SPECIAL PROJECT PROGRESS REPORT

Progress Reports should be 2 to 10 pages in length, depending on importance of the project. All the following mandatory information needs to be provided.

Reporting year	**2017**……………………………………………………
Project Title:	**Direct numerical simulation of wind wave fields in a rapidly changing environment** ……………………………………………………
Computer Project Account:	**SPGBSHRI** ……………………………………………
Principal Investigator(s):	**Prof V.I. Shrira** ………………………………………… ……………………………………………………
Affiliation:	**School of Computing and Mathematics, Keele University, Keele ST5 5BG UK** ………………………
Name of ECMWF scientist(s) collaborating to the project (if applicable)	…………………………………………………… ……………………………………………………
Start date of the project:	**01/01/2016**…………………………………………
Expected end date:	**31/12/2018**…………………………………………

Computer resources allocated/used for the current year and the previous one
(if applicable)
Please answer for all project resources

		Previous year		Current year	
		Allocated	Used	Allocated	Used
High Performance Computing Facility	(units)	**300000**	**300000**	**300000**	**10000**
Data storage capacity	(Gbytes)	**100**	**50**	**100**	**50**

Summary of project objectives

(10 lines max)

This project aims at developing new models and algorithms for wind wave modelling, applicable for situations with fast changes of the environment, which are beyond the limits of applicability of the classical kinetic equation. Rapid changes of wind can lead to the increased probability of extreme wave events. The generalized kinetic equation (GKE) and direct numerical simulations (DNS) of wind wave fields, based on the Zakharov equation, allow tracing the evolution of spectra and higher-order moments of the field for many thousands of wave periods (at least an order of magnitude longer than with other DNS approaches). In this project, a direct comparison of the DNS and the GKE is performed, with the aim to understand the role of coherent processes, resolved by the DNS but completely filtered out in the statistical approach. These processes result in the difference in the timescales of the evolution of spectra, and affect higher statistical moments.

Summary of problems encountered (if any)

(20 lines max)

No particular problems encountered ………………………………………………………………………

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Summary of results of the current year (from July of previous year to June of current year)

This section should comprise 1 to 8 pages and can be replaced by a short summary plus an existing scientific report on the project

In this project, we study numerically the long-term evolution of water wave spectra using three different models, aiming at understanding the role of different sets of assumptions. The first model is the classical Hasselmann kinetic equation (KE). Numerical simulations of long time evolution of random water wave fields are almost overwhelmingly based upon this model. The KE is based on two key assumptions: quasi-gaussianity, employed in the statistical closure, and quasi-stationarity, implied by the large-time limit. The specific role of each of these assumptions in the discrepancies between the kinetic theory and the reality is not clear. Since the spectral evolution occurs only when a wave field manifests a departure from gaussianity, the assumption of quasi-gaussianity implies that the evolution is slow. However, it is reasonable to assume that during a rapid transformation of a wave field (e.g. due to a rapid change of environment) both these assumptions, especially that of quasi-stationarity, can be violated.

Two other models are new. The second model is the generalised kinetic equation (gKE), derived without the assumption of quasi-stationarity. Thus, unlike the KE, the gKE is valid when a wave spectrum is changing rapidly (e.g.\ at the initial stage of evolution of a narrow spectrum). However, the gKE employs the same statistical closure as the KE. The third model is based on the Zakharov integrodifferential equation for water waves and does not depend on any statistical assumptions. Since the Zakharov equation plays the role of the primitive equation of the theory of wave turbulence, we refer to this model as direct numerical simulation of spectral evolution (DNS-ZE). The DNS-ZE method allows to study long-term spectral evolution (up to $O(10^4)$ periods), which was previously possible only with the KE. Thus, we are able to perform a direct comparison of spectral evolution with and without the statistical closure.

The project has been developing along a few different but closely related lines.

I. Spectral evolution of weakly nonlinear random waves

Our target is to trace the evolution of initially narrow (both in frequency and angle) spectra subjected only to dissipation at high frequencies. Such spectra are far from equilibrium, and at the initial stage of evolution they undergo relatively fast broadening, which is likely to put them at, or beyond, the limit of applicability of the quasi-gaussianity and quasi-stationarity assumptions, employed in the derivation of the classical kinetic equations. Thus, a meaningful comparison of spectral evolutions obtained with different sets of assumptions becomes possible.

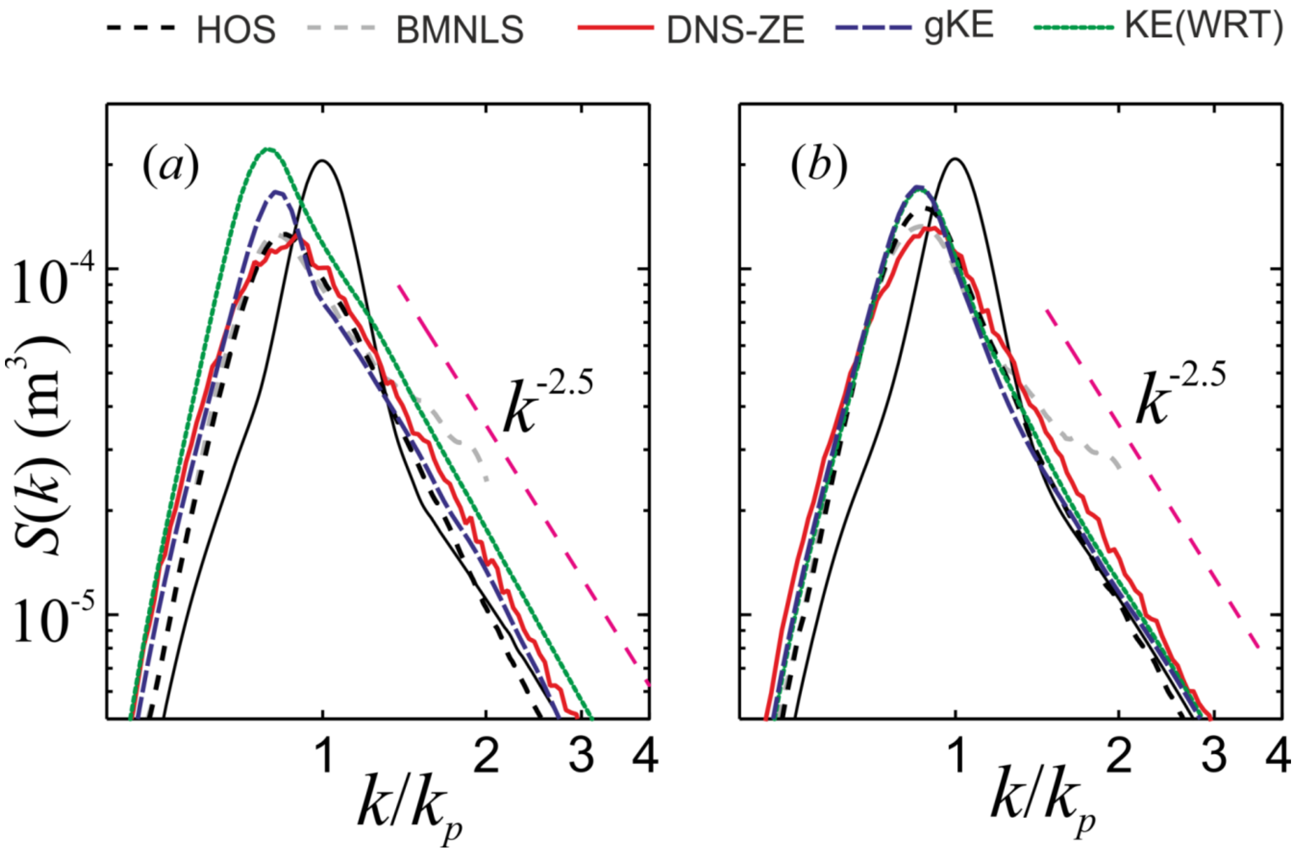

Figure 1. Short-term evolution of the initial spectra (*a*) A and (*b*) B, with a direct comparison of five different numerical approaches. Initial (black solid curve) and final (after 150 wave periods) omnidirectional energy spectra $S(k)$ are plotted. Evolution is simulated the high-order spectral method (HOS); Dysthe equation (BMNLS, both from Xiao *et al* 2013, cf their figure 7); DNS based on the Zakharov equation (DNS-ZE); standard kinetic equation, WRT algorithm, KE(WRT); and generalized KE (gKE)

As initial conditions, we choose two narrow-banded spectra with identical frequency distributions (a JONSWAP spectrum with high peakedness $\gamma = 6$) and different directional distributions. Spectrum A is very narrow in angle (corresponding to N=840 in the $\cos^N$ directional model), while spectrum B is initially wider in angle (corresponds to N =24). The reason for this particular choice of initial conditions is the vast amount of observational and numerical data relevant to these spectra. The same spectra were used as the initial conditions in the experimental study by Onorato *et al*

(2009) and numerical studies by Toffoli *et al* (2010) and Xiao *et al* (2013). In particular, Xiao *et al* (2013) performed numerical simulations of the evolution (only about 150 periods) of the same initial spectra using higher-order spectral method (HOS) and broadband modified nonlinear Schrodinger equation (Dysthe equation, BMNLS). Thus, we can consider the short-term evolution of these spectra (without wind forcing) with five different approaches, based on different sets of assumptions, and use the results for comparison and validation of our algorithms.

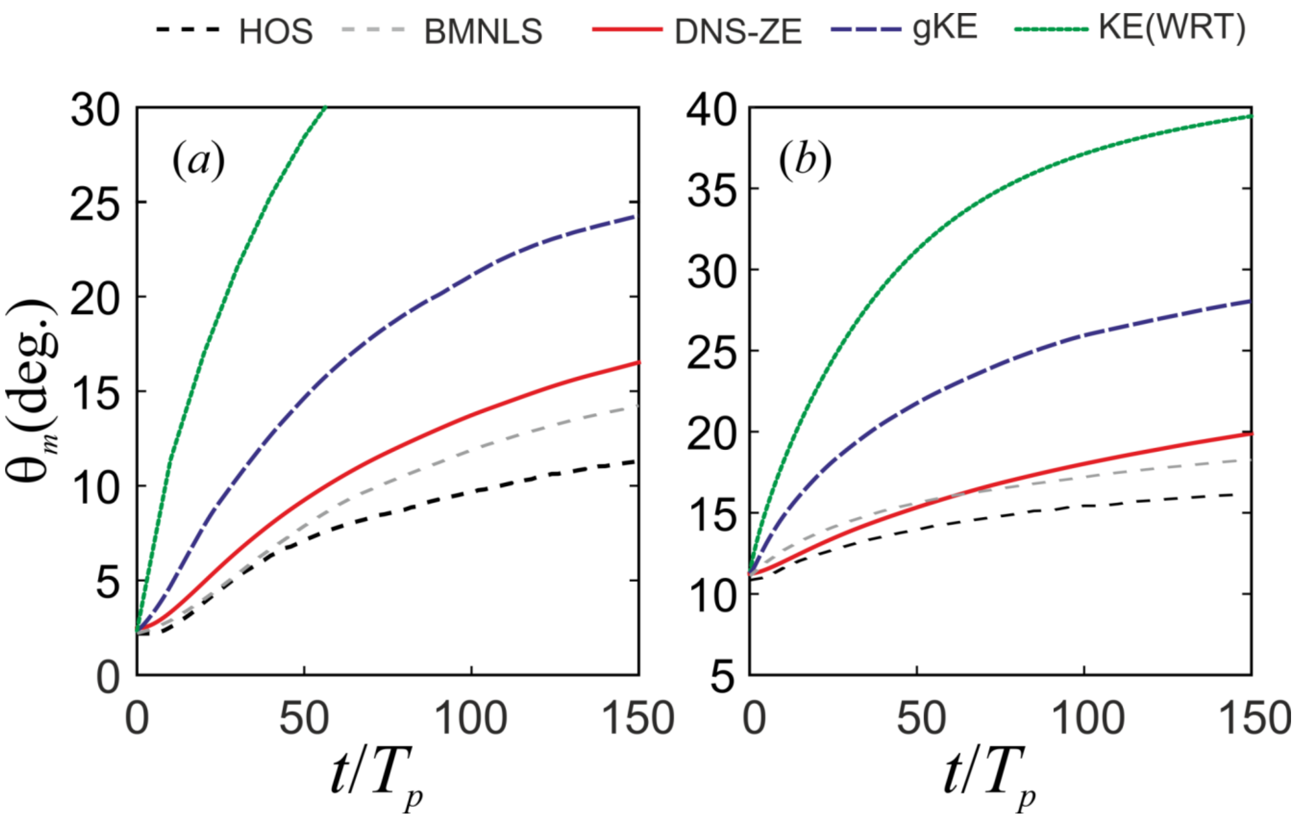

Figure 2. Evolution of directional spread θ_m (second moment of directional distribution) of two initial spectra (*a*) A and (*b*) B. As in figure 1, a direct comparison of five different numerical approaches is shown

In figure 1*a*,*b*, evolution of spectra A and B over the first 150 wave periods is shown, with a direct comparison of five numerical approaches: two kinetic equations (KE and gKE), two direct numerical simulations (DNS-ZE and higher-order spectral method (HOS) by Xiao *et al* (2013) and Dysthe equation (BMNLS, also from Xiao *et al* 2013). These numerical approaches fall into two categories, KE and gKE being statistical methods based on the same statistical closure, while DNS-ZE, HOS and BMNLS are dynamical methods free from statistical assumptions. To facilitate the comparison, we plot omnidirectional energy-wavenumber spectra $S(k)$, following Xiao *et al* (2013). Spectral evolutions predicted with two kinetic equations (KE and gKE) are practically identical for the case B with wider directional distribution, but for the initial spectrum A the KE noticeably overestimates the amplitude of the spectral peak. The simulations with three dynamical methods DNS-ZE, HOS and BMNLS are consistent with each other, but different from both kinetic equations. In general, the kinetic equations predict more narrow spectra, with a pronounced overshoot, while the DNS algorithms give wider spectra with lower amplitude of the peak.
In figure 2*a*,*b*, we show the evolution of mean directional spread $\theta_m = \overline{\theta_2}(k)$ for both initial spectra, defined, following Xiao *et al* (2013), as the average of the second moment of the directional distribution

$$\theta_2(k) = \left(\int_0^{\pi/2} \theta^2 D(k,\theta) \mathrm{d}\theta \right)^{1/2} \left(\int_0^{\pi/2} D(k,\theta) \mathrm{d}\theta \right)^{-1/2}$$

where $D(k,\theta)$ is the angular distribution function (Hwang *et al* 2010). Figure 2*a*,*b* shows a dramatic difference in the rate of angular broadening, which is consistent between DNS-ZE, HOS and BMNLS, much higher for gKE, and even higher for the KE.

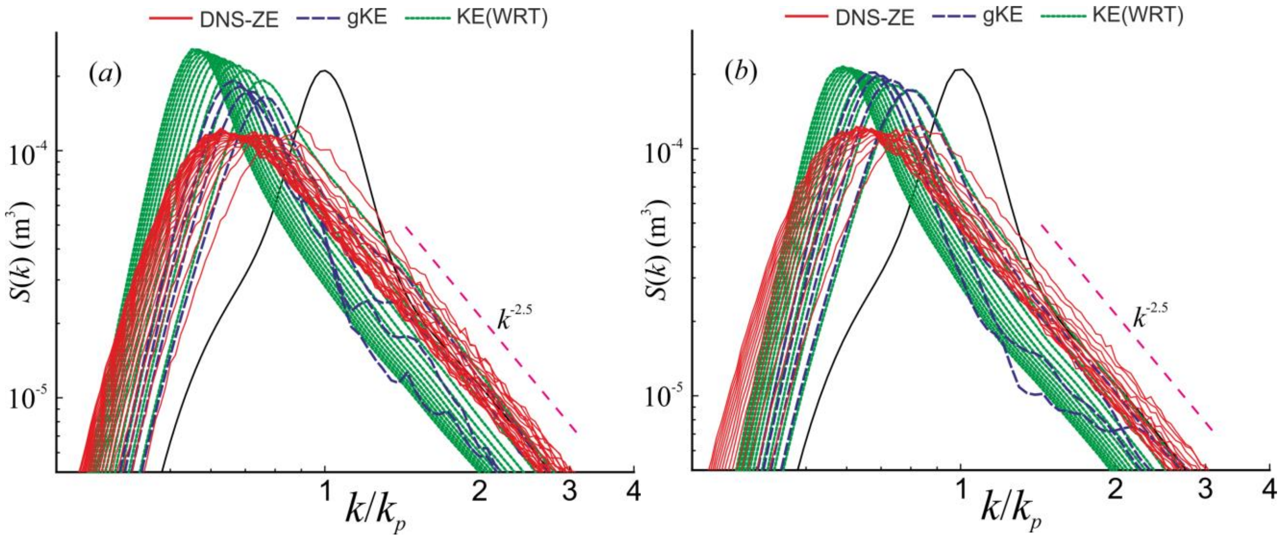

Figure 3. Long-term evolution of initial spectra (*a*) A and (*b*) B, with comparison of three different numerical approaches. Red solid curve - DNS based on the Zakharov equation (DNS-ZE), gKE and KE. Spectra are plotted every 150 periods, so that the first curves after the initial condition correspond to those shown in figure 1

In figure 3*a*,*b* we present the long-term evolution of both initial spectra obtained with the three models. Omnidirectional energy-wavenumber spectra are plotted every 150 peak periods of the initial spectrum. Thus, the curves plotted in figure 1 are shown as the first step after the initial condition. As can be seen from the figure, simulation of evolution of the same initial conditions by different models, based on different assumptions, leads to a number of interesting observations.

First, let us compare the two kinetic equations. In the case of spectrum B, which has moderate angular width, both kinetic models lead to nearly identical evolution, at least as far as the wavenumber (or frequency) spectrum is concerned. This is to be expected, since the KE and the gKE spectra for wind-generated waves were previously shown to be very close (Annenkov & Shrira 2016). However, for narrow directional spectra, it is reasonable to expect that the simulation results will be different, since the KE predicts no evolution in the limit of zero angular width, while the gKE is capable of modelling such situations. Indeed, in the case of spectrum A the two kinetic equations noticeably diverge, the spectral peak being appreciably higher in the case of the KE. The DNS spectral evolution in both cases predicts considerably wider spectra with lower peaks.

Although, as noted above, figure 3 demonstrates considerable differences between the predictions of the three employed models, it appears that the evolution of integral characteristics of the spectra (in particular, the wavenumber of the spectral peak, wave steepness, and therefore, the total wave energy) is close for all models. At large time, the evolution tends to be self-similar. From the theory of the KE (e.g. Badulin et al 2005) it is well known that the spectral evolution at large time approaches a self-similar state characterised by the downshift of the spectral peak $|\boldsymbol{k}_p| \sim t^{-2/11}$ and growth of the peak $n_p \sim t^{4/11}$. The DNS spectra have a peak with considerably lower amplitude than in the case of the KE and gKE spectra, and its time dependence does not tend to the KE asymptotic $n_p \sim t^{4/11}$, but the evolution of other integral characteristics is nearly identical.

The rates of change of the spectra obtained with the DNS-ZE scale as ε^4, where ε is the wave steepness. This agrees with our earlier findings for waves subjected to wind forcing and corresponds to the dynamical timescale of evolution, not to the kinetic one. This difference of growth rate scaling manifests itself, in particular, in the difference between self-similar spectral shapes and rates of angular broadening.

II. Evolution of higher statistical moments and comparison with experiment

In this work, we have proposed a new conceptual and numerical framework for modelling and forecasting of the higher moments of wave fields of arbitrary spectral width and, hence, of p.d.f. of elevation. In this framework, the evolution of wave spectra and higher moments is modelled by DNS-ZE, and/or by a kinetic equation (KE or gKE) with subsequent calculation of higher moments from spectra using the statistical closure. Both approaches, including the DNS-ZE one, allow long-term simulations of random wave fields. In addition to more practical implications, the role of the statistical closure, which is one of the most fundamental open problems in wave turbulence, can be studied.

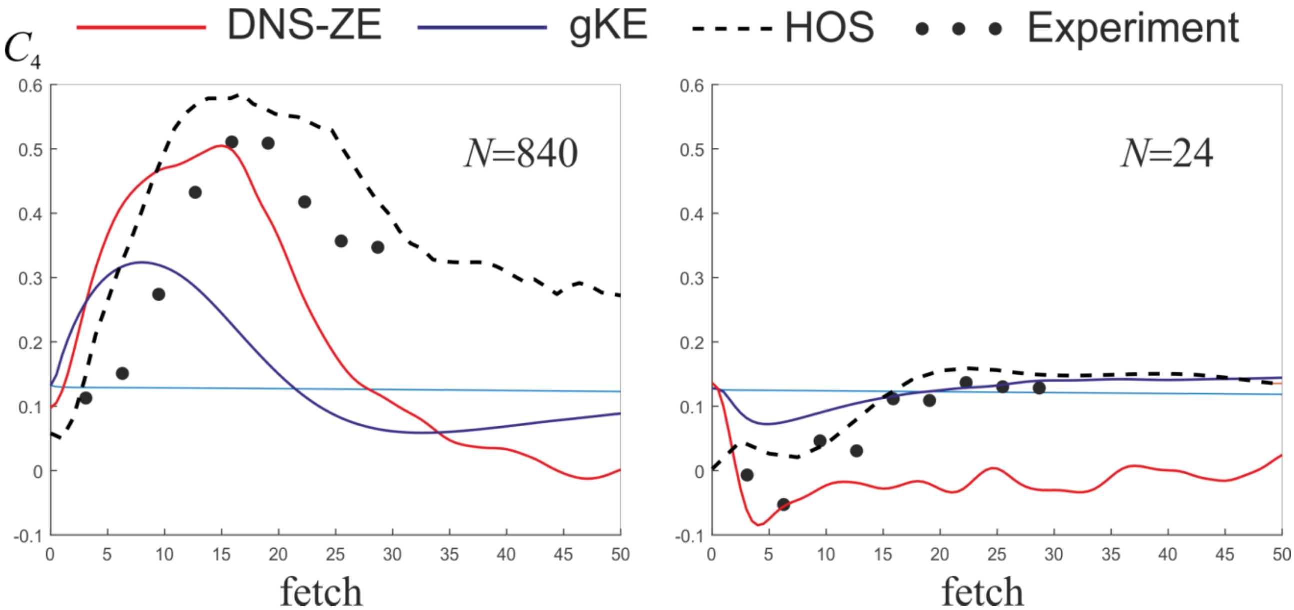

Figure 4. Evolution of the total kurtosis for spectra (*a*) A and (*b*) B vs fetch with different numerical approaches, and comparison with observations. Black dashed curve - high-order spectral method (HOS, Xiao *et al* 2013), red solid curve - DNS based on the Zakharov equation (DNS-ZE), blue dashed curve - gKE. Dotted blue curve shows bound harmonics kurtosis, calculated from the DNS spectra. Black dots - observations of the total kurtosis for the same initial conditions (Toffoli *et al* 2010)

Results demonstrate the dependence of the kurtosis on directional distribution. In figure 4, we show the comparison of the kurtosis evolution for initial spectra A and B over the first 50 wavelengths (corresponding to 100 wave periods in the duration-limited simulations). At the start of the evolution, the wave field is created as an ensemble of uncorrelated harmonics, and the dynamical kurtosis is zero, so that the total kurtosis is equal to the bound harmonics one. Observations by Toffoli *et al* (2010) and numerical experiments by Xiao *et al* (2013) show that the total kurtosis in both cases undergoes a rapid (over the first several dozen periods) evolution, attaining a positive or negative maximum and then decreasing in absolute value. DNS-ZE simulations reproduce well the initial maxima in both cases. After the initial evolution, the total kurtosis is found to have a small negative value in both cases, gradually decreasing in absolute value with time.

In figure 5, we study the effect of wind on the evolution of the dynamical kurtosis. These simulations show that the well-known phenomenon of large positive dynamical kurtosis during the evolution of initially one-dimensional narrowband spectra can be structurally unstable with respect to wind. Similar structural instability has been found in the case of interaction with shorter waves. On the other hand, simulations with wind show that wave fields of intermediate angular width can be characterized by moderate or even large negative dynamical kurtosis. This phenomenon is known from wave tank observations, but does not have an explanation within the framework of weakly nonlinear theory.

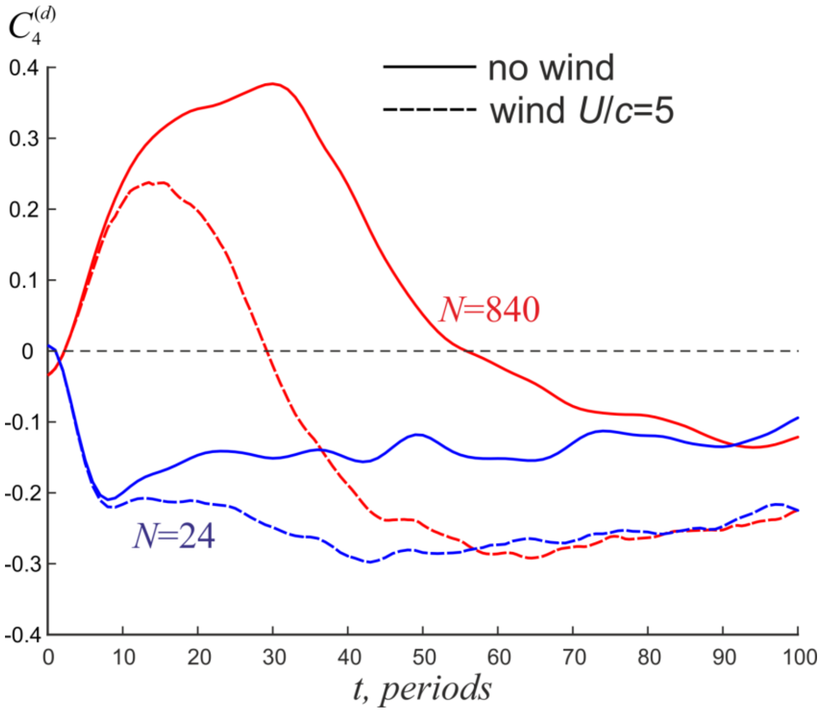

Figure 5. Evolution of dynamical kurtosis vs time by DNS-ZE. Initial conditions correspond to initial spectrum A (red curve) and B (blue curve). Dashed curves – wind U/c=5

III. Evolution of spectra under constant and changing wind and comparison with experiments

Simulations with DNS and kinetic equations demonstrate substantial differences between dynamical and statistical models with respect to spectral shapes, growth rates and timescales of evolution. Preliminary comparison with laboratory experiments performed in the large wind-wave facility in Marseille, and with the observations during the Tehuantepec experiment off the coast of Mexico show good agreement with the simulations performed with the DNS-ZE numerical model. A detailed comparison of numerical results and observations will be performed in the next year of the project.

References

Badulin, S. I., Pushkarev, A. N., Resio, D. & Zakharov, V. E. 2005 Self-similarity of wind-driven seas. *Nonlin. Proc. Geophys.* **12**, 891-945.
Hwang, P. A., Wang, D. W., Walsh, E. J., Krabill, W. B. & Swift, R. N. 2000 Airborne measurements of the wavenumber spectra of ocean surface waves. Part II: Directional distribution. *J. Phys. Oceanogr*. **30**, 2768-2787.
Onorato, M., Cavaleri, L., Fouques, S., Gramstad, O., Janssen, P.A.E.M., Monbaliu, J., Osborne, A.R., Pakozdi, C., Serio, M., Stansberg, C.T. & Toffoli, A. 2009 Statistical properties of mechanically generated surface gravity waves: a laboratory experiment in a three-dimensional wave basin. *J. Fluid Mech.* **627**, 235-257.
Toffoli, A., Gramstad, O., Trulsen, K., Monbaliu, J., Bitner-Gregersen, E. & Onorato, M. 2010 Evolution of weakly nonlinear random directional waves: laboratory experiments and numerical simulations. *J. Fluid Mech.* **664**, 313-336.
Xiao, W., Liu, Y., Wu, G. & Yue, D. K. 2013 Rogue wave occurrence and dynamics by direct simulations of nonlinear wave-field evolution. *J. Fluid Mech.* **720**, 357-392.

List of publications/reports from the project with complete references

1. Annenkov, S. Y., Shrira, V. I. 2016 Modelling transient sea states with the generalised kinetic equation. In: *Rogue and Shock Waves in Nonlinear Dispersive Media*, Springer, pp. 159--178
2. Annenkov, S. Y., Shrira, V. I. 2017 Modelling of kurtosis evolution for wind waves. The Onset of Rogue Waves, 7-8 April 2017, Northumbria University, Newcastle
3. Annenkov, S. Y., Shrira, V. I. 2017 DNS modelling of evolution of kurtosis for wind waves. 2017 WISE meeting, 14-18 May 2017, Victoria University, Victoria, B.C., Canada.
4. Annenkov, S. Y., Shrira, V. I. 2017 Spectral evolution of weakly nonlinear random waves: kinetic description vs direct numerical simulations. J. Fluid Mech., submitted.

Summary of plans for the continuation of the project

(10 lines max)

We will continue the numerical experiments with the dynamical (DNS-ZE) and statistical models, including the cases of constant and changing forcing, and interaction of two wave systems of different scales. We will consider both the evolution of spectra and of the higher statistical models, identifying situations with fast changes of probability of extreme events. We plan to perform a detailed comparison of numerical simulations with the available observations of spectral evolution in wind wave tank in Marseille, and in natural oceanic conditions (the Tehuantepec experiment)

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