}}{\partial t} = -2 \overline{w' q'_t} \frac{\partial \overline{q_t}}{\partial z} > 0 \text{ throughout the cloud layer}$$

CONSTRAIN Cold Air Outbreak (Field et al. 2017)



January 31, 2010 12:53 UTC

CONSTRAIN Cold Air Outbreak (Field et al. 2017)



January 31, 2010 12:53 UTC

Lagrangian Large-Eddy Simulations (6 participating groups)

Horizontal domain 100x100 km²

Horizontal grid size 200 m

Interactive radiation, SST increases with time, with and without ice microphysics

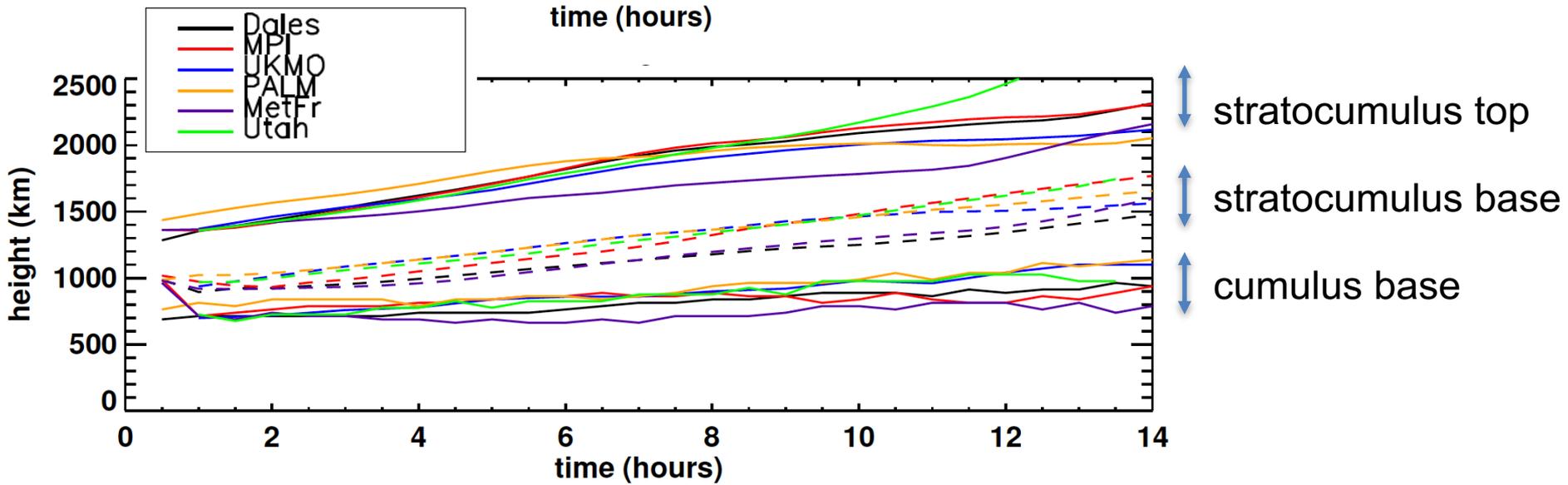
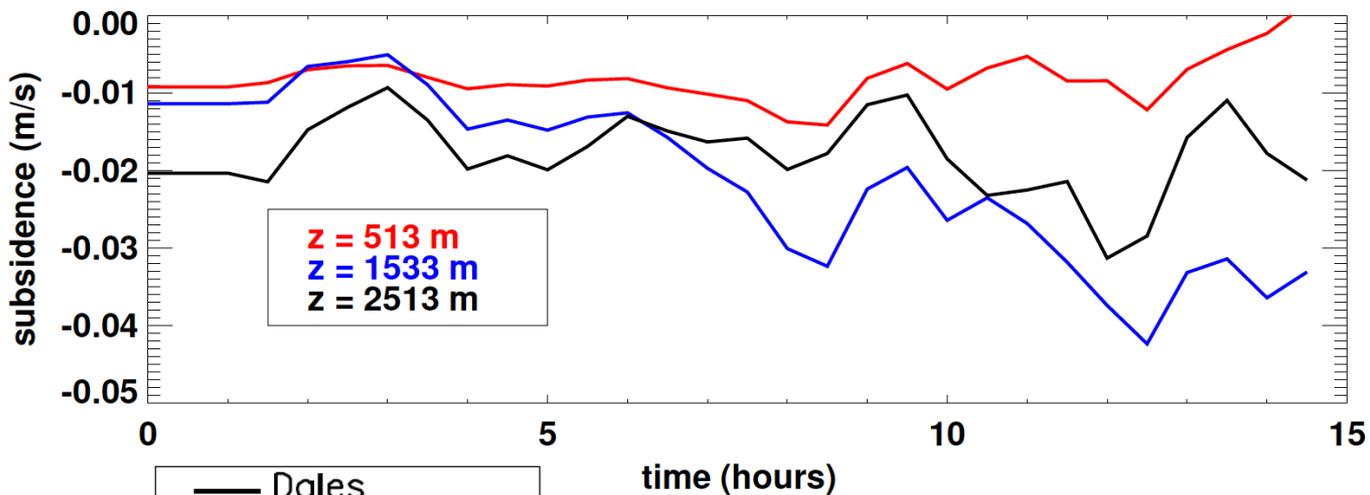
$\Delta z=25$ m up to 3 km, stretched grid above

Requested additional LES runs:

Varying cloud droplet concentration number

Coarsening horizontal grid size "NWP mode" (0.2, 0.4, 0.8, 1.6, 3.2 km)

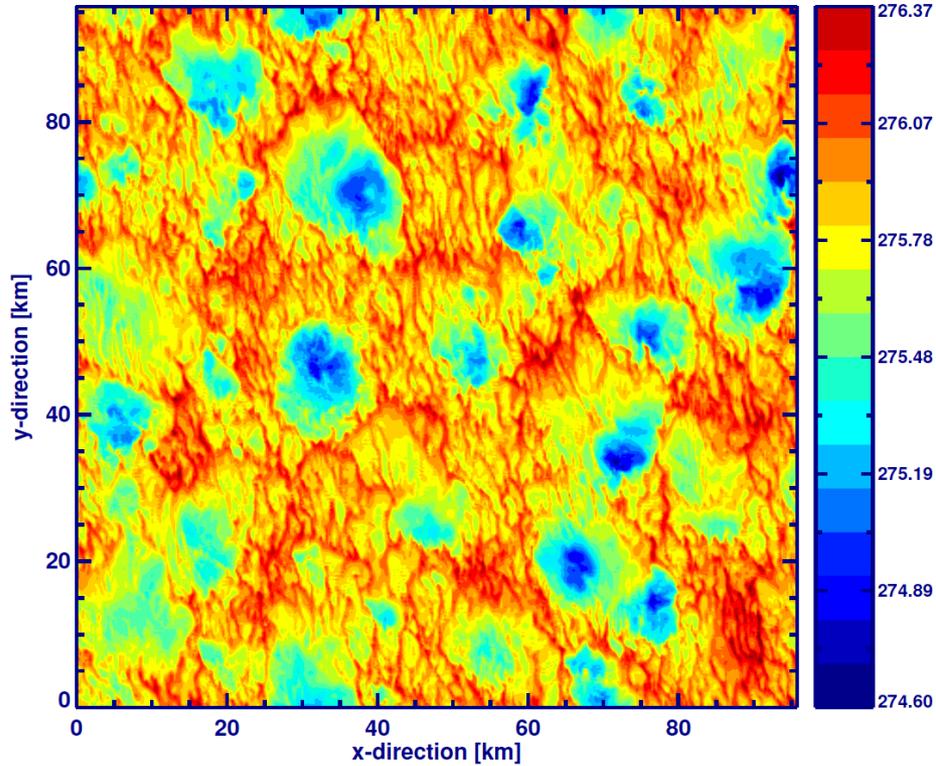
CONSTRAIN Cold Air Outbreak



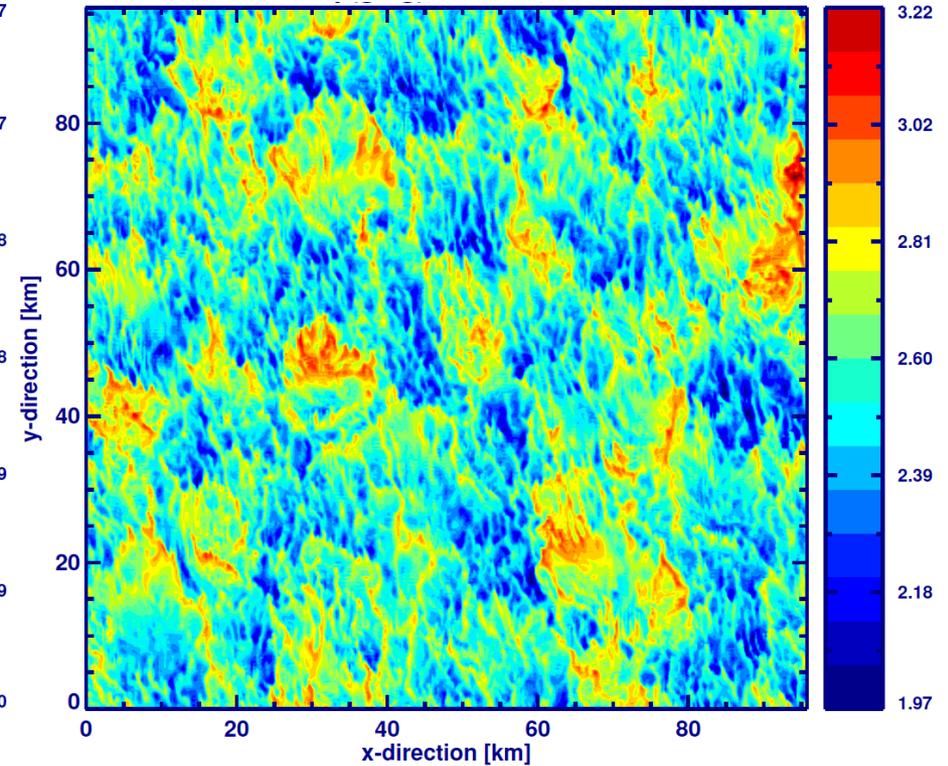
Cold pool formation (example from the UCLA-LES model)

$t = 12 \text{ h}, z = 100 \text{ m}$

potential temperature θ (K)



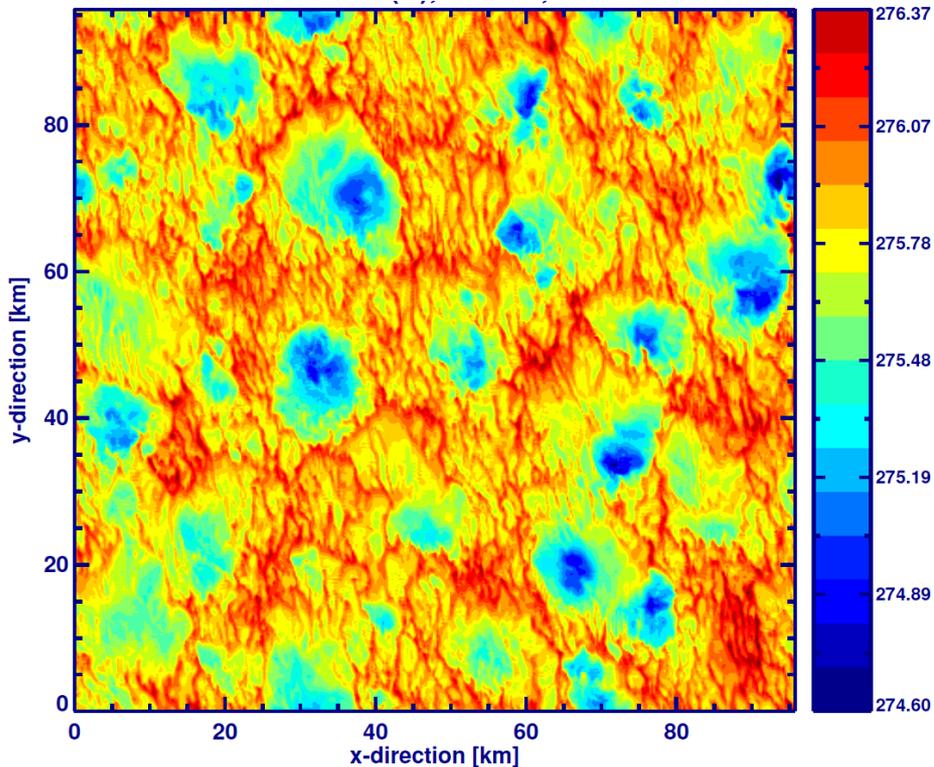
specific humidity q_v (g/kg)



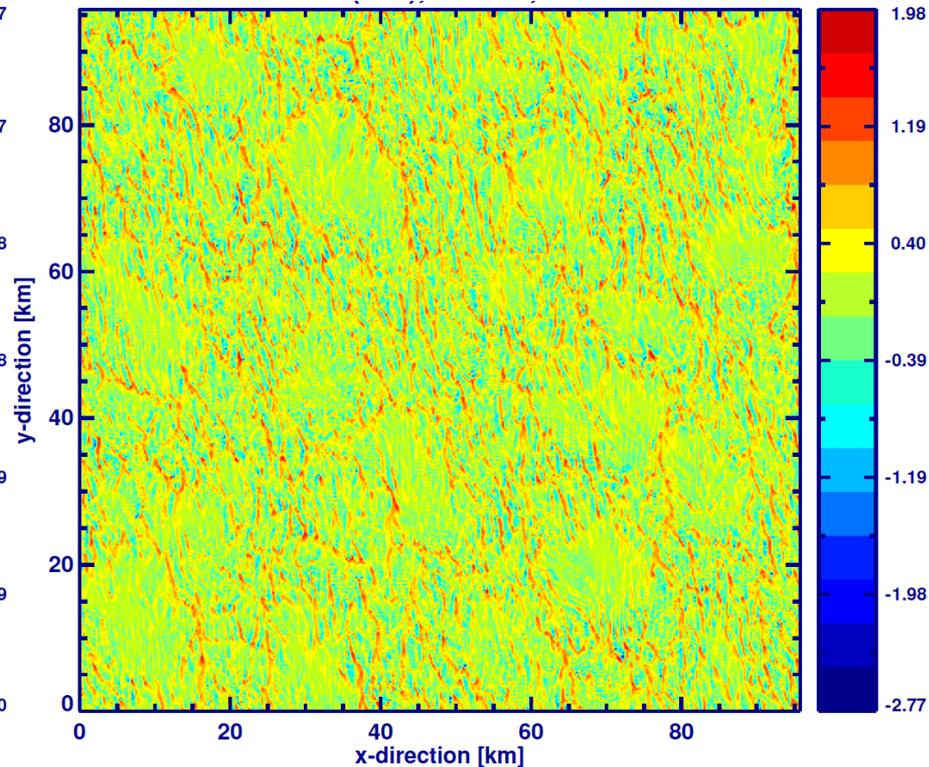
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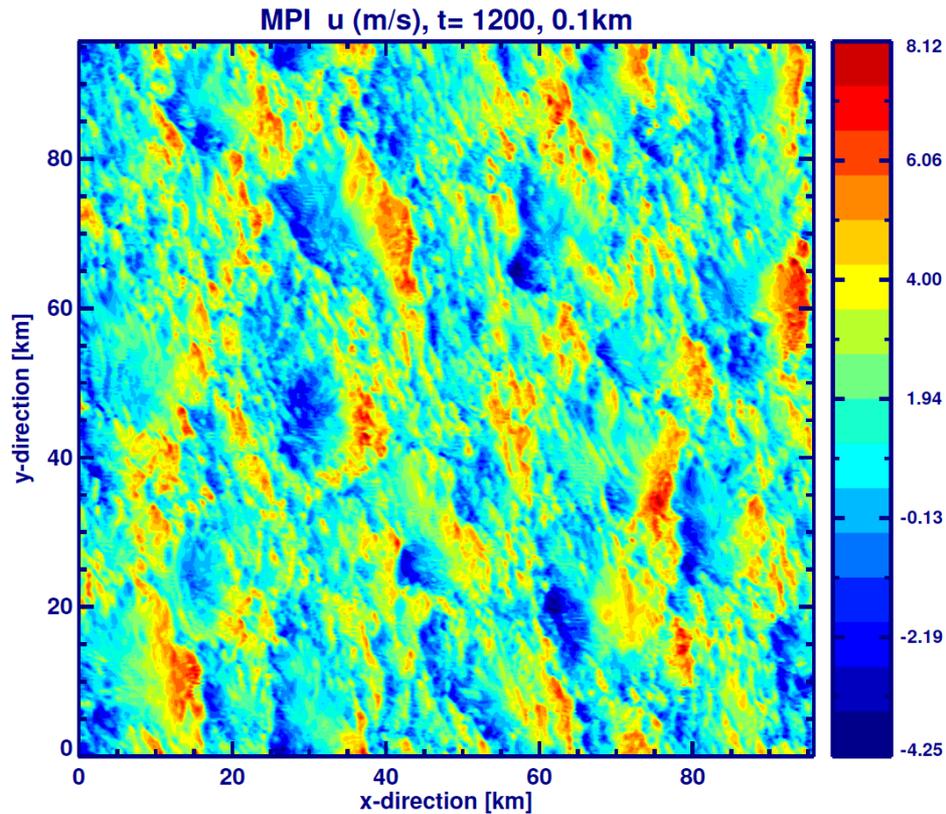
potential temperature θ (K)



vertical velocity w (m/s)



Horizontal wind velocity (note u ranges between -4 and 8 m/s)



Equally large variations are found in the other LES model fields

Turbulent/convective flux in traditional NWP

$$\overline{w'\phi'}_{\text{total}} = \overline{w'\phi'}_{\text{resolved}} + \overline{w'\phi'}_{\text{subgrid}}$$



zero for sufficiently large Δx

Turbulent/convective flux in very high resolution NWP

$$\overline{w'\phi'}_{\text{total}} = \overline{w'\phi'}_{\text{resolved}} + \overline{w'\phi'}_{\text{subgrid}}$$



becomes non-zero for sufficiently fine Δx

Turbulent/convective flux in very high resolution NWP

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should reduce accordingly

Turbulent/convective flux in very high resolution NWP

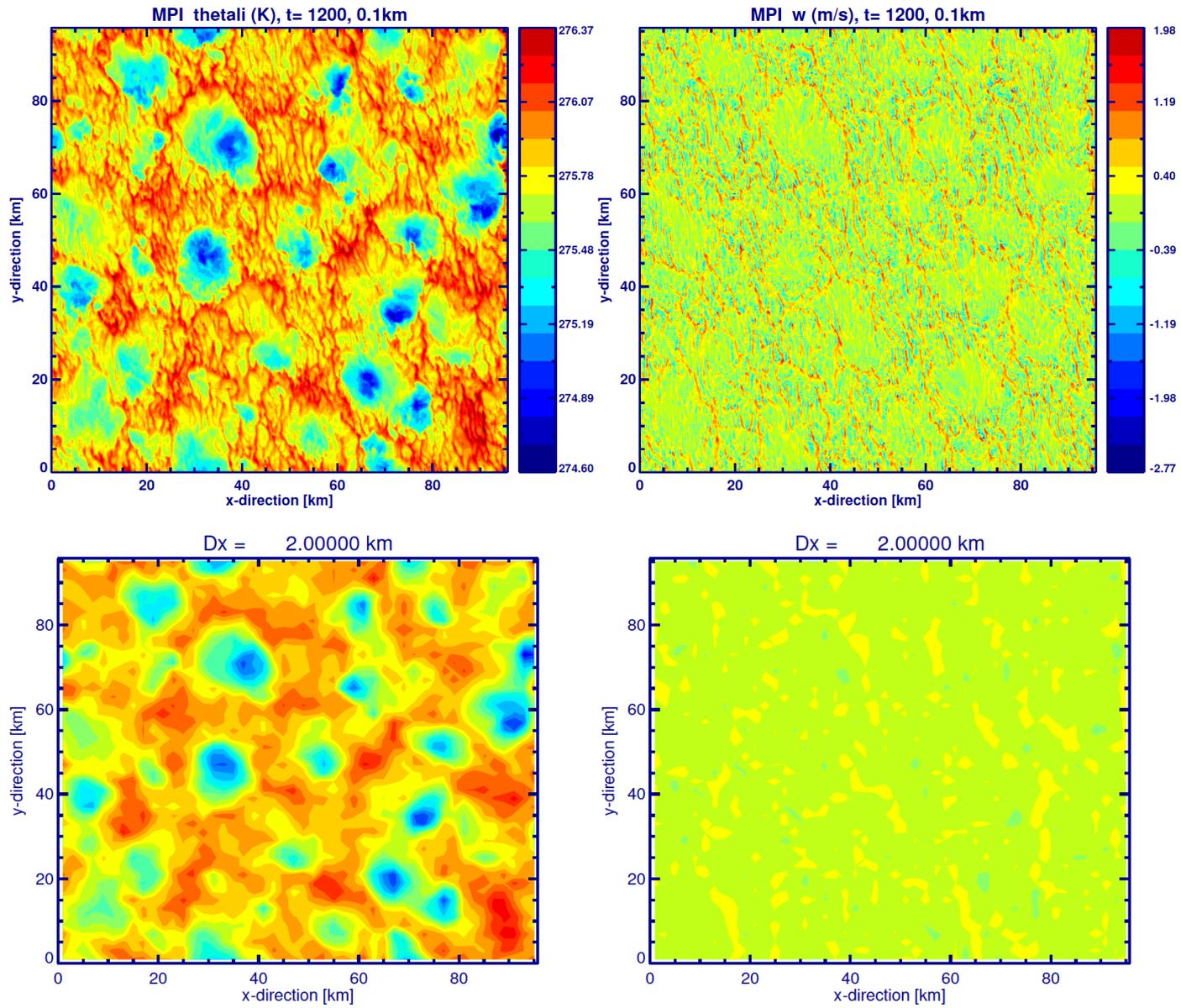
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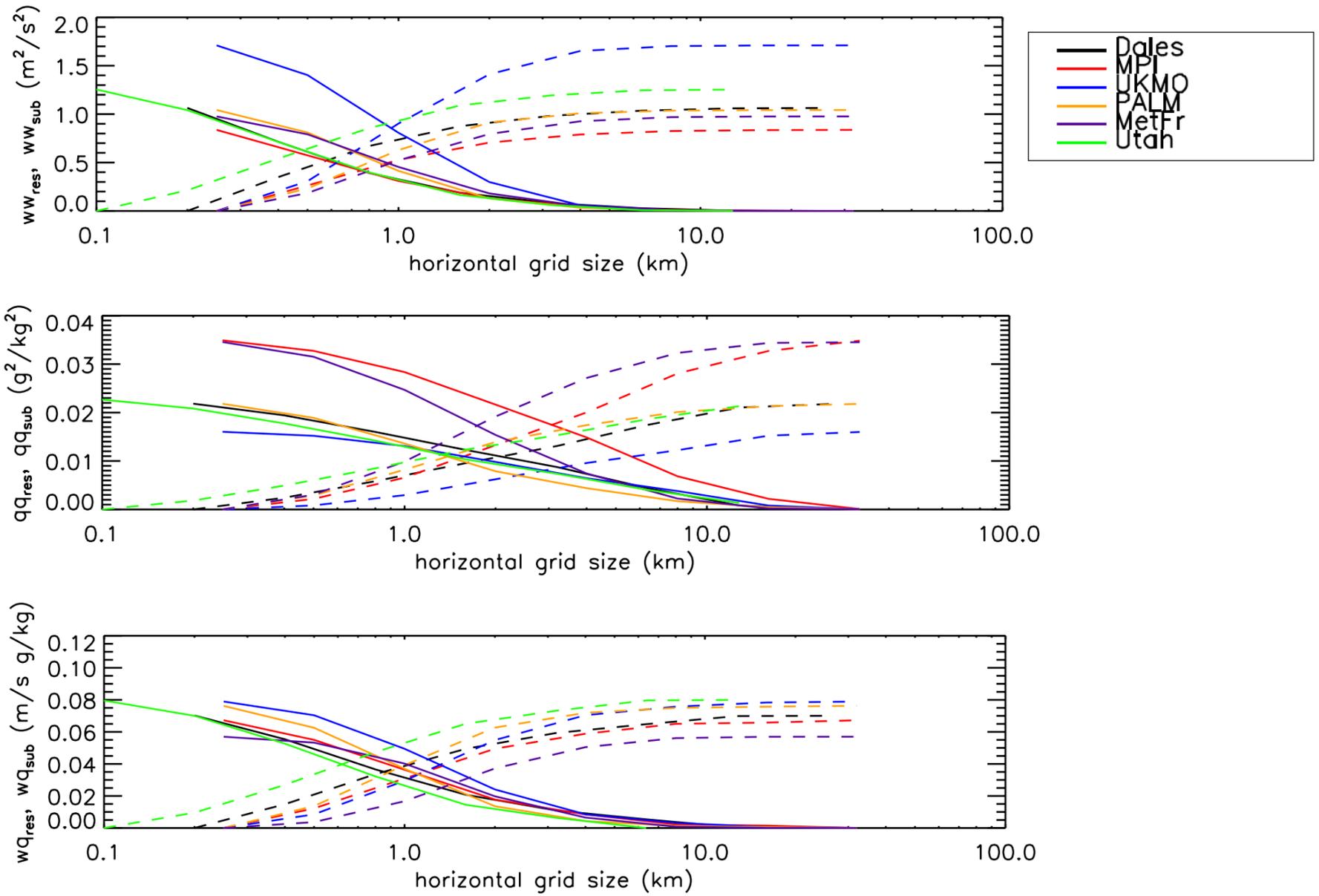
Diagnose resolved and subgrid flux from LES fields as a function of the horizontal grid size Δx that the NWP would use

Coarse graining the fields: example

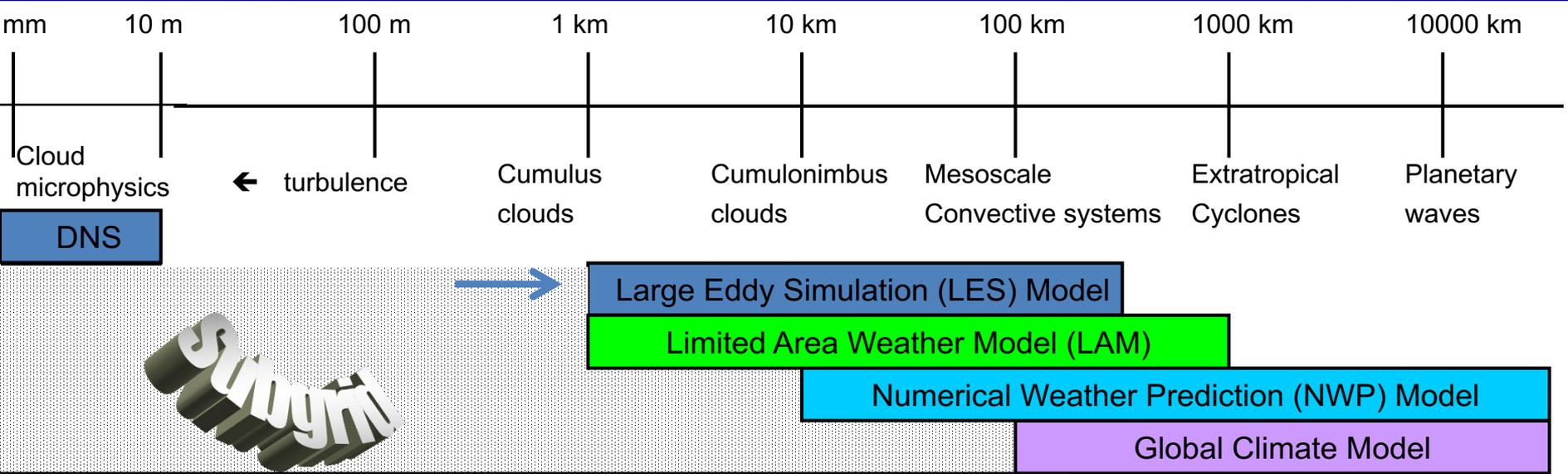


Coarse graining the fields

Resolved (res) and subgrid (sub) fluxes



Run LES in a "LAM" or "NWP" mode



NWP:
TKE all eddies

$$\frac{\partial E}{\partial t} = - \left(\overline{u'w'} \frac{\partial \bar{u}}{\partial z} + \overline{v'w'} \frac{\partial \bar{v}}{\partial z} \right) + \frac{g}{\theta_v} \overline{w'\theta'_v} - \frac{\partial}{\partial z} \left(\overline{w'E} + \overline{w'p'/\bar{\rho}} \right) - \varepsilon$$

LES:
TKE subgrid eddies

$$\frac{\partial e}{\partial t} + u_j \frac{\partial e}{\partial x_j} = \frac{g}{\theta_0} \widetilde{w''\theta''_v} - \widetilde{u''_i u''_j} \frac{\partial u_i}{\partial x_j} - \frac{\partial \widetilde{u''_j e}}{\partial x_j} - \frac{1}{\rho_0} \frac{\partial \widetilde{u''_j p''}}{\partial x_j} - \varepsilon$$

TKE closure for subgrid fluxes (example of HARMONIE)

$$\frac{\partial E}{\partial t} = - \left(\overline{u'w'} \frac{\partial \bar{u}}{\partial z} + \overline{v'w'} \frac{\partial \bar{v}}{\partial z} \right) + \frac{g}{\theta_v} \overline{w'\theta'_v} - \frac{\partial}{\partial z} \left(\overline{w'E} + \overline{w'p'/\bar{\rho}} \right) - \varepsilon$$

$$\overline{w'\psi'} = -K_\psi \frac{\partial \bar{\psi}}{\partial z} \quad \text{downgradient flux}$$

$$K_\psi = l_\psi \sqrt{E}$$

$$\frac{1}{l_{m,h}} = \frac{1}{c_n \kappa z} + \frac{1}{l_s}$$

length scale depends on **size of the eddies**

TKE (e) closure for subgrid fluxes (\sim) in an LES model

$$\frac{\partial e}{\partial t} + u_j \frac{\partial e}{\partial x_j} = \frac{g}{\theta_0} \widetilde{w''\theta_v''} - \widetilde{u_i''u_j''} \frac{\partial u_i}{\partial x_j} - \frac{\partial \widetilde{u_j''e}}{\partial x_j} - \frac{1}{\rho_0} \frac{\partial \widetilde{u_j''p''}}{\partial x_j} - \varepsilon$$

$$\widetilde{u_j''\varphi''} = -K_h \frac{\partial \varphi}{\partial x_j} \quad \text{downgradient flux}$$

$$K_{m,h} = c_{m,h} \lambda e^{1/2}$$

$$\lambda = l_\Delta \equiv (\Delta x \Delta y \Delta z)^{1/3} \quad \text{length scale depends on **grid size**}$$

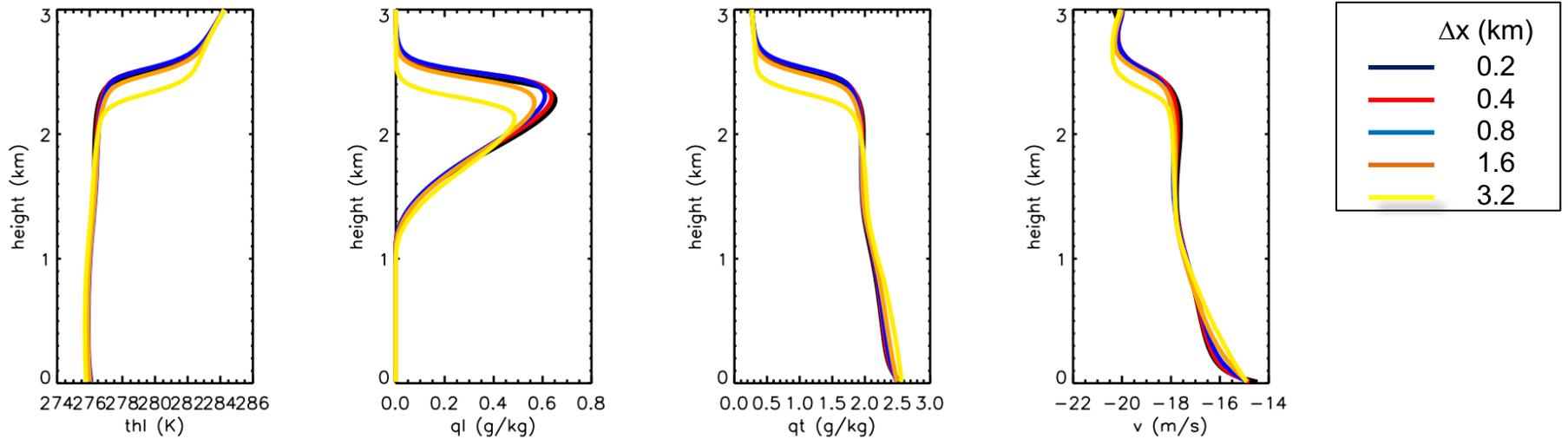
Analytical solution for LES subgrid TKE (steady-state, zero turbulent transport)

$$K_{m,\Delta} = c_s^2 \left[1 - \frac{Ri_g}{Ri_{C,\Delta}} \right]^{1/2} l_\Delta^2 S,$$

Eddy mixing depends on grid size $l_\Delta \equiv (\Delta x \Delta y \Delta z)^{1/3}$.

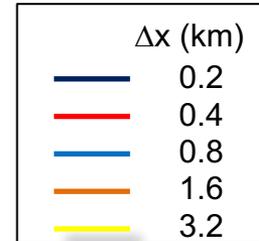
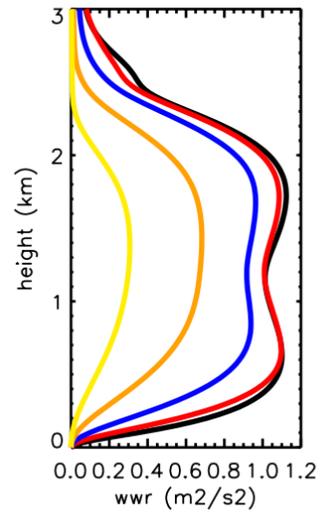
DALES results for $\Delta x = 0.2, 0.4, 0.8, 1.6$ and 3.2 km

Note: Stratocumulus clouds are dominating



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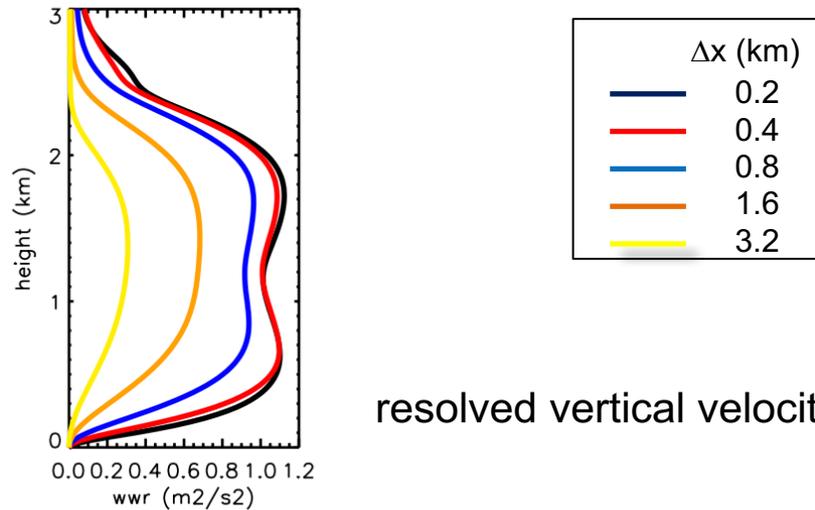
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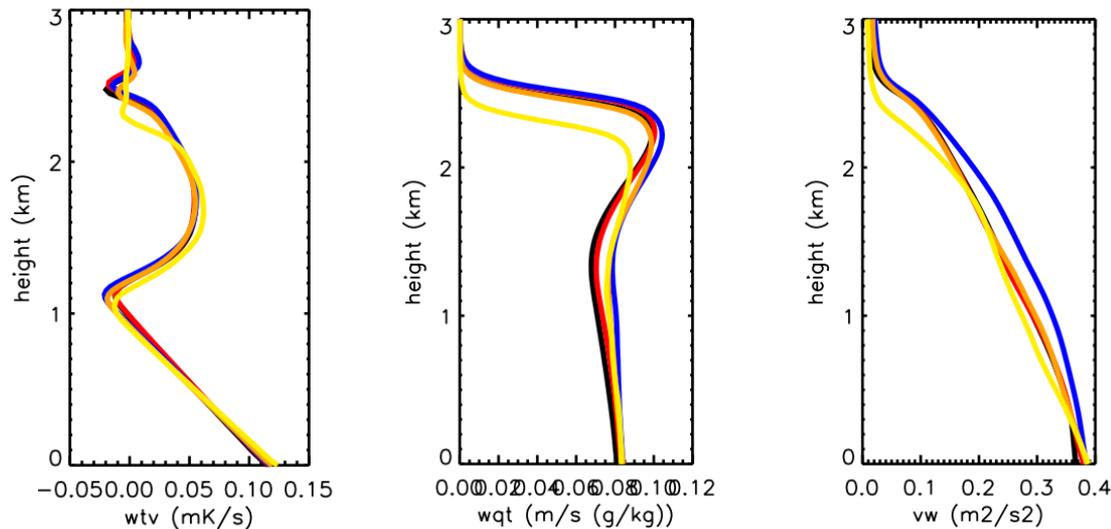
resolved vertical velocity variance

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resolved vertical velocity variance



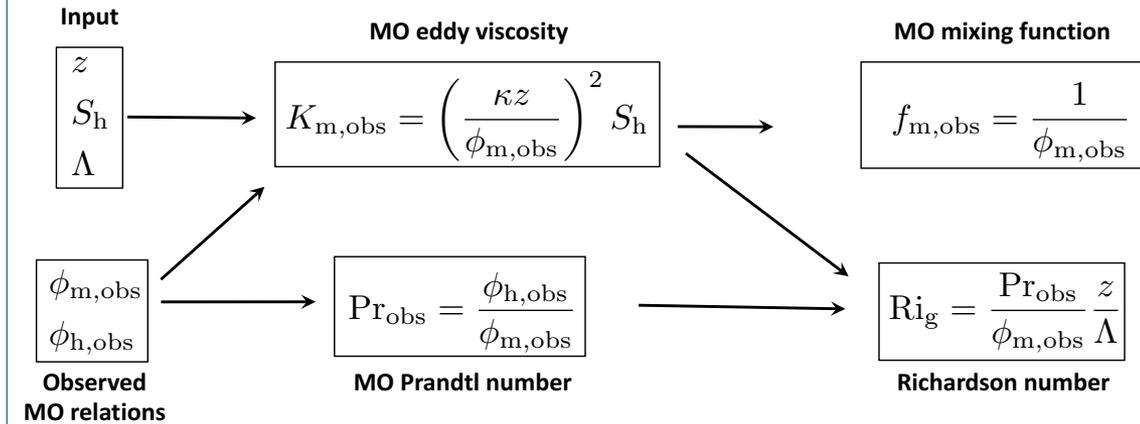
total fluxes (resolved + subgrid)

Analytical solution LES subgrid TKE model in terms of the mixing function f_m for a stable stratification

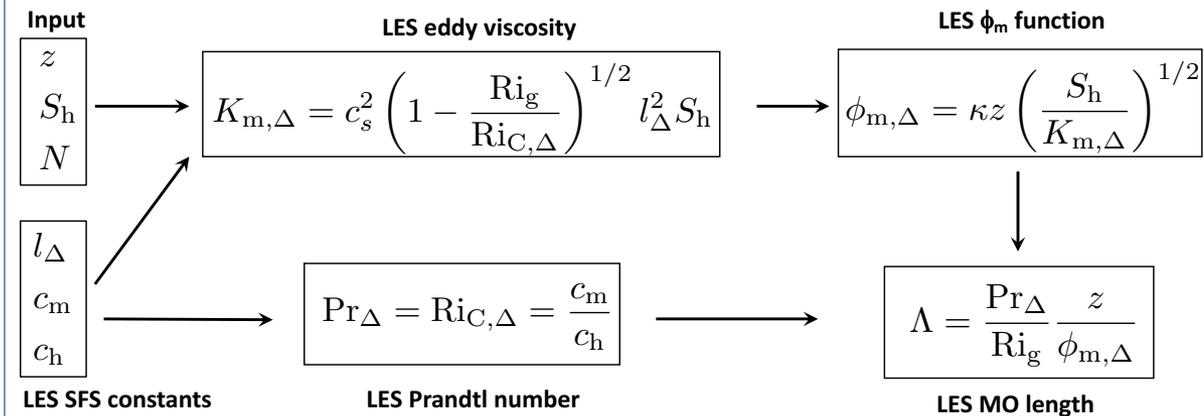
$$K_m = f_m^2 (\kappa z)^2 \left| \frac{\partial U}{\partial z} \right|$$

$$f_{m,\Delta} = c_s \left(1 - \frac{Ri_g}{Ri_{C,\Delta}} \right)^{1/4} \frac{l_\Delta}{\kappa z}.$$

From the observed MO-z/ Λ similarity relations to the Richardson number dependent mixing function



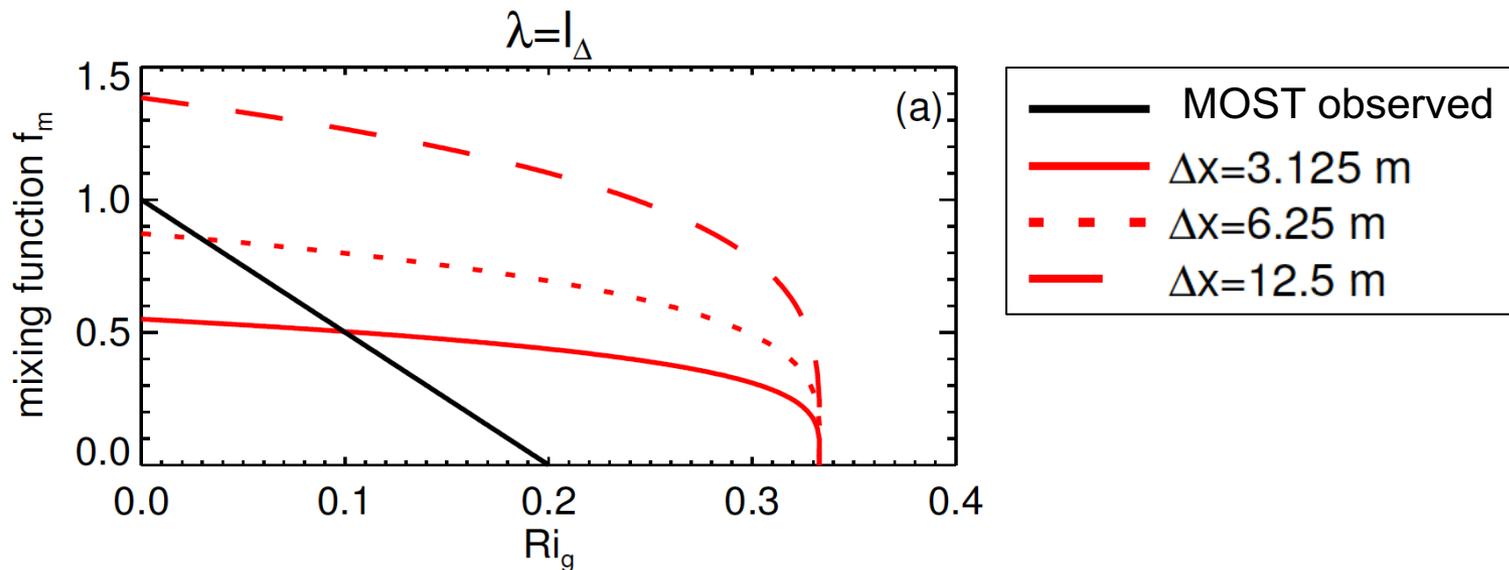
From the LES SFS TKE equation to its MO-z/ Λ similarity function



Analytical solution LES subgrid TKE model in terms of the mixing function f_m for a stable stratification

$$K_m = f_m^2 (\kappa z)^2 \left| \frac{\partial U}{\partial z} \right|$$

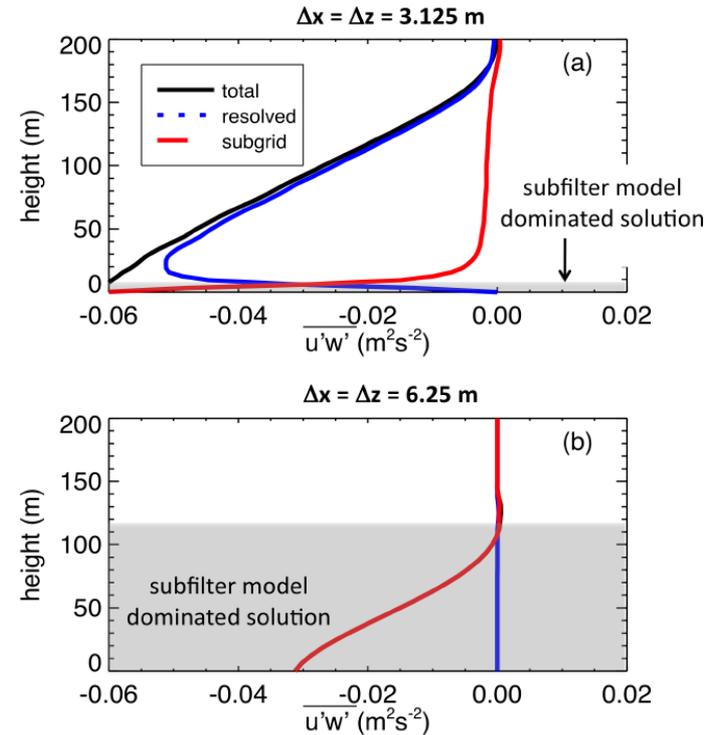
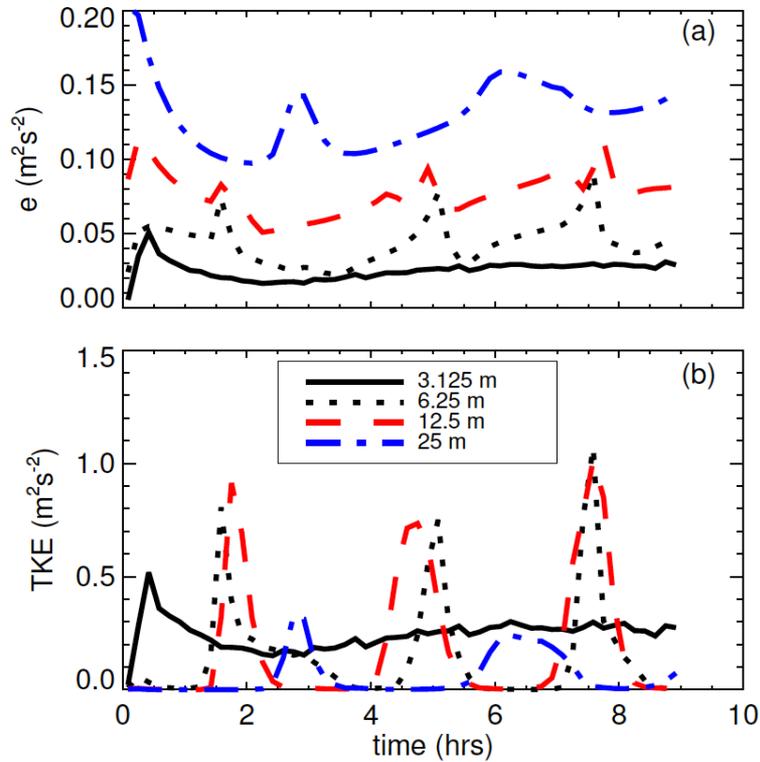
$$f_{m,\Delta} = c_s \left(1 - \frac{Ri_g}{Ri_{C,\Delta}} \right)^{1/4} \frac{l_\Delta}{\kappa z}$$



Larger $\Delta x/\Delta z$ leads to excessive mixing

Consequences of excessive mixing: resolved motions disappear and solution is controlled by subgrid TKE model

effect of changing Δx



Conclusions

CBL

- * Mesoscale fluctuations in buoyancy very small ("countergradient regime")
- * Mesoscale growth if scalar flux is down its mean gradient

Stratocumulus

- * Positive feedback $q_t \rightarrow \theta_v \rightarrow w$

Performance subgrid TKE equation for large Δx

- * Good for stratocumulus (because large eddies)
- * Danger of excessive mixing, e.g. in the stable boundary layer (small eddies)

Outlook

- * The dependency of horizontal turbulent fluxes on Δx

$$\frac{\partial \bar{\chi}}{\partial t} = - \frac{\partial \overline{u' \chi'}}{\partial x} - \frac{\partial \overline{w' \chi'}}{\partial z} + \dots$$